SUPPLEMENTARY PROBLEMS

for

ELECTRICITY AND MAGNETISM

BY

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New York

THE MACMILLAN COMPANY

1935

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Set up and electrotyped
Published September, 1931

SET UP AND ELECTROTYPED BY THE LANCASTER PRESS, INC.
PRINTED IN THE UNITED STATES OF AMERICA

CONTENTS

ELECTRICITY AND MAGNETISM

PREFACE

The material presented in the chapters which follow constitutes the content of a three-hour semester course which the author has offered for a number of years to juniors and seniors in a college of liberal arts. In this course the aim has been to give advanced students a reasonably thorough presentation of the fundamental principles upon which the extensive research and commercial application of electricity and magnetism rest. The physics of the subject has in all cases received the major emphasis. However in order that the student may come to appreciate the vital connection between fundamental theory and the applications of these basic principles much of the illustrative material has been drawn from actual research and engineering practice.

Because of difference in view point and experience teachers will naturally differ in judgment as to what topics should receive consideration in a text of this character. The topics which the author has elected to discuss have been deliberately chosen as a result of many years of teaching and engineering experience. It has been his primary object to prepare a text which shall serve as a basis for instruction in an advanced undergraduate course.

The introduction of solved illustrative problems is perhaps debatable practice from a pedagogical point of view; but it would appear that more is gained than lost by such a proceeding, particularly where advanced students are concerned, and hence such illustrative material has been incorporated.

It is highly important in a course of this character that the student should be led to develop analytical ability. To this end a number of analytical developments have been purposely left as exercises to be solved by the student. Individual instructors may find it advisable to add other similar problems. The author believes that it is highly important that the student acquire a clear understanding of the electrical units and of their inter-relations. This aspect of the subject has accordingly been emphasized throughout the text. Working formulae have been developed from basic concepts and in the end reduced to practical forms. In order that the student may acquire at least some

knowledge of the historical background of the subject and thus be enabled to develop a rational perspective, and also to serve as a stimulus for further independent reading, a limited amount of historical material has been included at certain points in the text.

In the assembling of material for the course covered by this volume a number of standard texts as well as specialized treatises have been freely consulted. Wherever possible original sources have been examined; a considerable amount of the material in the text is drawn from the author's personal research and engineering experience. Every effort has been made to give due credit for any material taken from original sources. I am indebted to a number of manufacturers for the loan of photographs, and to several periodicals for permission to reproduce illustrative material. Proper acknowledgment will be found in the appropriate places throughout the text. In the preparation of the chapter on Terrestrial Magnetism valuable assistance has been received from the Staff of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington. I am also under obligation to fellow teachers for constructive criticism, and to my wife for material assistance in the preparation of the manuscript.

It is not to be expected that the book will be found entirely free from errors, and the author will therefore appreciate information relative to any defects which those who use the text may chance to note.

CHARLES A. CULVER

NORTHFIELD, MINN.
August, 1930

ELECTRICITY AND MAGNETISM

CHAPTER I

INTRODUCTORY

1. Electron Theory of Matter.—In recent years the theories and laws underlying the wide and varied applications of electricity and magnetism have come to be based largely on the electron theory of matter. In view of this fact it will be useful to briefly review, at the beginning, our present understanding concerning the electrical nature of matter, leaving for a later chapter a more detailed discussion of this fundamental subject.

It now appears to be reasonably well established that the atom of a given element consists of an assembly of an equal number of positive and negative charges of electricity, the latter being known as *electrons*. All of the positive electrical entities and a part (about half) of the electrons form a relatively compact group designated as the nucleus. The remainder of the electrons are electrically associated with the nucleus but occupy positions at relatively great distances from the central group.

The evidence thus far uncovered appears to show that there are two fundamental entities, which serve as the building stones from which all matter is formed, and that these two basic entities are *electrical charges* of equal magnitude and of opposite sign. Both the positive and negative units are exceedingly minute; hence for practical purposes their dimensions may be neglected and they may be thought of as mere point charges. The inter-electronic spaces, particularly in the case of the outer electrons, are relatively great. The electrons are considered to be in rapid orbital motion. These orbits are probably not coplanar, and the electrons appear from time to time to shift from one orbit to another. Whether the moving electrons radiate energy continuously, or only when a change of orbit occurs, is still a matter of discussion.

Some of the outer electrons of certain atoms, particularly in the [illegible] of the metals, may be detached from the atomic structure of

which they form a part. As we shall see later, various agencies may be employed to artificially bring this about. For our present purposes, however, it should be noted that a limited number of electrons, in the case of metals, appear to move about from atom to atom in a more or less irregular manner. In certain other types of bodies, notably in the case of glass, porcelain, rubber, etc., this electronic migration does not occur. In the case of metals, certain liquids, and gases the application of an electric force will cause these "free" electrons to move in an orderly fashion; in short we may establish what is known as an electric current. Bodies in which there is no inter-atomic electronic movement are known as insulators, and those in which movement does or may obtain are classed as conductors. This explanation of electrical conductivity is not wholly satisfactory, due to the fact that the available number of "free" electrons appears to be insufficient to account for the high electrical conductivity of some bodies. However, the tentative theory of electrical conductivity in metals, just outlined, since it correlates a number of important facts, will serve our present purposes. It may be said in passing that the question of electrical conductivity is one upon which much research work has yet to be done, and it gives promise of being a highly important and fertile field for original investigation.

We shall however be concerned with a study of charges *at rest*, this part of our subject being known as *electrostatics*. At the beginning it is important that we arrive at a clear understanding of what is meant by the terms "charge" and "a charged body."

2. A Charge.—On the basis of the electron theory of matter, electrically neutral atoms are those in which there are present just enough electrons to neutralize the positive charges in the nucleus. If, by any means, one or more of the electrons of an atom be removed the remainder of the atomic structure will manifest a positive charge, and the body of which it forms a part will then exhibit a positive charge. Likewise, if one or more electrons be added to an atom or group of atoms, the atom or body will manifest a negative charge. In short, *a charged body is one in which the electronic equilibrium has been disturbed* Further, it may be noted as one of the fundamental laws of nature that like charges exert a repelling force on one another and unlike charges attract. Certain definite laws govern these force actions and we shall examine these relations in detail in a later section.

It is found that K is a constant depending upon the nature of the medium involved, and having values ranging from about 2 in the case of petroleum to approximately 10 for heavy flint glass. As implied above, this constant has a value, accurately speaking, of unity only in the case of a vacuum, but its value for air is so nearly unity that it is commonly so considered in practice. Faraday designated this constant by the term *specific inductive capacity;* he also spoke of non-conductors (insulators) as dielectrics. Hence K has come to be called the *dielectric constant.*

A practical problem will serve to illustrate the utility of the relation given by eq. 2 and also the significance of the dielectric constant.

Assume that we have two concentrated charges separated by a distance of 2 cm. in air ($K = 1$), each having a value of, say, 100 e.s.u. Substituting in eq. 2 we have

$$F = \frac{(+100) \times (-100)}{1 \times 2^2} = -2500 \text{ dynes,}$$

or a force of something like 2.5 grams will be acting to pull the plates together.

Now suppose, without changing the distance between the charges, we arrange to submerge them in oil having a dielectric constant of 3. We then have

$$F = \frac{100 \times 100}{3 \times 2^2} = 833.3 \text{ dynes.}$$

The effect of the dielectric in diminishing the force action thus becomes apparent.

Following is a table giving a selected number of the dielectric constants (specific inductive capacity) taken from the Smithsonian Physical Tables, Seventh Revised Edition. Only those have been included which are commonly used. In several cases limiting values are shown, the constant in a given case depending upon the particular type of sample. Temperature may modify slightly the value of K, and is therefore given in certain of the examples listed.

5. Field Strength.—In dealing with the force action between charges it will be convenient to have some method of indicating what the force action would be on a unit charge if placed at the point in question. Knowing this one could readily compute the

DIELECTRIC CONSTANTS

SUBSTANCE	TEMPERATURE °C.	CONSTANT K
Air	0	1.000000
Hydrogen	0	0.999674
Glycerine	15	56.2
Oils:		
Castor	11	4.67
Linseed	13	3.35
Olive	20	3.11
Sperm	20	3.17
Petroleum	—	2.13
Glass:		
Flint (extra heavy)	—	9.9
Flint (very light)	—	6.61
Crown (hard)	—	6.96
Mica	—	2.5 –6.62
Paper (telephone)	—	2.10
Paraffin	—	2.10–2.46
Porcelain	—	5.73–6.84
Shellac	—	2.95–3.73
Quartz	—	4.27–4.69

total mechanical force to which any charged body will be subjected when placed at a certain point in a given electrostatic field.

If, in the inverse square law embodied in eq. 2, we consider one of the charges to be in the nature of a test charge, and of unit value, our relation becomes

$$F = \frac{q}{Kd^2} = f, \qquad \text{Eq. 3}$$

and we have an expression for the force on unit test charge at the point in question. It will assist in avoiding confusion if we designate this particular force (force experienced by unit test charge) by a special letter, say f, as has been done in the above equation. The hypothetical test charge referred to above is commonly thought of as positive, and the force experienced by such a unit test charge at any given point is spoken of as the *field strength* or *electric intensity*. The unit of electric intensity, or field strength, has no particular name. It is however defined in terms of force, and is therefore a vector quantity, having both direction and magnitude. Accordingly it may be resolved and combined according to the general law of vectors.

From the foregoing it follows that the total mechanical force exerted upon any charge at a point in a field will be given by the

product of the field strength and the charge. This may be expressed by the simple relation

$$F = fq \quad \text{(in dynes)}, \qquad \text{Eq. 4}$$

where F represents mechanical force in dynes, f the electrostatic field strength, and q the charge.

A study of two or three typical examples will make the application of the relations given by eqs. 3 and 4 clear.

Suppose we have a charge of 100 e.s.u. at a distance of 25 cm. from a given point, in air. What will be the field strength at the point in question? Substituting in eq. 3 we have

$$f = \frac{100}{1 \times 25^2} = 0.16 \text{ e.s.u.}$$

If, in the above case, the charge had been surrounded by some dielectric such as oil the electric intensity would have been from one-half to one-third of the above value, depending on the particular value of K.

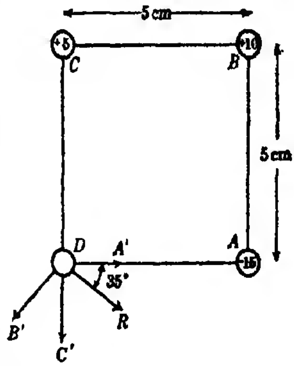


FIG. 1

Let us now assume that a charged body is placed at the point referred to above, the charge on the body being 30 e.s.u. To what force will the charged body be subjected? Substitution in eq. 4 will yield the desired result.

Again, suppose we have the condition illustrated in Fig. 1. If charges are located as shown, what will be the resultant field strength at the point D?

The field strength at D due to the charge at C may be found by substitution in eq. 3.

$$f = \frac{5}{1 \times 5^2} = 0.2 \text{ e.s.u.},$$

a repulsion, which may be represented by the vector component DC'.

Likewise, in the case of the charge at B,

$$f = \frac{10}{1 \times (\sqrt{50})^2} = 0.2 \text{ e.s.u.},$$

also a repulsion, and represented by DB'.

Similarly for the charge at A,

$$f = \frac{15}{1 \times 5^2} = 0.6 \text{ e.s.u.},$$

an attraction, and represented by the vector component DA'.

Combining the three vector components DC', DB' and DA' in the usual manner, we find the resultant field strength at D to have a value of 0.57 e.s.u. and in the direction indicated by DR, making an approximate angle with the direction of DA as shown. Hence if unit test charge were placed at D it would be acted upon by a force of 0.57 dyne, and tend to move in the direction DR.

6. Total Normal Electric Induction over a Closed Surface.—We now come to another concept having to do with the electrostatic field, and which, though difficult to visualize, is nevertheless perfectly definite, and of fundamental importance. We refer to what might be termed electric flux, and which is, in analytical discussions, referred to as total normal induction.*

We have already seen (Sec. 3) that the electric intensity has a definite value at every point in any given electric field. It should be possible, then, to find a relation between the field strength at all points of a surface inclosing a charge and the magnitude of that charge. In order to find such a relation it will first be necessary to set up a definition of what we have referred to above as electric induction or electric flux.

To assist in this procedure let us ascribe to our electrostatic "lines of force" an additional property or significance, namely, that the total number of lines which extend outward from a given charge shall be a function of the magnitude of the charge. We may think then of these lines, not only as representing direction of force action, but also as serving to indicate the magnitude of the strain which obtains in the medium surrounding the charge. On this basis, electric induction or electric flux may be thought of as referring to the strain in the medium, as represented by our lines of force.

Now suppose that we have a surface anywhere in an electric

* It will be recalled that in mechanics we have, for instance, the concept of momentum, which is given by the product of mass and velocity, two factors which taken conjointly give us a quantity concept which serves a very useful purpose. Though in no way connected with momentum, total normal induction is a concept of the same general type.

field, and that we desire to have available an expression for the total electric induction or flux extending through this surface. In order to formulate such an expression or definition let us divide our surface into a large number of very small areas, as shown in Fig. 2. In terms of the calculus these small areas may be designated by ds, and if the elemental areas be taken small enough, the electric intensity may be considered to be uniform over any given one. Now in general the direction of the electric induction or flux will not be normal to the surface in question, but in considering problems into which the concept of electric flux enters, it will be most convenient to deal with the value of the flux *normal* to the surface. We may therefore find the *component* of the flux normal to the surface. If now we take the product of the elemental area and the normal component of the flux, we will have what is known as the normal induction over the area ds. To find the normal induction over the surface *as a whole* we have, then, but to take the surface integral as given by the expression

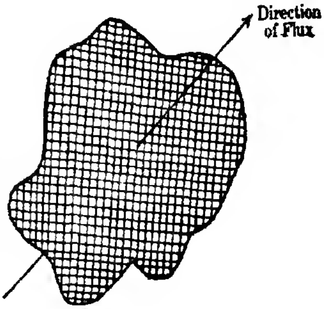


FIG. 2

$$I = \int N ds, \qquad \text{Eq. 5}$$

where I is the total normal induction or total electric flux, N the normal component of the electric intensity, and ds the elemental area.

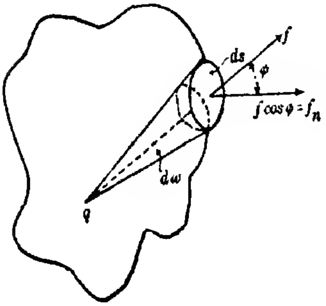


FIG. 3

7. Gauss' Theorem.—Having established a formal definition of electric flux we may now proceed to find a quantitative relation between the total normal induction over a closed surface and the magnitude of the charge which gives rise to it.

Imagine any closed surface in an electric field, as sketched in Fig. 3, and consider a small area ds. Let f = the electric intensity at a point on this elemental surface, due to the charge q.

Utilizing the relation (eq. 3) already developed for field strength we may write

$$f = \frac{q}{d^2},$$

where d is the distance from the charge to the surface area ds. The normal component of this intensity may be written

$$f \cos \phi = \frac{q}{d^2} \cos \phi = f_n.$$

Hence the normal induction, or flux, contributed by the element of surface ds would be given by

$$\left(\frac{q}{d^2} \cos \phi\right) ds,$$

which in turn might be written

$$q\left(\frac{ds}{d^2} \cos \phi\right).$$

But the expression

$$\frac{ds}{d^2} \cos \phi$$

is equivalent to the solid angle $d\omega$, subtended at the point where the charge q is located by the element of surface ds. Hence we may write

$$q\left(\frac{ds}{d^2} \cos \phi\right) = q d\omega.$$

This is the normal induction through $d\omega$ due to the charge q. Over the entire surface inclosing the charge the induction would then, by eq. 5, be given by the relation

$$\int q d\omega = 4\pi q. \qquad \text{Eq. 6}$$

Therefore we may say that the total normal induction over any closed surface in an electrostatic field is numerically equal to the product of 4π and the magnitude of the inclosed charge. In terms of lines of force one might say that there are $4\pi q$ lines terminating on any given charge. It is customary to say that if q is positive, the induction is directed outward, and if negative, it will be inward. Obviously, if there be no resultant charge within the surface the total normal induction will be zero.

The relation expressed by eq. 6 is known as *Gauss' Theorem*,

and will be found to be useful in the solution of a number of propositions which are to follow in our study of electrostatics.

8. Surface Distribution.—The distribution of a charge on the surface of a conductor depends upon the geometrical form of the conductor, and also upon the presence and distribution of surrounding charges. This follows from the fact that all charges give rise to fields of force and interact upon one another according to the general law of electrostatic force action as expressed in eq. 1. Further, there is abundant experimental evidence, as well as analytical reasons, for a statement to the effect that the charges in electrical equilibrium on a conductor are located entirely on its outer surface. Faraday, Webb and others strongly electrified a relatively large metal inclosure but could not detect any evidence of a charge within the inclosed region. In other words, the electrostatic field does not penetrate the surface of the conductor upon which the charge resides (Sec. 10). Since the charges are assumed to be at rest, the lines of force must at all points be normal to the conducting surface. If this were not so the electric force would have a component tangential to the surface and motion of the charges would result. As we proceed it will become evident that the distribution of the charge on a conductor is a matter of important consideration; hence an expression for surface density will be useful.

If a quantity of electricity, Q, be distributed over a surface whose area is A, the *average* surface density may be defined by the relation

$$\sigma = \frac{Q}{A}, \qquad \text{Eq. 7}$$

and the density *at a point* may be expressed as $\frac{dQ}{dA}\cdot$ In general the surface density will not be uniform. In a few special cases, however, σ may have a constant value, such as in the case of a sphere or a large plane far removed from other charges.

As a result of the electromechanical force action between the elemental charges which go to make up the charge as a whole, the surface density will be greatest at points of greatest curvature, approaching infinity in the case of sharp points.

Coulomb * carefully studied by means of direct experiment the

* Coulomb's fifth memoir, *Histoire de l'Academie*, 1787.

elementary laws governing the distribution of electricity on conductors, but found it exceedingly difficult to treat the general problem analytically. In fact a complete analytical treatment has been attained in only a few special cases, one of which is that of the ellipsoid.* In that particular case the surface density has been shown to be proportional to the distance of the tangent plane from the center. It is beyond the scope of this volume to review the mathematical analysis of this question, but it may be said that *the electrical distribution in any given case is such that the resultant electric force at all points within the conductor is zero.*

It will be shown in the next few paragraphs that the intensity of the field *near* a charged surface depends upon the surface density of the charge. Therefore as the surface density increases due to increasing curvature the field strength in the immediate vicinity of a point, for instance, will be relatively great. Any dust or water particles near or in contact with the charged body will become charged and then strongly repelled from the body under the action of the intense field which obtains in the region of the point. Thus the body tends to become discharged, the phenomenon often being referred to as a "brush discharge." In addition, due to the intense field existing in the region of a point, a condition known as ionization (Sec. 139) may come to obtain which may also result in a partial or complete discharge of the body. Indeed, under certain circumstances, the field intensity may become so great, due to the concentration of the charges on a sharply curved surface, that the stress will actually disrupt the dielectric. This effect becomes even more marked when a charged body of opposite electrical sign is near.

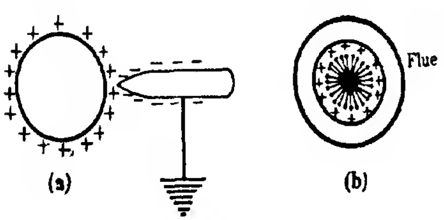


FIG. 4

The discharging action of pointed conductors is taken advantage of in a number of practical ways. The fundamental prin-

* First and second memoirs of Poisson, *Memoirs de l'Institut*, 1811. Green's *Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism*, 1828. *Papers on Electrostatics and Magnetism*, by Sir W. Thompson (Lord Kelvin), XV, p. 178. See also the second paper in the same collection. The student will find the consultation of original sources a fascinating and profitable pursuit.

ciple involved in two such applications may be illustrated by reference to Fig. 4.

If we have a charged body (Fig. 4*a*) near which, for instance, is situated an earthed conductor the latter will become charged as shown, and the charged air or dust particles will move between the body and the point, thus discharging the former. This is the principle of the lightning rod. The charged ball corresponds to the charged cloud, and the earthed conductor terminating in a point to the lightning rod conductor system. The function of the lightning rod is to *gradually relieve the electrostatic strain and thus prevent a disruptive discharge.**

In the *Cottrell precipitation process* (Fig. 4*b*) the discharging action of points is utilized for the purpose of precipitating dust and extremely fine particles of valuable metals from the gases which are given off in certain manufacturing and metallurgical processes. Along the center of the flue through which the dust-laden gases pass is suspended a conductor kept constantly charged by suitable means, and having points or sharp edges on its surface. Near this is another charged surface in the form of a tube or screen. The dust particles become charged and, as a result of the force action in the region of the points, move to one of the conductors, usually the positive, and form a deposit thereon. The material thus precipitated is removed from the "collector" at suitable intervals.

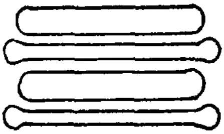


Fig. 5

In certain cases the discharging action of points and sharp edges results in the loss of considerable energy. In certain types of large air condensers (Sec. 26) the edges of the individual plates are given a rounded contour, the curved surface having a relatively large radius, as illustrated in Fig. 5.

9. Electric Intensity at a Point outside a Charged Sphere.—In order that we may have available certain analytical tools for use in the study of various instruments and devices, we will now proceed to derive several expressions for the value of the electric intensity, or field strength, under certain conditions.

We will first work out an expression for the field intensity at a point *outside* a uniformly charged sphere. Assume a charged

* To be effective a lightning rod should not be insulated from the building which it is designed to protect, and the conductor should be well "grounded." Common three-eighths gas pipe answers very well as a lightning rod conductor.

sphere as shown in Fig. 6. Our problem is to find the field strength at a point p outside the sphere. It is possible to derive two expressions for the T.N.I. (total normal induction); we may then equate these and solve for the field intensity, f. To derive these two expressions for the T.N.I., draw a spherical surface concentric with that of the sphere and passing through the point p.

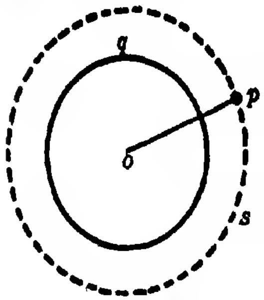


FIG. 6

Now the T.N.I. over this construction sphere is given by the product of the electric intensity and the area, or T.N.I. $= f \cdot 4\pi\overline{op}^2$. By Gauss' Theorem the T.N.I. is equivalent to $4\pi q$. Therefore $f \cdot 4\pi\overline{op}^2 = 4\pi q$, which reduces to

$$f = \frac{q}{\overline{op}^2} \text{ (e.s.u.).} \qquad \text{Eq. 8}$$

It is thus evident that, so far as conditions outside the sphere are concerned, the charge acts as if it were concentrated at the center.

Infinitely near the spherical surface, op becomes practically identical with the radius of the sphere, or

$$f = \frac{q}{r^2}. \qquad \text{Eq. 9}$$

But q = area $\times$ surface density = $(4\pi r^2)\sigma$; hence

$$f = 4\pi\sigma \quad \text{(e.s.u.).} \qquad \text{Eq. 10}$$

In other words the field strength at the surface of a charged sphere is equal to 4π times the surface density. Knowing the field strength, we may, as circumstances arise, find the mechanical force in dynes acting on any charged body.

EXAMPLE.—A sphere whose diameter is 10 cm. carries a charge of 100 e.s.u. uniformly distributed over its surface. (*a*) What is the surface density of the charge? (*b*) What is the field intensity 10 cm. from the surface of the sphere? (*c*) What is the field strength at a point infinitely near the surface?

(*a*) Area $= 4\pi R^2 = 4\pi 5^2 = 100\pi$ cm.^{2}.

By eq. 7, $\sigma = \frac{100}{100\pi} = \frac{1}{\pi} = 0.318$ e.s.u./cm.^{2}.

(*b*) By eq. 8, $f = \frac{100}{15^2} = 0.444$ e.s.u.

(*c*) By eq. 10, $f = (4\pi)\frac{1}{\pi} = 4$ e.s.u.

10. Value of Field Intensity at a Point inside an Electrified Spherical Shell.—In this instance we proceed as in the previous case. Referring to Fig. 7, the T.N.I. over the hypothetical surface through the point p is equal to

$$f(4\pi\overline{op}^2).$$

By Gauss' Theorem the T.N.I. over the construction surface is equal to $4\pi q$. But the charge inside this surface is zero. Hence $f(4\pi\overline{op}^2) = 0$. Therefore $f = 0$, which means that *there is no electric field at any point inside a uniformly electrified spherical shell.*

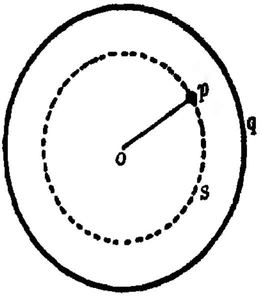


FIG. 7

11. *Electric Intensity at a Point outside a Uniformly Electrified Cylinder.*—Let us assume that we have an infinitely long cylinder having a charge q' per cm. of length. The problem is to find an expression for the field strength at any point p *outside* the conductor. Draw a concentric cylindrical surface (Fig. 8) of unit length through the point p; the planes forming the ends of this construction cylinder are perpendicular to the common axis. The direction of the field (being radial to the charged cylinder) will be parallel to these planes, and hence the normal electric flux over them is zero. We have then only to consider the area of the curved surface. The area of this surface will be $2\pi op$, and the T.N.I. over the curved area will be $f(2\pi op)$. The charge within this area is q'; hence by Gauss' Theorem the T.N.I. is also given by $4\pi q'$. Equating we have

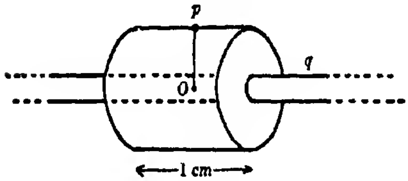


FIG. 8

$$f(2\pi op) = 4\pi q'$$

or

$$f = \frac{2q'}{op}. \qquad \text{Eq. 11}$$

We thus see that, in the case of a cylinder, the electric intensity varies inversely as the distance from the axis.

In a similar manner it may be shown that the electric intensity is *zero* at any point *inside* a uniformly electrified cylindrical shell.

It is found that even though the conducting surface be irregular in shape, there is no field on the inside. In practice, it is frequently necessary to shield conductors and instruments from the effects of extraneous charges. By placing the conductors or apparatus in a metal inclosure they are removed from the effects of outside fields. It is not, in all cases, essential that the shielding surface be continuous; frequently a wire screen will answer the purpose. The shield is commonly grounded.

Example.—Assuming that the presence of a neighboring charge does not appreciably change the surface distribution, what will be the force acting on a body carrying a charge of 200 e.s.u. if located 20 cm. from a long cylindrical conductor which bears a charge of 10 e.s.u. per cm. of length?

By eq. 11, $f = \frac{2 \times 10}{20} = 1$ e.s.u.

By eq. 4, $F = 1 \times 200 = 200$ dynes.

12. Electric Intensity at a Point in the Vicinity of a Uniformly Charged Infinite Plane.—Let BD (Fig. 9) be a charged plane of infinite extent, and let p be the point at which it is desired to find the field strength. Draw a right cylinder through the plane, making S' as far on one side of the plane as S is on the other. The right end of the cylinder includes the point p; the area, A, of each end is made one sq. cm. Consider the total normal flux over the surface of this cylinder. The direction of the field due to the charge on the plane is everywhere parallel to the axis of the cylinder, and therefore there will be no flux through the curved surface of the cylinder. There will however be induction through both ends of the cylinder. If f be the value of the electric intensity at any point p in the field, and A the area of either of the ends of the cylinder, the T.N.I. through these ends will be $2fA$. It is to be noted in this connection that the flux through the end S' is along the outward drawn normal and therefore will be of the same sign as that through S, hence the factor 2 in the above expression. If σ be the surface density of the charge on the plane,

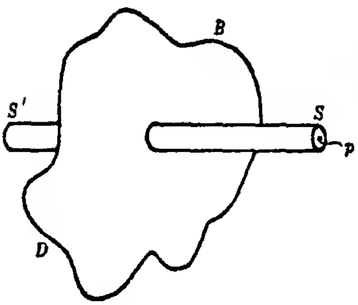


Fig. 9

it will also be the charge on the area A, inclosed by the construction cylinder, which, it will be recalled, was of unit cross section. Therefore by Gauss' Theorem the T.N.I. will be given by $4\pi\sigma$. Equating these values for the T.N.I., we have

$$2fA = 4\pi\sigma,$$

and since A is unity, this becomes

$$f = 2\pi\sigma \quad \text{(e.s.u.).} \qquad \text{Eq. 12}$$

In the above discussion σ has been taken as the charge *on both sides of the plane taken together.* If σ be thought of as the charge per unit area *on one side only,* the total charge inclosed by the construction cylinder will be 2σ, and our relation would then become

$$2f = 4\pi 2\sigma$$

or

$$f = 4\pi\sigma \quad \text{(e.s.u.).} \qquad \text{Eq. 13}$$

In practice this significance is sometimes given to σ, and the resulting relation is commonly known as *Coulomb's Law.* In words, this equation means that the electric intensity at a point very near a charged surface is numerically equal to the product of 4π and the charge per unit area *on one side* of the conductor. If the surrounding medium is other than air, eq. 1 shows that eq. 13 would become

$$f = \frac{4\pi\sigma}{k}. \qquad \text{Eq. 14}$$

It is to be noted that *the magnitude of the field strength (electric intensity) is independent of the distance of the point from the plane.* While we have assumed in the above discussion that the plane is infinite in extent, yet in practice essentially the same condition obtains when the point is very near a plane of any appreciable area. Later we will apply this relation in dealing with certain measuring instruments.

13. Mechanical Force on Unit Area of a Charged Conductor.— In dealing with certain problems involving electrostatic forces it becomes necessary to be able to determine the magnitude of the mechanical force on a charged conductor due to the charge on the conductor itself and on any other neighboring bodies. It will be found convenient to have such a relation in terms of surface density and field strength.

Suppose we have a charged conductor of any shape, and surrounded by air, as shown in Fig. 10. By Coulomb's Law (Sec. 12) the field strength, f, at a point, p, *outside* the conductor and very near to its surface would be $4\pi\sigma$, where σ is the surface density. At a point, p', just *within* the bounding surface the field intensity would be zero (Sec. 11).

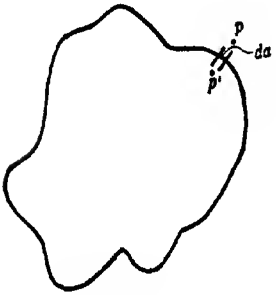


Fig. 10

Now the total or resultant field outside the conductor will be made up of two parts, that due to the charge on the elemental area da, which we will designate as f_1, and that due to the charge on the remainder of the conductor *and on all other neighboring bodies.* We will indicate the latter component by f_2. The total intensity at p will then be given by the relation

$$f = f_1 + f_2. \tag{i}$$

At the point p' the direction of the field (f_1) due to the charge on da will be reversed in sense and hence carry a negative sign. Since, however, p and p' are practically coincident in position so far as all charges *external* to the element da are concerned, the value and sign of f_2 will not be changed as we pass from p to p'. And since the intensity vanishes at points within the bounding surface it follows that

$$f_2 + (-f_1) = 0$$

or

$$f_2 = f_1. \tag{ii}$$

Hence from (i) and (ii) we have that

$$f_1 = f_2 = \tfrac{1}{2}f, \tag{iii}$$

where f is the *total intensity at p.*

For our present purposes we may think of the area da as detached from the remainder of the conductor but located in the field (f_2) due to the charge on the balance of the conductor and on any other bodies. The charge on the area da would be σda. The field (f_1) due to the charge on the element da will not figure in any possible force action on the element itself. We may therefore write, from eq. 4 and (iii),

$$F = \tfrac{1}{2}f\sigma da. \tag{iv}$$

Applying Coulomb's Law (eq. 13) we have

$$F = 2\pi\sigma^2 da.$$

If we elect to make da unity, our expression becomes

$$F = 2\pi\sigma^2, \qquad \text{Eq. 15}$$

which gives the mechanical force per cm.² acting outwardly along the normal to the surface of the conductor.

If the surrounding medium has a dielectric constant other than unity the use of eq. 14 instead of eq. 13 in making the above reduction will give

$$F = \frac{2\pi\sigma^2}{K}. \qquad \text{Eq. 16}$$

By eliminating σ from the last equation we get

$$F = \frac{f^2K}{8\pi}, \qquad \text{Eq. 17}$$

a relation which we shall find useful later.

If we are concerned with the effect of the charges on the conductor itself, and, for the moment, exclude from our consideration the field due to any neighboring charges, we have in eqs. 15, 16, and 17 means of determining the outward tension on the surface of a charged conductor due to the charges *on the conductor itself.* The existence of this outward mechanical force may be experimentally demonstrated.

Experience shows that at a barometric pressure of 76 cm. and a temperature of 15° C. the maximum value for σ in air will be about 8 e.s.u. per cm.². This corresponds to a value of 100 e.s.u. for f. Substituting this value in eq. 15 we get approximately 400 dynes as the maximum outward tension when the conductor is surrounded by air. Any attempt to increase the field intensity beyond this value by increasing the charge on the conductor will result in a brush discharge (Sec. 7).

While the above observations regarding the electrostatic pressure are important, we shall be chiefly concerned, in a later section, with the application of the equations developed above in connection with the forces which obtain in certain cases between adjacent charged conductors.

CHAPTER III

POTENTIAL

14. The Nature of Electrical Potential.—Thus far in our discussion we have learned to write down the specifications of a given electrostatic field in terms of the force action on unit test charge, and have also seen that we may graphically represent the status of the field by what we have termed lines of force or flux.

We come now to another method of describing such a field; a scheme which does not involve force actions; but a plan which involves a new and important concept; perhaps the most significant idea in the whole study of electricity. This concept was first introduced by the celebrated French mathematical astronomer, Laplace, and was given its name by the English theoretical physicist, Green. We refer to what is known as *electrical potential.*

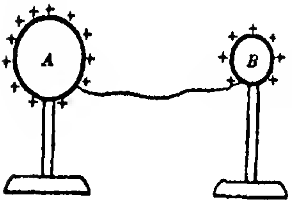


FIG. 11

In order to arrive at an understanding of this concept let us consider the simple case of two insulated charged spheres, preferably of different size, and each bearing a charge of the same magnitude, as sketched in *Fig. 11.*

If now these two charged bodies are connected by a wire there will be a migration of charges from the smaller to the larger body, and this transfer will continue until electrical equilibrium comes to obtain. The query at once presents itself, why this movement of charges? Evidently the body A was in a *different electrical condition* from B, even though the two bodies originally posse d equal charges. It would appear that there is some electr al condition which is, in a sense, analogous to temperature a d difference of temperature in the case of heat. It will be recalled that, in the study of heat, one may define temperature as the *thermal condition* of a body which determines the direction of the flow of thermal energy. In the corresponding electrical case we may similarly designate *the condition which determines the direction of the transfer of charges by the term electrical potential.* In other

words, we may say that whenever charges move in an orderly and definite manner along a conductor or through space, they do so because of the existence of an electrical condition known as difference of potential, the point *toward* which the electrons move being thought of as at the higher potential. It follows then that if there is no movement of electricity between two conductors in electrical contact or between two points on the same conductor we may assume that the two bodies or the two points are at the same electrical level or potential.

Why the two bodies, in our illustration, were at a different potential even though originally possessing equal charges will be discussed in Chapter V.

Up to this point in our discussion of potential we have referred chiefly to the potential *of a body*. We may however with equal propriety speak of the potential *at a point* in space, just as one may speak of the temperature at some point in the region of a heat source or heat sink.

It should be noted however that, though we speak of the potential at some given point, it is not possible to actually determine the absolute potential, as one may, for example, arrive at a knowledge of the absolute temperature of a body or a point in space. In this respect the electrical case is somewhat analogous to level in mechanical problems. There is no such thing as absolute height or level, and hence for convenience we have established a plane or level of reference which by common consent is taken as the level of the sea. In electrical practice the potential of the earth is taken as a reference value, and for many purposes its potential is considered to be zero. This does not mean however that the earth is actually at zero potential, but, owing to its relative size, any charges which we may artificially give to the earth produce only slight and transient changes in its potential. It is therefore thought of as being at a constant potential, and accordingly serves as a convenient condition of reference. Since the boundary of an electrostatic field is considered to be at infinity, in theoretical investigations infinity is taken as the point of zero potential

The foregoing observations concerning the concept of potential are of a general nature. We are now confronted with the question as to whether it is possible to establish a quantitative expression for potential. It is possible and also convenient to

express potential as well as difference of potential in terms of work.

Consider a positively charged body at A, Fig. 12, giving rise to a surrounding field of force. If we attempt to move a test charge in this field energy will be involved. Since work is a product of force and displacement a definite amount of work would be done in moving unit positive charge from infinity to the point C against the repelling force due to the charge on A, and the amount of energy expended in accomplishing this would be a function of the field strength. If a charge were thus moved in the field the potential energy of the electrical system, as a system, would be *increased by an amount equal to the work done in bringing the charge to the point C.* We may therefore set up a second definition of potential, namely, that *the potential at a point is numerically equal to the work done in bringing unit positive charge from infinity to the point in question.* Similarly *we may define difference of potential as the amount of work done in moving unit positive charge from one point to the other.* Thus we have another method of representing the condition of an electrostatic field.

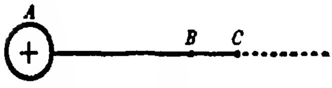


FIG. 12

In our illustration the difference of potential between C and B would be *equal* to the work done in transferring unit positive charge between these two points. It follows then that the difference of potential between two points will be unity when one erg of work is done in transferring unit charge from one point to the other. If the test charge is moved from a point of higher to one of lower potential the work will be negative. In other words work will be done by the charge and the potential energy of the system will be diminished.

From the foregoing it follows that if q electrostatic units of electricity are moved from a point whose potential is V_1 (e.s.u.) to a point where the potential is V_2 the work done, in ergs, will be given by $q(V_2 - V_1)$, or

$$W = q(V_2 - V_1). \qquad \text{Eq. 18}$$

The e.s.u. of potential has no particular name, though it is sometimes referred to as "ergs per unit charge." The practical unit of potential is the volt. For the present it may be said,

arbitrarily, that one e.s.u. is equal to 300 volts.* The expression "difference of potential" is frequently abbreviated as P.D.

If one coulomb of electricity (3×10^9 e.s.u.) is moved between two points whose P.D. is one volt $\left(\frac{1}{300}\text{ e.s.u.}\right)$ the work done would be given by applying eq. 18 thus,

$$W = 3 \times 10^9 \times \frac{1}{300} = 10^7 \text{ ergs.}$$

But 10^7 ergs = 1 joule; hence

$$W \text{ (joules)} = q \text{ (coulombs)} \times \text{P.D. (volts)}. \qquad \text{Eq. 19}$$

Equation 18 shows that the electrical work done in moving a charge between two points depends *only on the magnitude of the charge and the potential difference between the points.* The length of the path over which the displacement of the charge takes place is not a factor in the case.

15. Potential at Any Point in a Field.—Having defined potential we may now proceed to determine its value at any point in a given field. Suppose we have a charge as shown in Fig. 13. Consider a point p' located a very short distance, ds, from p.

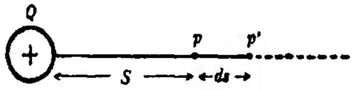


FIG. 13

Since ds is very small the intensity will be practically constant between p and p', and will be given by $\frac{Q}{Ks^2}$. Then the work necessary to transfer unit test charge from p' to p is given by the relation

$$dw = -Fds = -\frac{Q}{Ks^2}ds.$$

Therefore the work done in bringing our test charge from infinity to p might be set down as

$$\int_\infty^s dw = -\frac{Q}{K}\int_\infty^s \frac{ds}{s^2}.$$

* In electrical computations three systems of units are involved, the electrostatic system based on unit charge, the electromagnetic system based on unit magnetic pole, and the practical system derived from the electromagnetic and having magnitudes which are convenient in practice. In Chapter XXI the relations of the several systems will be discussed.

This, by definition, is numerically equal to the potential at p. Hence we may write

$$V_p = -\frac{Q}{K}\int_{\infty}^{s}\frac{ds}{s^2} = \frac{1}{K}\left(\frac{Q}{s} - \frac{Q}{\infty}\right) = \frac{Q}{Ks}, \qquad \text{Eq. 20}$$

or, in terms of the units involved,

$$V_p\,(\text{e.s.u.}) = \frac{Q\,(\text{e.s.u.})}{Ks}.$$

On the above basis we may also say that the potential difference between any two points in a field would be given by the expression

$$\text{P.D.} = \frac{1}{K}\left(\frac{Q}{s_1} - \frac{Q}{s_2}\right), \qquad \text{Eq. 21}$$

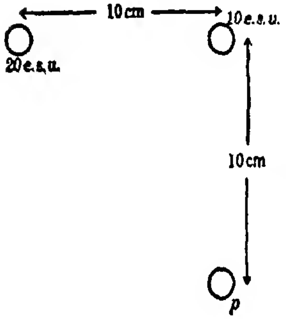


Fig. 14

where s_1 and s_2 represent the respective distances of the two points from the charge.

From eq. 20 it will be noted that the potential at a point is a ratio between charge and a length; direction is not involved. It therefore follows that potential is a scalar quantity. Hence the potential at any point in a field, due to the presence of a number of charges, is determined by finding the algebraic sum of the potentials due to each charge.

Example.—Concentrated charges are disposed as shown in Fig. 14. What is the potential at the point p?

$$V_p = \frac{10}{10} + \left(-\frac{20}{\sqrt{200}}\right),$$

$$V_p = -0.4 \text{ e.s.u.}$$

$$= -120 \text{ volts.}$$

16. Potential Gradient.—Referring to Fig. 15 and the conditions there represented, let p_1 and p_2 be two points so near together that the field strength will be sensibly constant over the distance x, and let the potential at these points be designated by V_1 and V_2 respectively. The work done in moving unit positive charge through the distance x would be fx. This is true because,

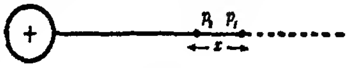


Fig. 15

if we are moving unit charge, the intensity is numerically equal to the mechanical force. By definition (Sec. 14) the work is equal to the P.D. Hence we may write

$$fx = V_2 - V_1,$$

or

$$f = \frac{V_2 - V_1}{x}. \qquad \text{Eq. 22}$$

The ratio $\frac{V_2 - V_1}{x}$ is known as the *potential slope or potential gradient.*

In terms of the calculus the same fact might be expressed thus,

$$f = -\frac{dV}{dx}, \qquad \text{Eq. 23}$$

where dV is the small P.D. between the points separated by the elemental distance dx. It is evident that we have in eq. 23 another definition of electric intensity or field strength. Translated into words it means that *the electric intensity is numerically equal to the space rate of the change in potential.* This intimate relation between P.D. and field strength is of much importance in both theoretical and practical computations. If the P.D. is in e.s.u. and the distance in cm. the intensity will be in e.s.u. of potential per cm.; if the P.D. is in volts, the intensity will be in volts per cm.

EXAMPLE.—Two points separated by 100 cm. have a P.D. of 220 volts. What is the potential gradient?

From eq. 22 we have

$$f = \frac{220}{100} = 2.2 \text{ volts per cm.}$$

17. Dielectric Strength.—Common observation discloses the fact that insulating media (dielectrics) of various kinds are widely used in the electrical industry. It has already been noted that a dielectric which is subjected to an electrostatic field undergoes an actual internal (molecular) distortion and if the field strength is sufficiently high physical rupture of the medium will result. The property of a dielectric by virtue of which it withstands physical rupture when subjected to electric stress is known as *dielectric strength*, and is expressed in terms of the field intensity

and unit thickness. In Sec. 16 we found that one may express field strength in terms of P.D. per unit of length. In dealing with the property of insulating media now under consideration it is customary to give the dielectric strength in volts (or kilovolts) per mm., cm., or mil.*

In the practical use of insulating materials it is obviously important to have a knowledge of the field strength which they will stand before rupture occurs, and also of the factors which tend to determine those values. The dielectric strength of a given specimen depends upon several factors, the most important of which are temperature, thickness of sample, time rate of applying the field, frequency,† whether a continuous or alternating field is applied, and shape and size of the electrodes.

In general increase in temperature tends to lessen the dielectric strength. There is reason for believing that the electrons which figure in whatever conductivity obtains in the case of insulators are liberated by heat ‡; hence such an effect might be expected. If the temperature is raised to such a point that the physical characteristics of the medium are modified the dielectric strength will of course be decidedly changed.

In many cases the dielectric strength of a medium does not vary directly as the thickness of the test specimen. Commonly the dielectric strength shows a smaller value for relatively thick samples than for those which are thin. This can be readily seen by examining the table of values given at the close of this section. Baur has proposed a law which has the form

$$V = d\,t^{3/2},$$

where V is the break-down potential, t the thickness of the specimen, and d the dielectric strength. This law gives approximate results in some cases. A more involved formula has been suggested by Eccles § which has the form

$$V = h + k\sqrt{(t + d)},$$

where h, d and k are constants. The non-linear relation which obtains between dielectric strength and thickness is probably

* The "mil," which is one thousandth of an inch, is frequently employed as a unit of length in electrical engineering practice.

† In alternating current practice the number of potential reversals per second is spoken of as the frequency. See Ch. XVIII *et seq.*

‡ See paper by H. Sagusa, Tohoku Univ., Sci. Reports, Dec. 1926.

§ *Wireless Telegraphy and Telephony*, W. H. Eccles, 2d ed., p. 31.

due to the fact that the thicker the specimen the less homogeneous it is. However thinly laminated materials give better values than non-laminated examples of the same total thickness.

The length of time during which the field is applied to the sample has a marked effect on the results. In general a specimen will withstand decidedly higher potentials for periods of the order of a second or less than for materially longer intervals. Some authorities hold that this effect is due to the fact that a certain minimum amount of energy is required in order to disrupt an insulating material. While it is true that the amount of energy required to produce rupture varies to some extent with the medium, yet it would appear that this lag phenomenon may also possibly be related to the rate of electronic movement in dielectrics.

If the applied voltage is alternating the dielectric strength of a given material is usually less than when tested with direct potential. The values in the latter case may run as high as two times the former. The frequency of the applied potential does not appear to figure appreciably in the region of audio frequencies, but at the extremely high frequencies used in communication engineering practice the dielectric strength is less than at ordinary commercial frequencies.

In the case of solid insulating materials, particularly when in the form of thin sheets, it has been found * that the dielectric strength is higher when the test is made with small electrodes than when larger terminals are employed. However if the electrodes are needle points the reverse is true.

Because of the fact that air plays an important part as a dielectric it is worth while to consider its properties somewhat in detail. It follows from what has been said concerning surface distribution in Sec. 8 that the size and form of the electrodes employed might be expected to have a marked effect on the dielectric strength in the case of air, and indeed we find such to be the case. Figure 16 is a graph plotted from data given in the Smithsonian Tables and shows the effect of the curvature of the electrodes on the dielectric strength of air. It will be noted that for sparking distances of the order of one or two millimeters, the curvature of the electrodes does not materially affect the

* F. M. Farmer, "The Dielectric Strength of Thin Insulating Materials," *Trans. A. I. E. E.*, 1913, p. 2097.

dielectric strength, but for distances of the order of a centimeter or more the breakdown potential increases with increasing radius of the electrode.

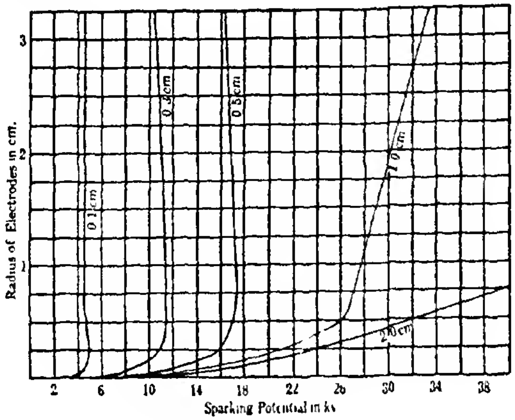


FIG. 16

Unfortunately there is no generally accepted method of testing the dielectric strength of insulating materials and hence the values given by various investigators vary widely. However in the case of air the American Institute of Electrical Engineers has established a series of values connecting sparking distance with

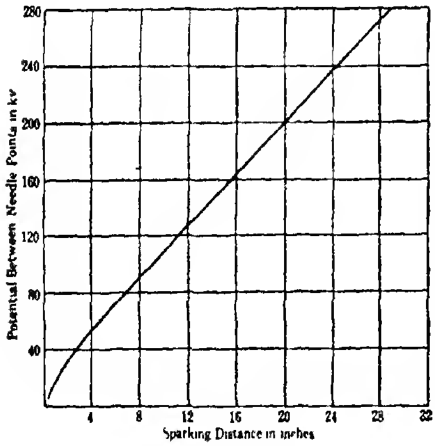


FIG. 17

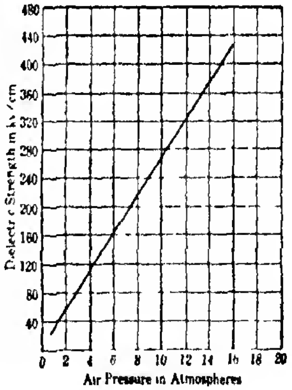


FIG. 18

applied potential (alternating) between sharp needle points (No. 00). Figure 17 shows this relation. The conditions are: temperature, 25° C.; barometric pressure, 760 mm.; relative

humidity, 80 per cent. The potential is given in root-mean-square values (see Sec. 115).

In utilizing a gas as a dielectric the pressure under which the gas is used is an important factor in determining its dielectric strength. Air under a pressure of several atmospheres is sometimes employed as a dielectric in condensers (Sec. 25). Figure 18 shows graphically the relation between pressure and dielectric strength for air as determined by E. A. Watson.

In the following table will be found the dielectric strength of various insulating materials commonly met with in practice. The data were taken from the Smithsonian Physical Tables,

Substance	Thickness	Kilovolts per Cm
Ebonite		300–1100
Empire cloth		80–300
Fibre		20
Glass		300–1500
Guttapercha		80–200
Linen, varnished		100–200
Mica		
Madras	0.1 mm	1600
"	1.0 "	300
Bengal	0.1 "	2200
"	1.0 "	700
Canada	0.1 "	1500
"	1.0 "	500
S. America		1500
Micanite		400
Paraffine, solid		350–450
Rubber		160–500
Papers		
Beeswaxed		770
Paraffined		500
Varnished		100–250
Xylol	0.2 mm.	140
"	1.0 "	80
Oils		
Castor	0.2 mm.	190
"	1.0 "	130
Linseed, raw	0.2 "	185
	1.0 "	900
Linseed, boiled	0.2 "	190
	1.0 "	80
Paraffine	0.2 "	215
"	1.0 "	160
Turpentine	0.2 "	160
"	1.0 "	110
Kerosene *	. .	50

* From the measurements made by Macfarlane and Pierce.

7th edition. For data concerning composite materials the student should consult any standard Handbook for Electrical Engineers.

18. Equipotential Lines and Surfaces.—In Sec. 15 (eq. 20) we found that the potential at any point in a field varies directly as the charge and inversely as the distance of the point from the charge. It will be evident from this relation that, in the case of an isolated concentrated charge, all points at a certain distance from the charge will be at the same potential. Using the distance from the charge to some definite point as a radius one may describe a surface about the charge, and all points on this surface would be at the same potential. Such a surface is known as an *equipotential surface.* If we are dealing with one plane only, lines might connect points of the same potential, and we would have *equipotential lines.* Figure 19 shows equipotential lines about a positive charge, the space between the concentric circles representing a P.D. of one e.s.u. (Why does the distance between the successive equipotentials gradually increase?)

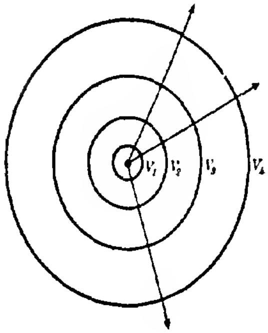


Fig. 19

Since, in many cases, the surface distribution is not uniform the equipotential surfaces will, in general, not be spheres, and the lines will not be circles. Figure 20 represents the equipotential lines, and the corresponding lines of force (dotted), in a typical case as determined experimentally. From the figure it will be observed that equipotential lines are somewhat analogous to contour lines, or lines of equal level, on a topographic map; they are also similar to isothermal lines in the case of heat. It should be noted in passing that the surface of an insulated charge conductor is an equipotential surface. This follows from the fact that if a difference of potential were to exist between any two points on the surface a movement of electrons would result and this movement would continue until electrical equilibrium obtained. In dealing with electrostatic problems, the fact that the surface of a conductor corresponds with an equipotential surface will be found to be useful.

From eq. 18 it will be evident that no work is required to move a charge along an equipotential surface. From this it follows

that the lines representing the electric intensity are everywhere normal to the equipotential surfaces.

Having considered equipotential lines and lines of force, it will be evident that we have at our disposal a method or system whereby we may completely describe the conditions which obtain in any given electrostatic field.

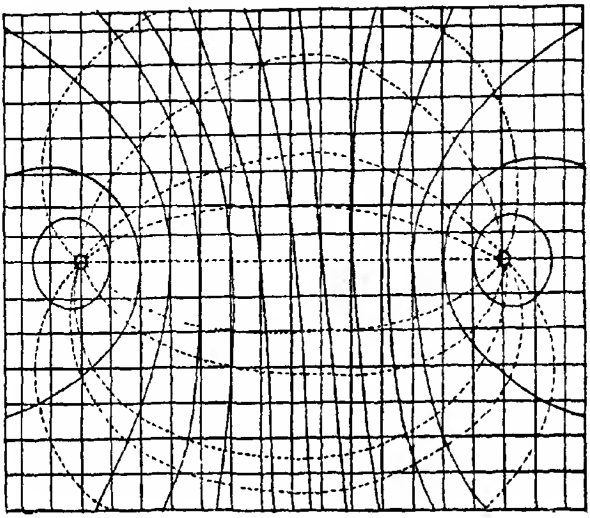


Fig. 20

19. Potential Energy of a Charge.—In charging a body, that is in causing a redistribution or rearrangement of the electrons, energy must be expended. The charge represents, therefore, a definite amount of potential energy, and it is frequently convenient to be able to compute the amount of this energy. If we can determine the amount of work done in placing the charge in its position we will have a measure of the potential energy represented by the charge. The method of analytical attack is similar to that followed in determining the magnitude of the potential energy of an elevated quantity of water.

Let us suppose that we bring a number of very small elemental charges, dq, from some point which is at zero potential and place them one at a time upon an insulated conductor, the initial potential of which is also zero. As the charge on the body increases its potential will rise (Sec. 15); therefore the amount of work done in transferring each succeeding charge will be greater than for the one preceding. In Sec. 14 it was pointed out that

the potential due to a charge is measured by the work required to transfer unit charge from a point at zero potential to the point in question, and it has also been shown (Sec. 14) that the work necessary to move any charge will be the product of the potential and the charge. In this case however the potential increases from zero to some value V. But the average potential would be $\frac{V}{2}$. The total work done in placing a total charge Q on the body would therefore be given by

$$W = \frac{V}{2}\int_0^Q dq = \tfrac{1}{2}VQ. \qquad \text{Eq. 24}$$

Since this represents the work done in charging the body it must also represent the potential energy of the charge. If V and Q are in e.s.u's., W will be in ergs. By introducing the necessary reduction factors we may express the potential in volts and the charge in coulombs. Doing this, the right side of eq. 24 would take the form

$$\tfrac{1}{2}(V \times 300)\frac{Q}{3 \times 10^9} = \tfrac{1}{2}\frac{VQ}{10^7}.$$

In order to preserve the equality in our original equation it will be necessary to divide the left side also by 10^7, which in turn will reduce the ergs to joules. Hence we may write

Potential energy represented by a charge

$$= W\text{ (joules)} = \tfrac{1}{2}V\text{(volts)} \times Q\text{(coulombs)}. \qquad \text{Eq. 25}$$

Example.—A conducting body has on it a charge of 10^6 e.s.u. It is found that the potential is 20 e.s.u. What potential energy does the charge represent?

By eq. 24, $\qquad W = \tfrac{1}{2}(20 \times 10^6) = 10^7$ (ergs).

If the same problem were stated in terms of practical units the quantities involved would be $\frac{1}{3000}$ coulomb and 6000 volts, and the solution would have involved eq. 25 thus,

$$W = \tfrac{1}{2}(6000)\frac{1}{3000} = 1 \text{ joule}.$$

CHAPTER IV

MEASUREMENT OF POTENTIAL

20. Absolute Electrometer.—The determination of the magnitude of potential in the various practical cases which arise is one of the most important electrical measurements which one is called upon to make. The calibration of all potential indicating and recording meters depends, in the last analysis, upon the absolute electrometer. In this instrument potential and difference of potential are determined in terms of two fundamental constants, namely mass and length. In its simplest form (devised by Sir William Harris) the instrument consists of a circular plate *A*, Fig. 21, suspended from one end of a balance beam with a second insulated plate *B* immediately below. If a difference of potential be established between the movable and fixed plates the former will be attracted to the latter. This attraction can then be compensated by adding weights to the other pan of the balance. The magnitude of the potential difference can then be computed in terms of a mechanical force as follows.

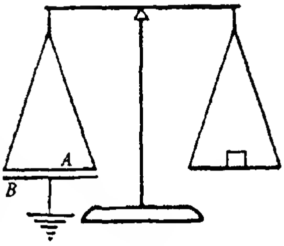


FIG. 21

In Sec. 13 we found that the mechanical force per unit area experienced by a charged body in an electric field is given by the relation (eq. 17)

$$F = \frac{f^2K}{8\pi},$$

where f is the field strength and K the dielectric constant. In this case air being the dielectric, $K = 1$.

For a pair of plates each of which has an effective area S, the force would be given by

$$F = \frac{Sf^2}{8\pi}. \qquad \text{(i)}$$

From Sec. 16, we have that the intensity is given by eq. 22, which, for this case, would be

$$f = \frac{V_A - V_B}{d}. \tag{ii}$$

Substituting eq. (ii) in eq. (i), we have

$$F = \frac{S(V_A - V_B)^2}{8\pi d^2} \qquad \text{Eq. 26}$$

as the mechanical force in dynes acting on the movable plate A. In practice the mechanical force is found in grams and is readily converted to dynes. The distance d between the plates, when mechanical equilibrium obtains, would be in cm., and $V_A - V_B$ in e.s.u. The working form of this relation would become

$$V_A - V_B = d\sqrt{\left(\frac{8\pi F}{S}\right)}, \qquad \text{Eq. 27}$$

where $V_A - V_B$ would be the difference of potential being measured.

Example.—It is found that 10 grams are required to restore equilibrium when the plates of an electrometer are connected to a source of potential difference. The distance between the plates is 0.5 cm., and the effective area of the movable plate is 100 sq. cm. Assuming $g = 980$ cm./sec./sec., what is the potential difference between the plates in volts?

Substituting in eq. 27, we have

$$V_A - V_B = 0.5\sqrt{\left(\frac{8\pi 10 \times 980}{100}\right)} = 24.8 \text{ e.s.u., or } 7440 \text{ volts.}$$

In the case of a simple plate the electrostatic field will not be uniform over the entire surface, the lines bending outward near the edges. As a result of this the simple form of the electrometer just described will not give accurate results. This defect is corrected in the instrument devised by Lord Kelvin. The essentials of the Kelvin absolute electrometer are shown in Fig. 22. In the Kelvin instrument a "guard ring" G has been added which is electrically connected to the movable disk and which is separated from the disk by the smallest possible space. As a result of the position of this guard ring the lines of electric flux are normal to the central disk even at the edges, and hence the field is uniform between the plates. Instead of weights being

used to counteract the attraction between the plates, the movable element is supported by a set of delicate springs, and provision is made for bringing about equilibrium by means of micrometer adjustments. In using the Kelvin instrument the spring system supporting the movable plate must first be mechanically calibrated in terms of a known force. For a detailed account of the construction and use of the absolute electrometer the reader should consult *Absolute Measurements in Electricity and Magnetism* by A. Grey.

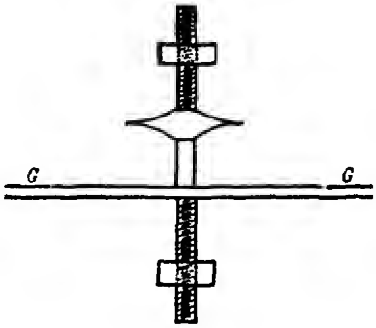


FIG. 22

21. Quadrant Electrometer.—An instrument which is widely used in scientific laboratories for measuring potential *by a method of comparison* is one due also to Lord Kelvin. The device takes

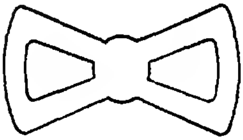


FIG. 23

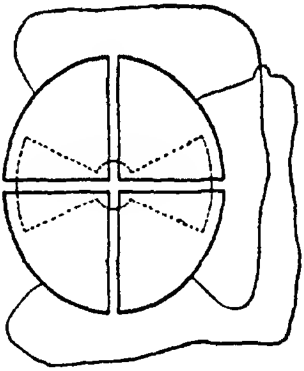
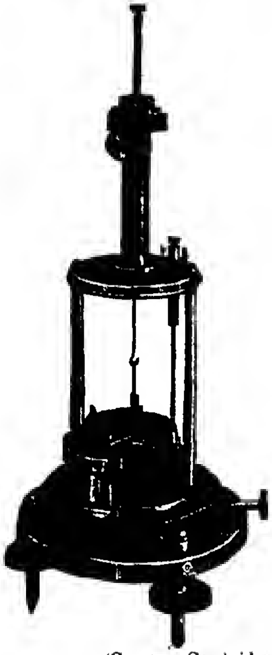


FIG. 24

its name from one of the mechanical features of its construction. The instrument in its modern form consists essentially of a specially shaped very light vane or "needle" supported by a metalized quartz fiber, the vane being suspended within a metal box. This box is divided into four parts or "quadrants," the opposite quadrants being electrically connected. Figures 23 and 24 indicate the movable and fixed elements of one form of this instrument. The quadrants are supported by insulating pillars made of quartz or amber. The movable vane is of very

thin aluminum or paper coated with thin metal foil. A mirror is attached to a light wire which is fastened to the vane.

This particular form of the quadrant electrometer is due to F. Dolezalek. In use the vane or needle is charged by being connected to a source of potential as, for example, the positive side of a 100-volt storage battery, the negative battery terminal being grounded. The source of the difference of potential to be measured is connected to opposite pairs of quadrants. With no charge on the quadrants the vane is adjusted to hang symmetrically with respect to the quadrants as shown in Fig. 24. With the quadrants oppositely charged the needle will be repelled by one set of quadrants and attracted by the other, the movement continuing until the electrostatic torque is balanced by the mechanical torque of the suspension. The deflection, as shown by a telescope and scale, is compared with the deflection due to a *known difference* of potential on the quadrants. With a suitable suspension a difference of potential of one volt on the quadrants may be made to give a deflection of 200 to 400 millimeters on a scale placed one meter from the mirror. Figure 25 shows a late model of this type of electrometer.



(Courtesy Cambridge Instrument Co.)

Fig. 25.—Quadrant Electrometer

Because of its wide utility in research and also because the theory of the electrometer so well illustrates the method of analytical attack in problems in electrostatics, it is worth while to review the deduction of the working formula of the instrument.

Referring to the diagrammatic sketch shown as Fig. 26, suppose the electrical conditions to be as indicated. The needle will then tend to move as shown by the arrow. Equilibrium will obtain when the mechanical couple equals the couple due to the electrostatic field. Our problem is to find the relation between the deflection and the potentials of the essential components of the instrument. To do this let us suppose the vane to be slightly

displaced. For small deflections the restoring or mechanical couple, L, is proportional to the deflection ϕ. Since we have displaced the moving system from its original neutral position, the total energy of the charged system must have been changed. If the magnitude of this change can be found we can develop an expression for the work done in effecting the displacement, which in turn will be proportional to the displacement.

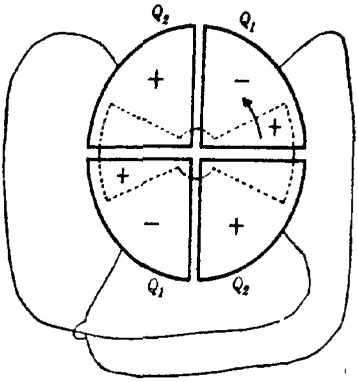


Fig. 26

We have shown (Sec. 19) that the potential energy of any electrified system is given by $\frac{1}{2}QV$, and also (Sec. 8) that $Q = \sigma A$, where A is the area involved and σ the surface density of the charge. Combining these two relations we have

$$\text{P.E.} = \tfrac{1}{2}\sigma A V. \tag{i}$$

By comparing the values for field intensity as given by eqs. 13 (Sec. 12) and 22 (Sec. 16) we find that

$$\sigma = \frac{V}{4\pi d}, \tag{ii}$$

where V is the P.D. between the two plates of a system and d their distance apart.

Substituting eq. (ii) in eq. (i) we get, as a general expression for the potential energy of the system,

$$\frac{1}{2}\frac{AV^2}{4\pi d}.$$

In our case, we have changed the effective area by an amount da; hence the change in energy (increase in this case) would be

$$\frac{V^2}{8\pi d}da.$$

In the electrical system being considered V is $V_N - V_{Q1}$. Thus the increase in energy is

$$\frac{da}{8\pi d}(V_N - V_{Q1})^2.$$

If desired, da may be expressed in terms of the actual dimensions of the vane, but since these, together with d, are constants of the particular instrument being used, we may leave this factor in its present form.

Similarly as the vane moves slightly away from the negative quadrants the system suffers a *decrease* in energy. This decrease will be, by analogy, equal to

$$\frac{da}{8\pi d}(V_N - V_{Q2})^2.$$

The total energy change will be the difference between the last two expressions, or

$$\frac{da}{8\pi d}[(V_N - V_{Q1})^2 - (V_N - V_{Q2})^2],$$

which reduces to

$$\frac{2da}{8\pi d}(V_{Q2} - V_{Q1})\left(V_N - \frac{V_{Q1} + V_{Q2}}{2}\right).$$

This should be equal to the work done in bringing about the change in position of the vane.

Now for small displacements the work done will be proportional to the deflection ϕ, or $W = B\phi$, where B is a constant. Therefore

$$B\phi = \frac{da}{8\pi d}(V_{Q2} - V_{Q1})\left(V_N - \frac{V_{Q1} + V_{Q2}}{2}\right);$$

hence

$$\phi = \frac{da}{8B\pi d}(V_{Q2} - V_{Q1})\left(V_N - \frac{V_{Q1} + V_{Q2}}{2}\right).$$

All of the factors in the term $da/4B\pi d$ are constants, and depend upon the construction of the particular instrument in use. In practice V_N is large compared with the potential of the quadrants; hence the second bracketed term does not differ materially from V_N. It therefore follows that the deflection ϕ is proportional to

$$(V_{Q2} - V_{Q1})V_N, \qquad \text{Eq. 28}$$

that is to the product of the potential of the vane and the potential difference between the quadrants. We thus have an ac-

curate and convenient method of *comparing differences of potential.*

It frequently becomes necessary to determine alternating difference of potential. By connecting one pair of quadrants with the vane the deflection will be in the same direction whether the P.D. is positive or negative. When connected in this manner the electrometer may be utilized in alternating current work. When so arranged the working equation reduces to a form which shows that the deflection is proportional to

$$\tfrac{1}{2}(V_N - V_{Q1})^2, \qquad \text{Eq. 29}$$

where V_N is the potential of the pair of quadrants which are connected to the vane. It should be noted that in this case the deflection is a function of the square of the P.D. When utilized in this manner the instrument is said to be used idiostatically. When the quadrants and the vanes are all at different potentials the instrument is said to be used heterostatically.

22. Compton Electrometer.—Probably the most sensitive form of the quadrant electrometer which has been devised is that developed by Dr. Karl T. Compton and Dr. Arthur H. Compton.* A high degree of sensitivity is secured by giving the needle a slight tilt about its long axis and also vertically displacing one pair of quadrants with respect to the other. Figure 27 is a diagrammatic side view of the Compton system. As a result of this asymmetrical arrangement of the needle and quadrants, electrostatic forces are brought into play which may be made to more or less completely neutralize the mechanical torque of the suspension. It is possible with this instrument to secure a sensitivity as high as 50,000 mm. per volt.† However the ordinary working range is 0–10,000 mm. per volt. Deflections are proportional to the P.D. over a relatively wide range. Figure 28 shows an exterior view of the Compton instrument.

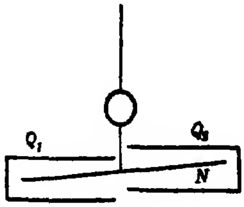
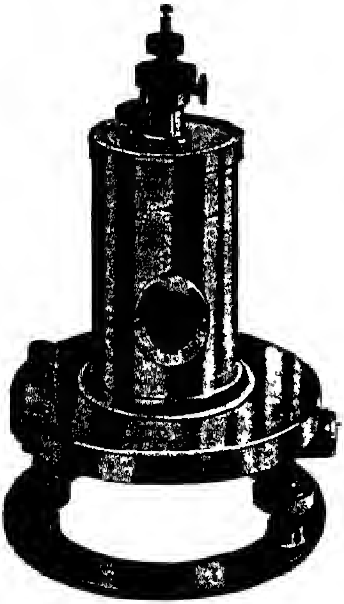


FIG. 27

* See a paper by the Messrs. Compton, "A Sensitive Modification of the Quadrant Electrometer," *Physical Review*, Aug. 1919, XIV, No. 2, 85.

† The term "sensitivity" as here used refers to the deflection as noted by a telescope and scale when a P.D. of one volt is applied to the quadrants, the scale being at a distance of one meter from the mirror. A similar meaning is often attached to the term "sensitivity" in other connections.

23. Electrostatic Voltmeters.—By utilizing the principle of the quadrant electrometer when used idiostatically it is possible to devise a portable voltmeter suitable for both direct and rapidly alternating potentials. Lord Kelvin designed such an instrument, one form of which is shown in Fig. 29. In this particular type of the instrument control is effected by means of an ad-



(Courtesy Cambridge Instrument Co.)
Fig. 28.—Compton Electrometer

Fig. 29.—Electostatic Voltmeter

justable weight attached to the movable vane, or by a delicate helixal spring attached to the shaft which supports the moving system. The scale is calibrated directly in volts, the range in some cases extending up to several thousand volts.

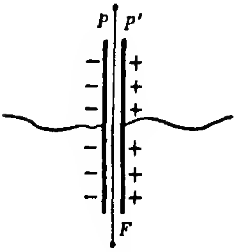
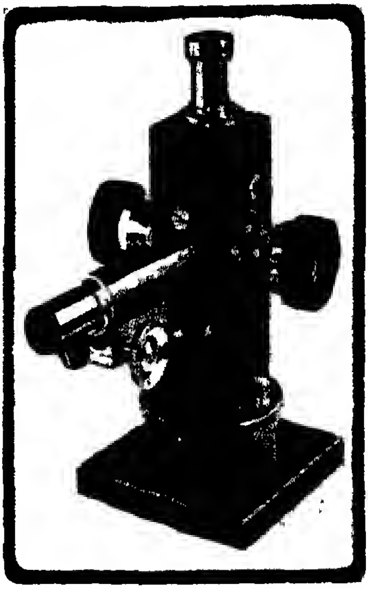


Fig. 30

For measuring potentials of the order of 100 volts Lord Kelvin developed what is known as a multicellular voltmeter. This differs from the above in that there are a number of vanes alternating with corresponding sets of quadrants. The vanes occupy a horizontal position and are attached to a light vertical spindle which in turn is suspended by means of a small metal wire or ribbon. The deflection is observed by means of a light

pointer attached to the movable system and moving over a horizontal scale. The advantage of the electrostatic voltmeter is that, once the moving system comes to rest, no energy is taken from the source.

24. String Electrometer.—For certain purposes, particularly in research work, the string electrometer is admirably adapted to the accurate measurement of small values of potential. The essential parts of an instrument of this type are shown diagrammatically in Fig. 30. The device consists of two carefully insulated plates P and P' between which is stretched an insulated quartz fiber F which is platinized. If the string is connected to a source of potential, and a difference of potential applied to the plates, the string will be slightly deflected as a result of the electro-static force action. The deflection is read by means of a micrometer microscope which forms a part of the instrument. The position of the plates and the tension of the string are controlled by means of micrometer screws. Dr. Joel Stebbins has recently designed an electrometer of this type which possesses a number of very desirable features. The Stebbins instrument has an extremely small temperature coefficient, high sensitivity, small electrostatic capacity and a short period. An exterior view of this instrument is shown in Fig. 31. In later chapters other methods of determining differences of potential will be described.



(Courtesy Pyroelectric Co.)

Fig. 31.—Stebbins String Electrometer

CHAPTER V

CAPACITANCE

25. Concept of Capacitance.—We have already shown (Sec. 14) that the potential of a conductor is measured by the work done in bringing unit test charge from infinity up to the conductor, and it has also been deduced (Sec. 9) that the field strength, and hence the opposing force, is proportional to the charge on the conductor. It therefore follows that the potential of an insulated conductor is proportional to its charge. This fact may be stated in another way by saying that the ratio of the charge to the potential is a constant quantity, thus,

$$\frac{Q}{V} = C. \qquad \text{Eq. 30}$$

If we rewrite our equation in the form

$$V = \frac{Q}{C},$$

it will be evident that the potential of a conductor depends not only upon the charge resident thereon but also upon a second factor, C, which we may designate as the capacitance of the conductor.

Equation 30 indicates that *we may define capacitance as the ratio of the charge on a conductor to its potential.* The same relation also shows that capacitance is *numerically* equal to the charge that will cause a conductor to have unit potential. In terms of mechanics it would be analogous to the number of gas molecules which it would be necessary to introduce into a given container in order to establish unit pressure. It is obvious that the amount of gas required to produce unit pressure would depend, in part, upon the volume of the container. In a somewhat analogous manner, the capacitance of a conductor is a function of its superficial area and of its geometrical form, as will be shown in discussions which follow.

Further, since the potential of a body depends not only upon its own charge but also upon the presence of other charged bodies in its vicinity, it follows that the capacitance of a conductor will depend upon its position with respect to other charged bodies.

This will become more apparent when we deal with the subject of condensers in the next section.

Referring to eq. 30, it may be noted that if Q and V are in e.s.u. C will also be in e.s.u. If however Q is in coulombs (3×10^9 e.s.u.) and V in volts ($1/3 \times 10^{-2}$ e.s.u.), the ratio Q/V would give

$$\frac{3 \times 10^9}{1/3 \times 10^{-2}} = 9 \times 10^{11}.$$

It is therefore convenient to have a practical unit of capacitance which shall be equal to 9×10^{11} e.s.u. This unit is known as the *farad*. In practical units eq. 30 becomes then

$$\frac{Q \text{ (coulombs)}}{V \text{ (volts)}} = C \text{ (farads)},$$

and we may say that a conductor has a capacitance of one farad if a charge of one coulomb gives it a potential of one volt.

The farad is a magnitude of capacitance which is rather large for common use and hence a fraction of this unit is ordinarily employed. The smaller practical unit has a value of one millionth of a farad and is known as the microfarad. On the above basis,

$$\text{one microfarad} = 9 \times 10^5 \text{ e.s.u.}$$

In certain lines of communication engineering where capacitance values of very small magnitudes are encountered a still smaller practical unit is sometimes employed, which is known as the micro-microfarad, the latter unit having a value of one millionth of a microfarad, or

$$\text{one micro-microfarad} = 0.9 \text{ e.s.u.}$$

Microfarad is frequently abbreviated as mf. or mfd., and micro-microfarad as $\mu\mu$f. or mmf.

To summarize,

$$\text{Farads} = \frac{\text{e.s.units}}{9 \times 10^{11}},$$

$$\text{Microfarads} = \frac{\text{e.s.units}}{9 \times 10^{5}},$$

$$\text{Micro-microfarads} = \frac{\text{e.s.units}}{0.9},$$

$$\text{Farads} = \frac{\text{microfarads}}{10^6}.$$

Examples.—(*a*) A conductor having a capacitance of 4×10^5 e.s.u. is found to have a potential of 10 e.s.u. What is the magnitude of the charge on the conductor?

Changing eq. 30 to fit our needs and substituting we have

$$Q = CV = 4 \times 10^5 \times 10 = 4 \times 10^6 \text{ e.s.u.}$$

(*b*) A conductor having a capacitance of 0.5 mf. is given a charge of 2×10^{-3} coulombs. What is its potential?

From eq. 30 we have

$$V = \frac{Q}{C} = \frac{2 \times 10^{-3}}{0.5 \times 10^{-6}} = 4 \times 10^3 \text{ (volts).}$$

26. Condensers.—Speaking broadly, it may be said that any arrangement or device by means of which the electrostatic capacitance of a conductor is augmented is a condenser. We have already seen (Sec. 19) that a charged conductor represents potential energy. Hence a condenser may function as a storehouse of electrical energy. A condenser consists essentially of one or more pairs of electrical conductors, separated by some form of dielectric.

The condenser is one of the oldest known pieces of electrical apparatus and is, in its modern forms, in extensive use at the present time. It is said that the principle of the condenser was originally discovered by a German named Von Kleist in 1745. The device used by Kleist consisted of a glass receptacle filled with water and held in the hand. Kleist's experiment was repeated by Cuneaus at Leyden and the early form of condenser came to be known as a "Leyden Jar." A few years afterwards Benjamin Franklin designed condensers having tinfoil coatings pasted on both glass jars and glass plates. The type of condenser developed by Franklin continued to be used with little if any change in form for something like one hundred and sixty years. It was not until the advent of radiotelegraphy that improved types of condensers came to be produced.

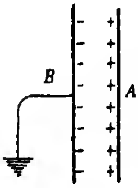


Fig. 32

However, before discussing the various forms of modern condensers it will be well to examine the basic theory of this device. Let us first note what condition must be fulfilled in order to bring about an increase in the capacitance of a conductor.

Referring to Fig. 32, suppose we have a plate *A* charged

positively as shown. The potential of A would be equal to the amount of work required to bring unit positive charge from infinity to the plate, and the work, in turn, would be determined by the field strength due to the charge on the body A.

If now we place a second conductor B near to A and connect it to earth a negative charge will be induced on this second conductor and, *because of this negative charge, the resultant electric intensity at all points in the field will be less than formerly* (Sec. 4). This means that the work required to bring our test charge from infinity will, under the changed conditions, be less and hence it must follow that the potential of A has been lowered because of the presence of the grounded conductor. Remembering eq. 30, it will be evident that if the potential due to a given charge has been decreased it must follow that the capacitance of A has been increased as a result of the presence of the second earthed conductor. In other words, a greater charge will now be required to raise the potential of the body A to unity. Such an electrical organization constitutes a condenser. The effect would have been the same had we connected B to the negative side of the source which gave A its positive charge. Indeed this is usually done in practice.

The capacitance of a condenser may be defined in much the same way as we defined the capacitance of a simple conductor. On this basis it may be said that the capacitance of a condenser is the ratio of its charge to its potential or, otherwise stated, it is the quantity of electricity necessary to establish unit P.D. between the plates.

In practice there are many instances where two or more conductors are so related to one another electrically that, in effect, they constitute a condenser, and it is important to be able to compute the capacitance in such cases. We shall therefore proceed to develop several working relations by the use of which one may predetermine the capacitance in the more important cases. We will begin with the simple case of the sphere.

27. Capacitance of a Sphere.—Assume a sphere surrounded by air as shown in Fig. 33. Let the sphere possess a charge whose value is Q e.s.u. The potential, V, at some point p outside the sphere will be Q/d, where d is the distance of the point p from the center

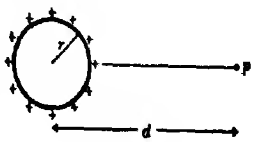


FIG. 33

of the sphere. At the surface of the sphere $d = r$ and the potential would be Q/r. This may be written

$$r = \frac{Q}{V}.$$

Now Q/V is by definition (Sec. 25) equal to the capacitance; hence

$$r = \frac{Q}{V} = C. \qquad \text{Eq. 31}$$

Since the radius would be measured in units of length we see that the *capacitance of a sphere is numerically equal to its radius in centimeters.* It thus turns out that the e.s.u. of capacitance is in the nature of a unit of length. A sphere thoroughly insulated and well removed from other charged bodies may be used as a standard of capacitance.

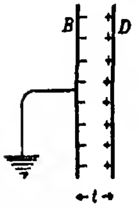


Fig. 34

28. Capacitance of a Parallel Plate Condenser.—Assume two parallel plates arranged as shown in Fig. 34, D being charged and B connected to earth. By Coulomb's Law (eq. 13) the field intensity in the region between the plates would be $4\pi\sigma/K$, where σ is the charge per unit area on one of the plates. The work done in moving unit test charge from one plate to the other would measure the P.D. between the plates. Hence we may write

$$W = V_B - V_D = -\int_B^D f dx = \int_B^D \frac{4\pi\sigma}{K} dt = \frac{4\pi\sigma t}{K},$$

where t is the thickness of the dielectric. But $Q = \sigma A$, where A is the total effective area of one of the plates, and

$$C = \frac{Q}{V};$$

hence

$$C = \frac{\sigma A}{V}.$$

Substituting we have, as an expression for the capacitance of a parallel plate condenser,

$$C = \frac{KA}{4\pi t}. \qquad \text{Eq. 32}$$

If A and t are in centimeters C will be in e.s.u. By introducing

the necessary transformation factors, eq. 32 becomes

$$C = \frac{KA}{1.131 \times 10^7 t}\,\text{mf.} \qquad \text{Eq. 33}$$

Owing to the fact that the field near the edges of the plates is not uniform, the formulas just developed will give only approximate values. These relations can however be made to apply rigorously if a "guard ring" similar to that used in the case of the Kelvin absolute electrometer (Sec. 20) is provided for one of the plates.

EXAMPLES.—A condenser consists of two parallel plates each 12 × 18 cm. The plates are separated by glass ($K = 6$) 2 mm. thick. What is the capacitance of the condenser in e.s.u.? In microfarads? In micro-microfarads? Using eq. 32 we have

$$C = \frac{6 \times 12 \times 18}{4\pi \times 0.2} = 516 \text{ e.s.u.}$$

Employing eq. 33,

$$C = \frac{6 \times 12 \times 18}{1.131 \times 10^7 \times 0.2} = 0.000573 \text{ mf.}$$

Changing the result in e.s.u. to mmf.,

$$\frac{516}{0.9} = 573 \text{ mmf.}$$

29. Capacitance of Coaxial Cylinders.—Referring to Fig. 35, assume two long coaxial cylinders of radii a and b, and let q = charge per unit length on the inner cylinder; $-q$ = charge per unit length on inside of outer cylinder; V = potential of the inner cylinder. The field intensity at a distance x from the axis of the inner cylinder will be $2q/Kx$ (Sec. 11). The work done on unit test charge in passing from the outside cylinder to the inside cylinder would be equal to

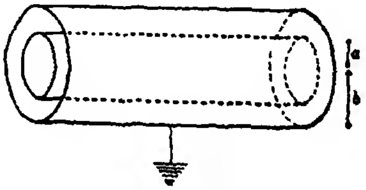


FIG. 35

$$W = -\int_b^a \frac{2q\,dx}{Kx},$$

or

$$W = V = -\frac{2q}{K}\int_b^a \frac{dx}{x} = \frac{2q}{K}\log\frac{b}{a}.$$

Transposing we have

$$\frac{q}{V} = \frac{K}{2 \log \frac{b}{a}}.$$

But

$$\frac{q}{V} = C;$$

hence

$$C = \frac{K}{2 \log \frac{b}{a}} \text{ e.s.u. per unit length.} \qquad \text{Eq. 34}$$

When using the above relation in calculations it should be remembered that the logarithm in the denominator is the natural logarithm having the base $e = 2.718$. Since we may transform logarithms using 10 as a base to logarithms having the base e by multiplying by 2.3026, we have for any length L

$$C = \frac{KL}{2(\log_{10} b/a)\, 2.3026 \times 9 \times 10^5} \text{ mf.,} \qquad \text{Eq. 35}$$

when L is in centimeters.

This relation is important because it serves as a basis for the calculation of the capacitance of submarine cables. In such a case the surface of the metallic conductor acts as the inner cylinder and the water as the outer cylinder. The insulating sheathing serves as the dielectric, the constant being, in cable practice, of the order of 3. Since 1 mile = 160934.4 cms., and since the constant

$$\frac{160934.4}{2 \times 2.3026 \times 9 \times 10^5} = 0.0388,$$

we may write for the capacitance of a cable in microfarads per mile,

$$C = \frac{0.0388K}{\log_{10} b/a}. \qquad \text{Eq. 36}$$

Example.—A submarine cable consisting of a stranded conductor as the core and gutta percha insulation is 2200 miles long. The mean diameter of the core is 250 mils and the diameter outside the insulation is 500 mils. If the gutta percha has a dielectric constant of 3.6, what is the capacitance of the cable?

Since b/a is a ratio it is immaterial as to what units are used in expressing a and b. It is also to be noted that, in this relation, the logarithm of b/a is used as an actual number.

Substituting we have

$$C = \frac{0.0388 \times 3.6 \times 2200}{\log_{10} \frac{250}{125}} = \frac{0.0388 \times 3.6 \times 2200}{0.3010} = 1020.6 \text{ mf.}$$

30. Capacitance of a Single Vertical Wire.—To simplify the problem we will neglect the proximity of the earth. Assume a long cylindrical wire, as shown in Fig. 36, of circular cross section, and radius r. Consider a thin element of width dx, distant x from a point p on the axis. First let us find an expression for the

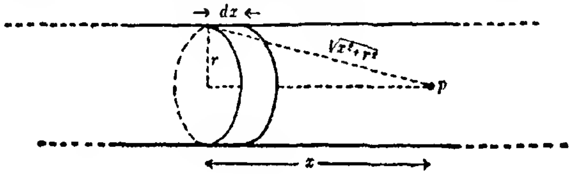


FIG. 36

potential at p. The charge on the element of surface will be $2\pi r\sigma dx$, where σ is the surface density. The distance of all parts of the charge from the point p is $\sqrt{(x^2 + r^2)}$; hence the potential at p (eq. 20) due to the charge on the elemental surface will be

$$dV = \frac{2\pi r\sigma dx}{\sqrt{(x^2 + r^2)}}.$$

The potential due to the charge on the whole wire of length h will be

$$V = 2\int_0^{h/2} \frac{2\pi r\sigma dx}{\sqrt{(x^2 + r^2)}},$$

which gives

$$V = 4\pi r\sigma \left[\log_e \{h/2 + \sqrt{(r^2 + h^2/4)}\} - \log_e r\right].$$

In general, the radius of the wire will be small compared with the length, and hence the above expression reduces to

$$V = 4\pi r\sigma \log_e h/r.$$

The total charge $Q = 2\pi r\sigma h$, or

$$2\pi r\sigma = \frac{Q}{h}.$$

Therefore

$$V = \frac{Q}{h} 2 \log_e h/r.$$

And

$$\frac{Q}{V} = \frac{h}{2 \log_e h/r} = C \quad \text{(in e.s.u.)}. \qquad \text{Eq. 37}$$

In microfarads, the capacitance would be

$$\frac{h}{4.6052 \times 9 \times 10^5 \times \log_{10} h/r}, \qquad \text{Eq. 38}$$

where h and r are expressed in centimeters.

This equation may be used to compute the approximate capacitance of a single vertical wire used as an antenna (Sec. 166). Owing however to the fact that the lower end of the wire is comparatively near the earth the calculated value will be something like 10 per cent too small.

EXAMPLE.—What is the capacitance of a single vertical wire 100 ft. in length, size No. 10, B. & S.?

One foot = 30.48 cm. and wire No. 10, B. & S., has a diameter of 0.2588 cm. Substituting in eq. 38 we have

$$C = \frac{100 \times 30.48}{4.6052 \times 9 \times 10^5 \times \log_{10} \frac{100 \times 30.48}{0.1294}} = 0.000168 \text{ mf.}$$

31. Capacitance of a Pair of Telephone, Telegraph, or Power Wires.—Let Fig. 37 represent a sectional view of the two wires of a telephone or power transmission line. In order to determine the capacitance of such a pair of wires it will first be necessary to find the potential difference between the wires due to a positive charge on one and a negative charge on the other.

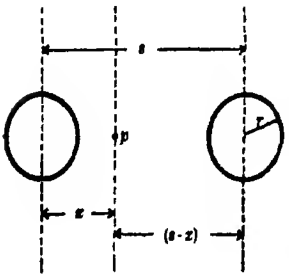


FIG. 37

Consider a point p on a line joining the centers of the two wires. The field strength at p will be the sum of the intensities due to $+Q$ on one wire and $-Q$ on the other, where Q is the charge per unit length. It has previously been shown (Sec. 11) that the intensity at any point due to the charge on a uniformly electrified cylinder is given by the expression $\frac{2Q}{d}$, where d is the distance

from the cylinder to the point in question. In this case the total field intensity will be

$$f = 2Q\left(\frac{1}{x} + \frac{1}{s - x}\right) = 4\pi r\sigma\left(\frac{1}{x} + \frac{1}{s - x}\right).$$

By definition (eq. 23, Sec. 16),

$$f = \frac{dV}{dx} \qquad \text{or} \qquad dV = f dx.$$

The total difference of potential between the two wires will then be given by

$$\int dV = \int_{x=r}^{x=s-r} f dx = 4\pi r\sigma \int_{x=r}^{x=s-r} \left[\frac{s}{x(s - x)}\right] dx.$$

Integrating we get

$$V = \left(4\pi r\sigma \log_e \frac{x}{s - x}\right)_r^{s-r}.$$

Since r is small compared to s this reduces to $8\pi r\sigma \log_e \frac{s}{r}$. Since the charge per unit length is $Q = 2\pi r\sigma$,

$$V = 4Q \log_e \frac{s}{r},$$

and

$$\frac{Q}{V} = \frac{1}{4 \log_e \frac{s}{r}} = C \quad \text{(in e.s.u./cm.).} \qquad \text{Eq. 39}$$

For any length of two-wire line, L, we may then write, as the capacitance in microfarads,

$$C = \frac{L}{4 \times 2.3026 \times 9 \times 10^5 \log_{10} \frac{s}{r}}$$

$$C = \frac{0.0000001208L}{\log_{10} \frac{s}{r}}, \qquad \text{Eq. 40}$$

L being in cm., and s and r in the same units. Since 1 mile = 160943.4 cm., the capacity per mile in mf. of two parallel

wires is

$$C = \frac{0.0194}{\log_{10}\frac{s}{r}}. \qquad \text{Eq. 41}$$

In deriving this relation we have assumed for the sake of simplicity that the wires were far enough apart so that the mutual interaction of the charges would not disturb the uniform perimetrical distribution of the charge. We further assumed that the wires were high enough from the earth to preclude any influence from that source. In practice neither of these ideal conditions obtains; hence the above formula yields only approximate numerical results. The measured values are, in general, higher than the computed values, the difference depending upon local conditions.

EXAMPLE.—What is the capacitance of a pair of telephone wires one mile in length, the size of the wire being No. 10, B. & S., and the spacing one foot?

One foot = 30.48 cm.; No. 10 B. & S. wire has a diameter of 0.2588 cm. Substituting in eq. 41,

$$C = \frac{0.0194}{\log_{10}\frac{30.48}{0.1294}} = 0.008176 \text{ mf.}$$

A special case which may be deduced from the foregoing is that of a single wire parallel to and at a height above the surface of the earth.

In order to arrive at a working relation in this case we may assume the earth's surface to be a good conductor and at zero potential. Fleming has pointed out that "the difference in potential between the charged wire at a height h above the earth would be half of that between the wire and a similar oppositely charged wire at a depth h below the surface of the earth, supposing all the earth then removed. Hence the capacity of the single wire at a height h above the earth must be double that of two parallel wires at a distance $2h$ apart." * Therefore from eq. 39 the capacitance of such a wire would be given by the relation

$$C = \frac{2L}{4 \log_e \frac{2h}{r}} \text{ (e.s.u.)}, \qquad \text{Eq. 42}$$

* J. A. Flemming: *Propagation of Electric Currents*, p. 213.

where L is the length of the wire, h its mean height above the earth, and r the radius of the wire in the same units as h.

In microfarads this expression becomes

$$C = \frac{L}{2 \times 2.3026 \times 9 \times 10^5 \times \log_{10}\frac{2h}{r}}, \qquad \text{Eq. 43}$$

and the capacitance per mile in microfarads would be

$$C_1 = \frac{0.0388}{\log_{10}\frac{2h}{r}}. \qquad \text{Eq. 44}$$

Example.—A single horizontal wire 100 ft. in length is suspended 35 ft. above the earth. If the wire is size No. 14, B. & S., what is its capacity in mf.?

Equation 43 is applicable in this case; substituting we have

$$C = \frac{100 \times 30.5}{2 \times 2.3026 \times 9 \times 10^5 \log_{10}\frac{2 \times 35 \times 30.5}{0.163}} = 0.000179 \text{ mf.}$$

Formulas have been developed for the capacitance of various forms of radio antennas, but as they give only roughly approximate results they are not included here. Such formulas are however useful, and the reader who is interested in this particular aspect of capacitance is referred to such standard works as *Principles of Radio Communication,* by J. M. Morecroft, or *Radio Engineering,* by J. H. Reyner, or *Wireless Telegraphy and Telephony,* by W. H. Eccles.

32. Capacitance of Condensers in Series and Parallel.—It frequently becomes necessary to design condensers for use at potential differences which are much higher than the dielectric strength (Sec. 17) of the insulating material which it is desired to use in a given case. In order to meet this situation it is customary to connect several condenser units in series, thereby distributing the P D. between the several units which go to make up the condenser as a whole. The danger of a breakdown of the dielectric is thus minimized, but by such an arrangement of condenser components the total capacitance is less than the capacitance of the individual units which compose the complete condenser. It naturally becomes important to be able to compute the total capacitance of a group of condenser units connected in this manner.

Referring to Fig. 38, let C_1, C_2, and C_3 represent condensers connected in series and which, in general, would have different capacitance values. Our problem is to find an expression for the total capacitance in terms of the capacitance of the individual units. By the laws of electrostatic induction, if these condensers are charged by connecting the points 1 and 4 to a source of potential difference each condenser in the series will acquire an equal charge, which we may call Q. Designating the potentials at the points 1, 2, 3, etc., by V_1, V_2, V_3 respectively we may write (eq. 30)

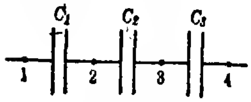


FIG. 38

$$V_1 - V_2 = \frac{Q}{C_1},$$

$$V_2 - V_3 = \frac{Q}{C_2},$$

$$V_3 - V_4 = \frac{Q}{C_3}, \text{ etc.}$$

For the total difference of potential between 1 and 4 we would have

$$V_1 - V_4 = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \cdots\right).$$

But $\dfrac{Q}{V_1 - V_4} = C$, the total capacitance; therefore

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \qquad \text{Eq. 45}$$

It is thus evident, as implied above, that *the combined capacitance of condensers in series is less than the capacitance of any one of the units composing the series.* Because of this fact condensers are sometimes connected in series in order to secure a capacitance value less than that of any unit which may be readily available for some particular purpose.

When however condensers are connected in series for the purpose of reducing the P.D. which each individual unit will be required to stand or when for any other reason it is desired to arrange for a relatively large capacitance, condensers may be connected in parallel. Let us develop an expression for the capacitance in this case also.

A parallel arrangement is shown in Fig. 39. In this case all condenser units are charged to the same potential; hence, from our fundamental definition of capacitance (Sec. 25), it is evident that each condenser unit will acquire a charge proportional to its individual capacitance, and the total charge will of course be equal to the sum of the charge on the several units. Hence we may write

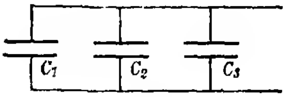


FIG. 39

$$Q = Q_1 + Q_2 + Q_3, \text{ etc.,} \tag{i}$$

where Q is the total charge on the bank of condensers as a whole.

From eq. 30 it follows that

$$Q_1 = C_1V, \qquad Q_2 = C_2V, \quad \text{etc.} \tag{ii}$$

Substituting the values given in (ii) in eq. (i) and reducing, we have

$$C = C_1 + C_2 + C_3, \text{ etc.,} \qquad \text{Eq. 46}$$

where C is the capacitance of the group of condensers as a whole. Thus we see that the total capacitance equals the sum of the capacitance of the several individual units.

In order to secure the desired capacitance and at the same time keep down the P.D. across the elemental units it is sometimes necessary to assemble a series-parallel organization as shown in Fig. 40. In order to compute the capacitance in such a case one first proceeds to find the capacitance of each series group by the use of eq. 45, and then deals with the values thus secured as a parallel combination by the aid of eq. 46.

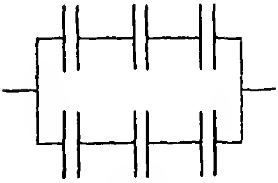


FIG. 40

In dealing with condensers in practice it should always be remembered that, for a given charge, the potential across a condenser will vary inversely as the capacitance.

EXAMPLE.—Eight condensers are connected in a series-parallel arrangement similar to that shown in Fig. 40, four 1-mf. units being in each series group. What is the total capacitance?

For each series group we would have

$$\frac{1}{C} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 4,$$

$$C = \frac{1}{4}\text{ mf.}$$

The parallel arrangement of the two groups would then give

$$C = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}\text{ mf.}$$

33. Condenser Type of Insulating Bushing.—An interesting application of the series arrangement of condensers when utilized for the purpose of distributing the electric stress is found in a certain type of insulating bushing used in the construction of high potential transformers (Sec. 124). Such transformers are inclosed by metallic casing and it becomes necessary to carefully insulate the lead-in terminals from the case. If the bushings were made of dielectric material only, the electric field intensity in that part of the bushing next to the lead-in wire would be very great, while the electric stress in the outer part would be comparatively low. In order to distribute the electric stress uniformly throughout the cross-section of the insulating material, and thus minimize the danger from breakdown, the bushing is made up of alternate layers of insulating material and some conductor such as tin foil, as indicated in the longitudinal section shown in Fig. 41. The alternating layers of insulating material and tin foil are of diminishing length, the reduction being so proportioned in relation to the increase in circumference that each succeeding layer has the same area. These layers of tin foil and dielectric constitute a number of condensers of *equal capacitance* connected in series. By means of this arrangement the total P.D. between the transformer terminal and the case is subdivided into equal parts throughout the entire cross-section of the bushing.

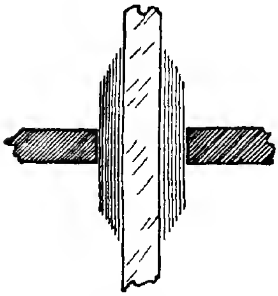


Fig. 41

34. Energy Stored in a Charged Condenser.—Reference has already been made to the fact that condensers are utilized for the purpose of storing or absorbing energy. It therefore becomes

necessary to derive an expression for the energy resident in a charged condenser in terms of its capacitance and the potential to which it is charged.

It has already been shown (eq. 24, Sec. 19) that the work done in charging a body is given by the relation

$$W = \tfrac{1}{2}QV.$$

But by eq. 30,

$$Q = CV;$$

hence

$$W = \tfrac{1}{2}CV^2. \qquad \text{Eq. 47}$$

Assuming then that there has been no loss of energy by leakage, this relation gives us the value of the energy in a condenser of capacitance C when charged to a potential V. It is to be noted *that the energy content varies as the square of the potential.* It should be borne in mind that this energy resides in the medium separating the plates of the condenser, that is, in the electrostatic field.

If C and V are in e.s.u., the energy will be expressed in ergs. If we recall that a farad is equivalent to 9×10^{11} e.s.u. and that a volt represents 1/300 of an e.s.u., one may change eq. 47 so that practical units may be employed. Introducing these factors into the right hand side of our equation, we get

$$\frac{C \times V^2 \times (300)^2}{9 \times 10^{11}} = \frac{CV^2}{10^7}.$$

To preserve equality we must divide W by 10^7. Such a process reduces ergs to joules. Hence we may write

$$W \text{ (joules)} = \frac{1}{2} C \text{ (farads) } V^2 \text{ (volts)}. \qquad \text{Eq. 48}$$

By a similar procedure it may be shown that

$$W \text{ (joules)} = \frac{1}{2} \frac{C \text{ (mf.) } V^2 \text{ (volts)}}{10^6}. \qquad \text{Eq. 49}$$

EXAMPLE.—If a condenser having a capacitance of 0.005 mf. is charged to a potential of 5000 volts, what is the energy content?

Substituting in eq. 49,

$$W = \frac{1}{2}\frac{0.005(5000)^2}{10^6} = 0.0625 \text{ joule.}$$

35. Types of Condensers.—Leyden jars, which were originally utilized as condensers, are now practically obsolete. Modern condensers may be roughly classified under four types, depending upon the character of the dielectric used, viz. mica condensers, paper condensers, oil condensers, and condensers having a gas as a dielectric, commonly air.

Most condensers used as secondary standards of capacitance are made up of alternate layers of very thin metal and carefully tested mica, the whole unit being compressed, and impregnated in vacuo with a suitable non-hydroscopic insulating compound. Mica has a high dielectric strength and also a relatively high constant. By means of a series-parallel arrangement of the units a mica condenser can be assembled which will withstand a potential of 25,000 volts or higher. Condensers built to operate at high pressures are usually made only in capacitance of the order of a few hundredths, or less, of a microfarad.

Condensers consisting of alternate layers of tin foil and paper, commonly made by rolling together long strips of foil and paper, are widely used in telephone and telegraph practice. They are usually made in units of 0.5 to 2 microfarads, and are designed to be operated at comparatively low potentials of a few hundred volts.

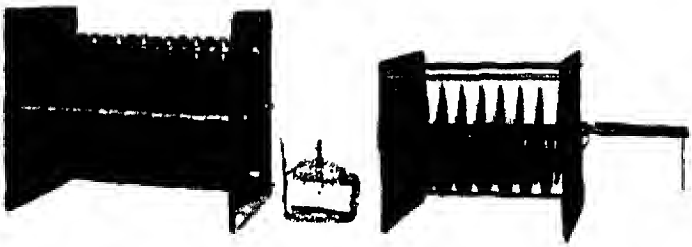


Fig. 42.—Fixed and Variable Condensers

In communication engineering, particularly in radio and guided-wave (Sec. 170) practice, air condensers are extensively employed. One advantage of the air type of condenser is that the capacitance may be readily made variable. Another advantage is that the dielectric is "self-healing" in the event of rupture. Still another advantage will be touched upon later in our discussion. An obvious disadvantage of air condensers is their relatively large size. Air condensers are made in values ranging

from 100 to 500 mmf. Figure 42 shows a fixed and a variable air condenser used in high frequency power work. For purposes of comparison there is included in the same illustration a mica condenser of the same order of capacitance and operating potential.

Variable condensers are of several types, depending upon the shape of the movable plates. If it is desired to have a condenser whose capacitance varies directly as the angular displacement of the movable plates they are made semicircular in form. Figure 43 shows a typical curve for a condenser having semicircular plates.

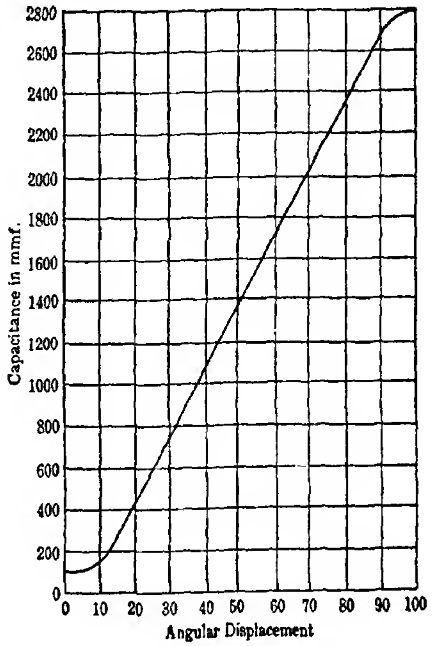


Fig. 43

When a variable condenser forms a part of a wave meter the use of semicircular plates results in a wave length scale which is non-uniform, being crowded too closely at the lower readings. To avoid this it becomes necessary, for reasons which we shall see later, to so design the movable plates that the capacitance shall vary as the square of the displacement.

In order that the effective area between the fixed and movable plates shall vary as the square of the angle of rotation the movable plates must have a shape, the boundary curve of which is given by the equation

$$r = \sqrt{(4a\phi + r_2^2)},$$

where r is the radius of the movable plate, r_2 the radius of the circular area which must be cut from the fixed plate in order to provide clearance for the shaft supporting the movable plates, a being a constant relating ϕ to the capacity. Figure 44 shows the approximate contour of the plates of such a condenser.

In certain high frequency electrical measurements it becomes necessary to have available a type of variable condenser such that a given angular displacement shall produce the same *per cent* change in capacitance at all parts of the scale. This condition will obtain if the boundary curve of the movable plates is given by the equation

$$r_1 = \sqrt{(2C_0\alpha\epsilon^{\alpha\phi} + r_2^2)},$$

where C_0 represents the capacitance when the angular displacement ϕ is zero, α (a constant) the per cent change in capacitance per scale division, and r_2 has the same meaning as in the previous case. A pair of plates conforming to these conditions are shown in Fig. 45. For an authoritative discussion of air condenser design the student is referred to the circular of the Bureau of Standards, No. 74, Section on "Air Condensers."

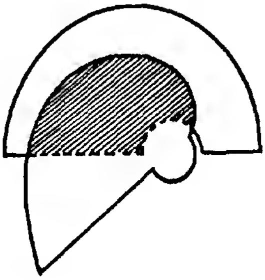


FIG. 44

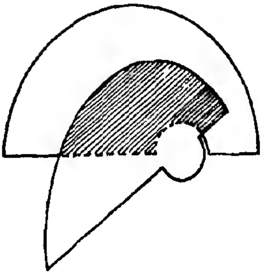


FIG. 45

If a fixed condenser of the air dielectric type is to be subjected to high electric potentials compressed air is sometimes employed; the dielectric strength of air increases with pressure. In both fixed and variable condensers oil may be substituted for air. Both the dielectric strength and the constant of oils are greater than for air; thus capacitance, as well as the potential at which the condenser may be operated, is increased. In addition the oil dielectric is "self-healing" in the event a discharge takes place between the plates. However if the discharges are at all frequent the oil soon becomes partially carbonized and thus becomes a conductor. Pure mineral oil, such as that sold for medical purposes, is suitable for use in condensers. Castor oil is also sometimes used, the latter having a dielectric constant nearly twice that of the mineral oils.

36. Electrolytic Condensers.—A special type of condenser known as the electrolytic also utilizes a gas as the dielectric. Several metals, notably aluminum, tantalum, and magnesium, possess the property, when immersed in an electrolyte, of permitting the current to flow only in one direction. A certain critical potential difference, depending upon the particular electrolyte, must not be exceeded; otherwise conduction will take place. For example with aluminum as the electrode the critical voltage for sodium sulphate is 400. If two electrodes, say of aluminum, are used, practically no current will pass, and we have in effect a condenser. The rectifying effect of such combinations is probably due to a very thin gas layer which forms on the plate or on the oxid or hydroxid layer, which is usually present in such cases. This layer of gas acts as a dielectric, and the metal electrode and the electrolyte function as the two plates of the condenser.

Electrolytic condensers show a higher energy loss than dry condensers even when operated below their critical potential. However, owing to the extremely thin dielectric layer, units of comparatively large capacitance can be arranged in small compass.

37. Energy Losses in Condensers.—It has been pointed out that one of the uses of a condenser is to serve as an energy storage device. When utilized for such a purpose it should be remembered that there are certain ways by which the energy due to the charge may be dissipated.

Due to a defective dielectric actual conduction through the insulating material or along its surface may occur. The author has found that certain of the widely used synthetic dielectric materials may exhibit comparatively poor insulating properties, due apparently to surface leakage. If such material is used as insulating bushings in air condensers serious losses may occur. Quartz is probably best for such purposes, with hard rubber ranking next.

If the potential difference impressed upon the condenser is excessive, ionization (Sec. 139) will occur, resulting in a brush discharge at the edge of the plates, thus giving rise to what is known as corona losses. This can be avoided, to some extent, by the method suggested in Sec. 8.

Another cause of energy loss in a condenser is what is known as "dielectric viscosity" or "dielectric hysteresis." When a poten-

tial difference is impressed upon the plates of a condenser the first rush of charging current is followed by a small but decreasing flow. A measurable length of time is thus required in order to establish a fixed electrical state. The dielectric appears to absorb a part of the charge. A similar phenomenon is manifest when a condenser is discharged, instantaneous discharge being followed by a diminishing movement of the electrons. It is thus apparent that the complete charge and discharge lags somewhat behind the applied potential, hence the term "dielectric hysteresis." It follows also that the charge resident in a condenser is, to some extent, a function of the frequency of the applied P.D. This absorption or dielectric hysteresis results in an energy loss which manifests itself in the form of heat. As the temperature of the dielectric increases not only the hysteresis loss but the other loss tends to increase. It is therefore important in designing a condenser, or in selecting one for a particular use, to make proper allowance for the rate at which the unit will be expected to handle energy without excessive heating. In an air condenser the dielectric loss is absent.

38. Determination of Dielectric Constant.—In Sections 28 and 29 it was shown that the specific inductive capacity or dielectric constant enters as a factor in the calculation of capacitance. We have also seen that this constant must be taken into consideration in several other connections. It is therefore important to be able to determine the value of the dielectric constant for the various insulating materials. Fortunately this involves only a comparatively simple procedure.

In eq. 32 we have a relation giving the capacitance of a parallel plate condenser. Writing this equation in a form to serve our present purposes it would become

$$C_a = \frac{K_a A}{4\pi t}, \tag{i}$$

where C_a is the capacitance, as determined by appropriate methods, of such a condenser with air as the dielectric, and K_a is the constant for air.

Without changing the area (A) of the air condenser plates or their distance apart (t), suppose we completely fill the space between the plates with some of the material under test. Let us now determine the capacitance as before. Equation 32

would now become

$$C_x = \frac{K_x A}{4\pi t}, \tag{ii}$$

where C_x is the capacity of the condenser with the dielectric being studied occupying the space between the plates, and K_x the constant of this medium.

If we now divide eq. (ii) by eq. (i) there results

$$\frac{C_x}{C_a} = \frac{K_x}{K_a}.$$

But K_a is unity; hence

$$K_x = \frac{C_x}{C_a}. \qquad \text{Eq. 50}$$

Therefore we see that the dielectric constant is given by the ratio of the capacitance of a condenser with the test medium as the dielectric to its capacitance with air as the medium. Indeed this ratio frequently serves as a definition of specific inductive capacitance or dielectric constant. Methods of accurately measuring capacitance are dealt with in treatises on electrical measurements.

PROBLEMS

1. What is the total number of lines of e.s. flux emanating from a concentrated positive charge of 5 coulombs?

2. A plate 5 × 10 cm. carries a total charge of 2 coulombs. What is the field strength at a point opposite the center of the plate and distant 1 cm.? What is the potential at the same point? What would be the force acting on a concentrated charge of 0.5 coulomb if placed at the point in question?

3. What is the field intensity in volts per cm. at a distance of 100 cm. from a charged sphere of 10 cm. diameter, the charge on the sphere being 0.0005 coulomb?

4. A potential difference of 100,000 volts exists between two parallel wires which are 100 cm. apart. What is the potential gradient?

5. Three concentrated charges are located on the circumference of a circle at equal distances apart, the radius of the circle being 10 cm. The magnitude of the charges are + 4, + 6, and − 10 e.s.u. respectively. What is the field intensity at the center of the circle? The potential?

6. Two metal plates each 10 × 15 cm. and separated by 2 cm. carry equal and opposite charges. The potential difference between the plates is 10,000 volts. What is the force of attraction in grams weight?

7. Deduce eq. 18 from eq. 22 or 23.

8. Prove that the energy resident in an electrostatic field is equal to

$$\frac{Kf^2}{8\pi},$$

where f is the field intensity.

9. How may six 1-mf. condensers be arranged in order that their combined capacitance shall be 2/3 mf.?

10. A condenser of 350 mmf. is at hand, but it is desired to have available a condenser whose capacitance shall be 200 mmf. What will be the capacitance of the condenser which must be connected in series with the 350 mmf. unit in order to give the required capacitance?

11. An air condenser made of two circular plates and having a capacitance of 0.0002 mf. is to be constructed. The potential difference which will be applied is of such a magnitude that the plates must be at least 1 cm. apart. What should be the diameter of the plates?

12. A condenser having a capacitance of 500 mmf. is connected in parallel with one of 250 mmf. How much work will be done in charging the combination to a potential of 10,000 volts? Express the result in joules.

13. Assuming that the mean diameter of the earth is 7912 miles, what is its capacitance in farads? In microfarads? How much energy would be involved if the earth were charged to a potential of 10,000 volts?

14. It is desired to construct a condenser having an approximate capacitance of 4 mf. The mica available has a thickness of 0.5 mm. and a constant of 6.2. The metal foil to be used is so cut that the sheets will overlap 6 × 8 cm. How many sheets of metal and mica will be required?

15. A telegraph circuit is 100 miles long and consists of two wires spaced 15 inches apart supported on poles at a mean height of 18 ft. The wires are No. 12 B. & S. What is the capacitance of the two wires with respect to one another? What would be the capacitance of one of these wires with respect to the earth?

CHAPTER VI

CURRENT, ELECTROMOTIVE FORCE, AND RESISTANCE

39. The Electric Current and Electromotive Force.—Thus far in our study we have considered the case of charges at rest. We are now to examine the phenomena which are manifest when charges are in motion. We shall also consider the laws which obtain in such cases.

One or more electrons in orderly motion may be thought of as constituting what we term an electric current, and the time rate at which the electrons or group of electrons pass some reference point determines what is known as the *current strength.* This may be represented by the relation

$$I = \frac{Q}{t}, \qquad \text{Eq. 51}$$

where I is the current strength, Q the quantity of charge, and t the time. It follows then that the total quantity which passes any given point will be given by the product of the current strength and the time. It should also be noted that the current has the same value at all points in any simple conductor along which the electronic movement may be taking place.

If, in eq. 51, Q be measured in e.s.u. and t in seconds, the current strength I will be in e.s.u. However, as we shall see later, the electrostatic unit of current is not of such a magnitude as to be convenient for practical use; hence another unit has been adopted which is known as the ampere, and which, for the time being, may be defined as the equivalent to 3×10^9 e.s.u. Hence, if Q is in coulombs and t in seconds, I will be in amperes.

This orderly movement of electrons may take place in a solid, a liquid, a gas, and even in a region free from gas molecules. For the present we will confine our attention to the case of electronic movements or conduction in solids.

However, before entering into a discussion of this important subject it will be well to account for the electronic movement to which reference has just been made. In mechanics, any motion involves a force and likewise in the realm of electricity some type

or form of force must obtain if charges are to be maintained in a state of motion. Such a driving force may be brought into being in various ways, but regardless of the nature of its genesis it is designated by the term *electromotive force.* For purposes of reference we may say that electromotive force, commonly abbreviated E.M.F., *is that which moves or tends to move electrons from one place to another.* In due season we shall investigate the chemical, mechanical, and other means whereby electromotive force may be established. For the present we need only note the fact that such a force may exist and proceed to study its relation to what we have designated as current strength.

40. Ohm's Law.—The relation most frequently employed in dealing with the electric current was disclosed by G. S. Ohm, a German mathematician-physicist, in a paper published in 1826. Dr. Ohm found that in solids the magnitude of the current is strictly proportional to the electromotive force. Another way of stating the same fact is to say that the ratio of the E.M.F. to the current is constant, or

$$\frac{\text{E.M.F.}}{I} = \text{a constant.}$$

This constant is known as the resistance of the conductor, and is commonly designated by R. This important law may then be written

$$\frac{\text{E.M.F.}}{I} = R. \qquad \text{Eq. 52}$$

For many purposes the readily obtainable forms

$$\text{E.M.F.} = RI, \qquad \text{Eq. 53}$$

and

$$I = \frac{\text{E.M.F.}}{R}, \qquad \text{Eq. 54}$$

will be found useful. For brevity we shall usually write E for E.M.F. The foregoing relation, known as Ohm's law, is applicable only to those cases in which the E.M.F. has a fixed value and direction. Where the E.M.F. varies in magnitude and direction the law must be modified. There are certain notable exceptions, even in the case of a constant E.M.F., where the law does not hold. We shall examine these cases in a later chapter.

41. Resistance.—We have seen that the ratio $\frac{E}{I}$ defines what we have been pleased to call resistance, *a quantity which indicates the property of a conductor by virtue of which it opposes the movement of electrons.* The inverse ratio, $\frac{I}{E}$, or the reciprocal of resistance, $\frac{1}{R}$, is frequently referred to as *conductance.*

If in Ohm's law E.M.F. and I are expressed in electrostatic units, resistance will be also expressed in e.s.u. Changing the quantities in the ratio $\frac{E}{I}$ to volts and amperes respectively we would have $\frac{E}{I} \times 9 \times 10^{11}$. In order to preserve equality in the expression for Ohm's law it would therefore be necessary to multiply R by the factor 9×10^{11}. This means that we would have a new unit of resistance whose value would be $\frac{1}{9 \times 10^{11}}$ of an e.s.u. This unit is known as the ohm, and *a conductor is said to have a resistance of an ohm when an E.M.F. of one volt applied at its terminals will give rise to a current of one ampere.* By international agreement arrived at in 1908 there was established what is known as the *international ohm.* This standard unit is the resistance of a column of mercury of uniform cross-section at 0° C. and having a length of 106.300 cm. and a mass of 14.4521 grams.

In the course of his studies Ohm found that the resistance of a conductor is a function of its length and its cross-sectional area. Expressed mathematically this fact may be written

$$R = \rho \frac{L}{A}, \qquad \text{Eq. 55}$$

where ρ is proportionately constant depending upon the nature of the material involved. This constant is known as the *specific resistance.* If we make L and A unity in eq. 55, we have a definition of specific resistance, namely, the resistance of a conductor whose length is 1 cm. and whose cross-section is 1 cm.^{2}. In engineering work the foot is taken as the unit of length and the

circular mil * as the unit of cross-sectional area, and such a portion of a conductor is referred to as a mil-foot. The specific resistance, ρ, is therefore expressed in two different ways, depending upon the units employed. Specific resistance is frequently spoken of as *resistivity*.

In dealing with resistivity, it must be borne in mind that there are a number of factors which tend to modify the value of the specific resistance. One of these factors is temperature. For ordinary temperatures the relation which obtains between resistance and temperature may be expressed by the equation

$$R_t = R_0(1 \pm at), \qquad \text{Eq. 56}$$

where R_t is the resistance of the conductor at a temperature t, R_0 its resistance at 0° C., and a the temperature coefficient. By temperature coefficient is meant the change in resistance brought about per unit of resistance per degree of temperature. For wider ranges of temperature the relation

$$R_t = R_0(1 + at + bt^2) \qquad \text{Eq. 57}$$

will give more accurate results, where b is a second constant which may be determined by three temperature resistance measurements.

Both of the above laws however fail at extreme temperatures. For instance, it has been found that at the temperature of liquid helium (− 267° C.) the resistance of certain pure metals is extremely low; in fact much lower than might be expected from the foregoing relations. It has been found, for example, that a current set up in a ring of pure lead which is maintained at the temperature of liquid helium will continue for a period of time measured in hours, and requires several days to drop to half value.

It would thus appear that at the temperature approaching absolute zero the ordinary resistance-temperature law does not hold. Recently a considerable amount of work has been done on the resistance of certain metals in a crystalline form, and while it is too early to generalize from the results at hand, the data thus

* A mil is one-thousandth of an inch. A circle whose diameter is one mil has an area of one circular mil. Because of the fact that the area of any circle when expressed in circular mils is numerically equal to the square of the diameter of the circle in mils this method of expressing areas is found to be convenient in practical engineering.

far uncovered would appear to indicate that some interesting and useful facts in regard to metallic conduction will be brought to light as a result of this study.

Temperature, however, is not the only factor which has a bearing on the value of the resistance of a conductor. For instance, there are several elements, the most notable example being selenium, which change resistance when subjected to radiation, particularly in the visible part of the spectrum. When strongly illuminated, selenium has a much lower resistance than when in the dark. This property is taken advantage of in the design of various photometric devices and also in connection with certain electrical relays. Later we shall also note that resistance may be modified by one or more other factors.

Before leaving the subject of resistance it might be well to point out that most conductors show an increase in resistance with increase in temperature. There are, however, one or two notable examples where the change in resistance is negative, the

Resistivity and Temperature Coefficient *

Material	Composition or Condition	Resistivity		Temperature Coefficient, in Ohm per Ohm per Degree C.
		Microhm Cm.	Ohms Mil-ft	
Advance	Cu-Ni	48.8	294	+0.000020
Aluminum	Wire, annealed	2.74	16.5	+0.0039
Copper	" "	1.724	10.4	+0.0040
Carbon	Arc lamp	0.0005	—	−0.0003
Constantin	Copper (60%) Nickel (40%)	50.0	300	−0.0001
Climax	Nickel-Steel	87.2	525	+0.00054
German Silver	Cu-Ni(30%)Zn	48.2	290	+0.00020
Gold	Pure	2.06	12.42	+0.0037
Iron	Pure-ann.	9.96	59.9	+0.0045
Ia Ia	Cu-Ni	49.0	295	−0.000005
Manganin	Cu-Mn-Ni	41.4 to 73.8	249 to 443	±0.000011 to ±0.000039
Mercury	at 0° C.	94.3	—	+0.00088
Nickel	Ni	10.67	64.3	+0.004–6
Nichrome I	Ni-Cr	99.6	600	+0.00044
Nichrome II	Ni-Cr	109.5	660	+0.00016
Platinum	Pt at 0° C.	8.98	54.0	+0.0038
Silver	Ag at 0° C.	1.5–1.7	9–10.2	+0.0040
Tungsten	W at 0° C.	7.0	42.0	+0.0039

* Values are for 20° C. unless otherwise specified.

most conspicuous example being that of carbon. On page 71 is a table giving the resistivity and temperature coefficients of a number of elements and alloys.

An examination of the foregoing table will disclose the fact that it is possible to produce alloys having an extremely low temperature coefficient, a very desirable feature in wire which is to be used in the manufacture of standard resistance units. Manganin, an alloy of copper, manganese and nickel, is widely used in making resistance coils which are to be used for resistance measurements. An alloy extensively used in heating devices is known as nichrome, having a composition and constants as indicated in the table. Other alloys such as Ia Ia and constantin, also alloys of copper and nickel, find wide commercial use in rheostats and other similar devices.

42. Difference of Potential as Applied to Circuits.—It has been previously pointed out (Sec. 14) that the difference of potential between two points is numerically equal to the work done in transferring unit charge between the two points. If this transfer of electricity takes place along a conductor the energy thus expended will be manifest in the form of heat, and the work thus transformed into heat in unit time, when unit current flows, is to be thought of as a measure of the *potential difference* between the ends of the conductor. The magnitude of this energy dissipation is independent of the direction of the electronic movement. It follows then that, if a current flows, there must be a difference of potential between any two points in the conductor, or, to put the case another way, there is a *fall of potential* along a conductor carrying a current. This is a highly important concept and the student should fix the idea clearly in mind.

Further, since a current always flows in a complete circuit, and since the potential *cannot fall throughout the entire circuit,* it is obvious that there must be some means whereby the potential may be stepped up at some point or points in the circuit. This naturally involves a *source of energy* at such point or points, as for example a battery or a dynamo. In other words a source of E.M.F. must obtain. Numerically, the time rate at which the source supplies energy to the circuit system as a whole, when unit current is flowing, gives the magnitude of what we have called the electromotive force. We may express this fact thus:

$$\text{work done} = (\text{E.M.F.})\, It$$

or

$$\frac{\text{work done}}{t} = (\text{E.M.F.})\, I.$$

If I = unity, then

$$\frac{\text{work done}}{\text{time}} = \text{E.M.F.} \qquad \text{Eq. 58}$$

The distinction between E.M.F. and P.D. should never be lost sight of. The term E.M.F. is restricted to the source of the energy in the circuit. Differences of potential are developed in the various parts of the circuit as a result of the movement of the electrons (current flow). An E.M.F. may exist even when the circuit is open and hence no current flowing, but a *P.D. exists only when a current flows in the conductor.*

Further, it should be noted that the *direction of the E.M.F. is fixed by the nature of the source*, while a P.D. has a direction which depends upon the direction of the current in that particular part of the circuit under consideration.

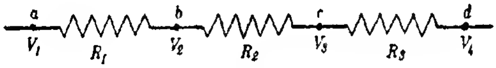


FIG. 46

43. Resistances in Series and Parallel.—A circuit may be made up of several components or parts connected in series as shown in Fig. 46. Suppose current flowing in the circuit from a toward d.* The value of this current will be the same at any point between a and d; but there will be a fall in potential as we pass from a to d Applying Ohm's law we may then write

$$V_1 - V_2 = R_1 I,$$
$$V_2 - V_3 = R_2 I,$$
$$V_3 - V_4 = R_3 I.$$

Adding the above three equations, we have

$$V_1 - V_4 = (R_1 + R_2 + R_3) I,$$

or

$$\frac{V_1 - V_4}{I} = R_1 + R_2 + R_3 = R, \qquad \text{Eq. 59}$$

* In dealing with this and related problems it should be borne in mind that Ohm's law applies not only to a circuit as a whole but also to each component part.

where R is the total resistance between a and d, and $V_1 - V_4$ is the total drop (fall in potential) over the combined resistances. Hence it may be said that the total resistance of several resistances in series is equal to the sum of the individual resistances.

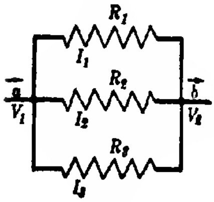


FIG. 47

A second arrangement of resistance components is sketched in Fig. 47, three resistances being taken as typical. Such a network is spoken of as a parallel, or multiple, arrangement of resistances. If a current I flows in the main circuit it will divide between the points a and b, a part flowing through each resistance. Let R be the combined resistance, between a and b, of the several conductors. Since the fall in potential will be the same between a and b *along each branch*, we have, by Ohm's law,

$$I_1 = \frac{V_1 - V_2}{R_1},$$

$$I_2 = \frac{V_1 - V_2}{R_2},$$

$$I_3 = \frac{V_1 - V_2}{R_3},$$

etc., for any number of conductors. Now

$$I = \frac{V_1 - V_2}{R},$$

and since the sum of the current in the several branches must equal the current in the main circuit we may write

$$\frac{V_1 - V_2}{R} = \frac{V_1 - V_2}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_1 - V_2}{R_3}, \text{ etc.},$$

or

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}, \text{ etc.} \qquad \text{Eq. 60}$$

In words this means that the reciprocal of the total or combined resistance, in such cases, equals the sum of the reciprocals of the resistances of the individual branches, or the total conductance equals the sum of the conductances of the individual branches. For three branches, as illustrated, the above relation reduces to

the more convenient form

$$R = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_1R_3}. \qquad \text{Eq. 61}$$

In the case of two branches this becomes

$$R = \frac{R_1R_2}{R_1 + R_2}. \qquad \text{Eq. 62}$$

Again referring to Fig. 47, and considering for our present purposes that there are only two branches instead of three as shown, we have a case frequently encountered in practice. For instance a current by-pass or "shunt" may be connected about the internal winding of a current-indicating instrument such as a galvanometer or ammeter. The question arises as to the value of the current in each branch.

As outlined above,

$$I_1 = \frac{V_1 - V_2}{R_1},$$

$$I_2 = \frac{V_1 - V_2}{R_2},$$

or

$$I_1R_1 = V_1 - V_2 = I_2R_2.$$

Therefore

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}, \qquad \text{Eq. 63}$$

which shows that the current will divide inversely as the resistance of the individual branches. Using this equation and the relation $I = I_1 + I_2$, it thus becomes possible in any given case to compute the current in either the main or shunt circuit.

For instance, suppose we have a galvanometer whose resistance is 80 ohms and it is desired to arrange a shunt in parallel with the instrument winding so that only one-tenth of the total current shall pass through the galvanometer.

Adapting eq. 63 to this case we have

$$\frac{I_g}{I_s} = \frac{R_s}{R_g},$$

where the subscripts g and s indicate galvanometer and shunt respectively. Since the galvanometer winding is to carry one-

tenth of the total current the ratio of the galvanometer current to the current in the shunt, that is, $\frac{I_g}{I_s}$, will be $\frac{1}{9}$, and hence we have

$$\frac{1}{9} = \frac{R_s}{80},$$

or

$$R_s = 8.9 \text{ ohms}$$

as the resistance of the shunt.

44. Kirchhoff's Laws.—Two useful corollaries, known as Kirchhoff's laws or rules, may be deduced from Ohm's law. The first of these generalizations is to the effect that in any given network where several conductors meet at a common point, the sum of the currents flowing toward the point is equal to the sum of the currents flowing away from the point. In other words, *the algebraic sum of the currents entering and leaving a common point is zero.* It is the convention to use the positive algebraic sign for the currents flowing toward the point and the negative sign for those which flow away. In the case illustrated by Fig. 48 we may write

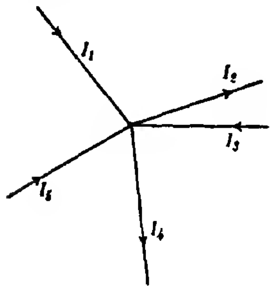


Fig. 48

$$I_1 - I_2 + I_3 - I_4 + I_5 = 0.$$

This law follows from the fact that, in the case of a constant current in a circuit of negligible capacitance, the charges do not accumulate at any one point.

Kirchhoff's second law is a statement of the fact that, in any closed circuit, the algebraic sum of the products of the resistances of, and the currents in, the several parts of the circuit is equal to the sum of the E.M.F.'s acting in that particular path. The application of this law may be illustrated by reference to Fig. 49. Let the resistance of the connecting wire $ab = R_1$, and that of $cd = R_4$, the other resistances and currents being as shown. In the loop consisting of R_1, R_3 and R_4

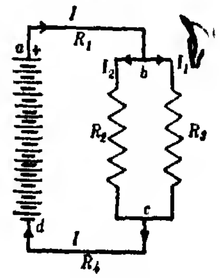


Fig. 49

$$E = IR_1 + I_1R_3 + IR_4.$$

In the loop R_1, R_2 and R_4

$$E = IR_1 + I_2R_2 + IR_4.$$

In the loop consisting of R_2 and R_3

$$0 = I_2R_2 - I_1R_3.$$

By the first law, considering say the point b,

$$I - I_1 - I_2 = 0.$$

It would be possible, for example, by the aid of the four equations just deduced to determine three unknown quantities.

Kirchhoff's laws are among the most useful generalizations available in electrical study. By the aid of these and Ohm's laws it is possible to solve any problem involving a direct current network, no matter how complicated. Careful attention must however be given to the algebraic signs of the terms representing the currents and the corresponding "drops." If, in first analyzing a given problem, the direction of some particular current is not known one may assume it to flow in a definite direction. If the numerical result shows a positive algebraic sign our assumption was correct; if a negative sign, it means that the actual direction of the current is the reverse of that which we assumed.

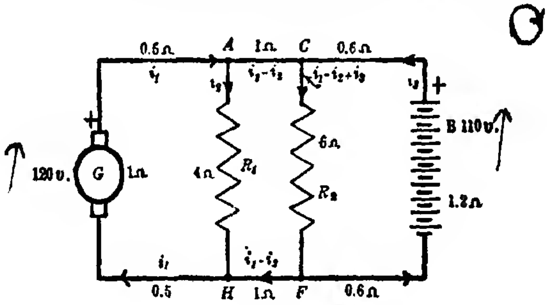


FIG. 50

The utility of Kirchhoff's laws may be illustrated by applying them to the solution of the problem involved in a circuit organization frequently met with in practice, viz., the operation of a storage battery in parallel with a dynamo to supply a common load. Such a network is shown in Fig. 50, where R_1 and R_2 represent load resistances.

Suppose that it is required to find the current delivered to the network by the dynamo and by the storage battery; also to find

the current through R_1 and R_2. First assume that the currents in the several branches flow as indicated in the figure. Applying Kirchhoff's second law to the loop GAH, we have

$$120 = 0.5i_1 + 4i_2 + 0.5i_1 + i_1,$$

which yields

$$i_1 = 60 - 2i_2. \tag{1}$$

Next considering the loop $GACFH$ we may write

$$120 = i_1 + 0.5i_1 + 1(i_1 - i_2) + 6(i_1 - i_2 + i_3) + 1(i_1 - i_2) + 0.5i_1.$$

Reducing, we get

$$120 = 10i_1 - 8i_2 + 6i_3. \tag{2}$$

Eliminating i_1 by combining (1) and (2) we have

$$240 = 14i_2 - 3i_3. \tag{3}$$

By applying the second law to the loop BCF,

$$110 = 1.2i_3 + 0.6i_3 + 6(i_1 - i_2 + i_3) + 0.6i_3,$$

which reduces to

$$110 = 8.4i_3 + 6i_1 - 6i_2. \tag{4}$$

Eliminating i_1 by combining (1) and (4),

$$-125 = 4.2i_3 - 9i_2. \tag{5}$$

We may now solve for i_3 by means of (3) and (5), which gives

$$i_3 = 12.89 \text{ amperes},$$

which is the current supplied by the storage battery to the load network.

The value of i_2 may be found by substituting the value of i_3 in (3) thus,

$$240 = 14i_2 - 3i_3,$$
$$i_2 = 19.9 \text{ amperes},$$

which is the current through the load resistance R_1.

Using the value of i_2 in (1) we find

$$i_1 = 20.2 \text{ amperes},$$

which is the current supplied by the generator to the load circuit.

The value of the current through R_2 will be given by applying the first law, which will yield

$$i_1 - i_2 + i_3 = 13.19 \text{ amperes}.$$

CHAPTER VII

FUNDAMENTAL MEASUREMENTS

45. The Wheatstone Bridge.—The determination of resistance values is probably the most important electrical measurement that one is called upon to make, and with suitable equipment it is easily possible to make measurements of this character to an accuracy better than one one-hundredth of one per cent.

The most widely used method of measuring resistance was devised by Wheatstone and consists of a network made up essentially of four conductors as shown in Fig. 51. In the diagram G represents a current-detecting device, such as a galvanometer; B is a source of E.M.F. Keys, K_1 and K_2, are provided for closing the galvanometer and battery circuits as shown.

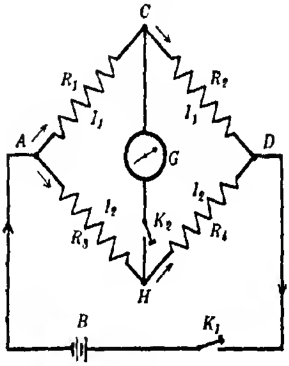


FIG. 51

Suppose a difference of potential is applied between the points A and D of the network. Current will flow along the paths ACD and AHD. For a given P.D. between A and D the magnitude of the current I_1 which will flow through the branch ACD will depend upon the resistances R_1 and R_2. Likewise the current I_2 will depend upon the value of R_3 and R_4. The potential drop over AC will be given by I_1R_1 and likewise for AH by I_2R_3. Now if and when the several resistances comprising the network are so adjusted in value that no current passes through the galvanometer G, it follows that the points C and H are at the same potential, or

$$I_1R_1 = I_2R_3.$$

When this condition obtains it also follows that

$$I_1R_2 = I_2R_4.$$

Dividing one of these equations by the other and simplifying,

we have

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$ Eq. 64

If, then, three of these resistances are known, the fourth can be readily determined.

In practice there are two common forms of the Wheatstone bridge. One, known as the slide wire type, consists of a resistance wire of uniform cross-section, corresponding to R_3 and R_4, on which rests a sliding contact corresponding to H in Fig. 51. By this arrangement it is possible to vary the ratio of R_3 to R_4. Since the resistance constituting R_3 and R_4 is of uniform cross-section, and since resistance is proportional to length, length may be substituted for resistance in eq. 64, thus giving

$$\frac{R_1}{R_2} = \frac{L_1}{L_2},$$ Eq. 65

where L_1 and L_2 correspond to R_3 and R_4 respectively. The resistance R_1 may be the unknown and R_2 a standard resistance coil or set of such coils, usually in the form of a so-called resistance box. Figure 52 is a diagrammatic sketch of an assembly of this type. The resistance wire is commonly a meter in length and is stretched over a meter stick. Heavy copper bars of negligible resistance serve to connect the ends of the slide wire to the openings provided to receive the standard and unknown resistances R_s and R_x. In practice provision is made for conveniently interchanging the standard and unknown resistances, thus eliminating, in part, the effect of contact resistance between these resistances and the connecting bars.

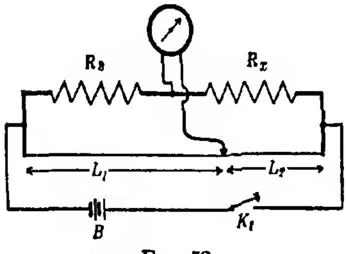
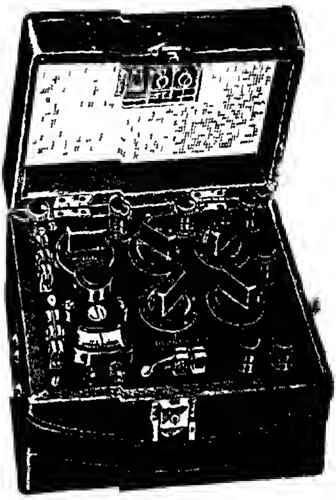


FIG. 52

A second form of the Wheatstone bridge finds wide application in engineering practice. Because it was originally made for use in the English * postal service it is known as the "post office" box or bridge. In this form of the bridge the resistances R_3 and R_4 consist of a set of "ratio coils" of fixed values. By means of

* In England the telegraph and telephone service is operated by the government post office department.

contact plugs or a wiping switch decimal ratios from 1 to 100 and from 100 to 1 may be cut into the bridge circuit. The resistance R_2, for instance, consists of a set of accurately made resistance coils having values which commonly range from 1 to 10,000 ohms. Any one or a combination of these coils may be introduced into the bridge network by means of contact plugs or a rotating switch. It thus becomes possible to measure resistance over a wide range of values. In the portable form of the post office bridge, or "testing set" as it is sometimes called, a portable galvanometer and a dry battery are housed in the same case as the resistance coils. A portable unit of this type is shown in Fig. 53.



(Courtesy Leeds and Northrup Co.)

FIG. 53.—PORTABLE WHEATSTONE BRIDGE OR TESTING SET

46. Carey-Foster Bridge.—Another and somewhat more accurate method of measuring resistances, particularly slight changes in resistance, was devised by Professor Carey-Foster. This arrangement consists of a network made up essentially of six resistances, one of which is a standard resistance of high accuracy. From the following description it will be apparent that the Carey-Foster method makes it possible to determine *the difference between two resistances*, one of which is of standard value, to a very high degree of accuracy.

The unique feature of the Carey-Foster modification of the more simple Wheatstone bridge consists in the interchanging of the standard resistance and the resistance under test in such a manner as to eliminate the resistance of the connecting bars.

Referring to Fig. 54, S_1 and S_2 are the two resistances being compared; R_1 and R_2 are two nearly equal auxiliary resistances. The exact value of these auxiliary resistances need not be known as they disappear from our final relation. Let r_1 and r_2 be the resistances of the connecting strips MN and $M'N'$ respectively, and ρ be the resistance of unit length of the bridge wire. NN' is

uniform resistance wire similar to that incorporated in the Wheatstone bridge. It is to be noted that the resistances represented by S_1, r_1 and a constitute one "arm" of the bridge, while S_2, r_2, and b together form the other corresponding arm. In practice a standard resistance unit, say S_1, is selected which has a value very nearly that of the unknown, S_2. The galvanometer contact p is then adjusted until zero deflection obtains, and the

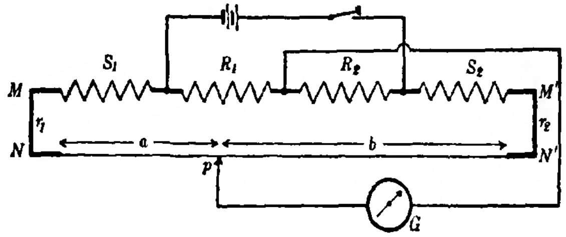


FIG. 54

readings a and b noted. By means of a convenient switching arrangement the standard and unknown resistances are then interchanged and another "balance" secured. In general p will now show a different setting, and the resistance of that portion of the bridge wire between the first and second settings will equal the difference between the values of the standard and unknown resistances.

For the first setting we may write

$$\frac{R_1}{R_2} = \frac{S_1 + r_1 + \rho a_1}{S_2 + r_2 + \rho b_1}, \tag{i}$$

where a_1 and b_1 represent the lengths of the bridge wire to the left and right of the contact point p.

When S_1 and S_2 are interchanged, if a_2 and b_2 be the readings on the bridge wire for the new balance, we have

$$\frac{R_1}{R_2} = \frac{S_2 + r_1 + \rho a_2}{S_1 + r_2 + \rho b_2}. \tag{ii}$$

It is desired to eliminate the factors R_1, R_2, r_1, and r_2 from our equations. To do this we may add unity to both sides of eqs. (i) and (ii). This gives

$$\frac{R_1 + R_2}{R_2} = \frac{S_1 + r_1 + \rho a_1 + (S_2 + r_2 + \rho b_1)}{S_2 + r_2 + \rho b_1}$$

and

$$\frac{R_1 + R_2}{R_2} = \frac{S_2 + r_1 + \rho a_2 + (S_1 + r_2 + \rho b_2)}{S_1 + r_2 + \rho b_2}.$$

It is obvious that the right sides of the above two equations constitute an identity.

Further, since changing the position of the galvanometer contact, p, does not alter the total resistance of the bridge, it follows that $a_1 + b_1 = a_2 + b_2$. By rearranging terms, and utilizing the above relation, it will be evident that the numerators of the above fractions are equal, and hence

$$S_1 + r_2 + \rho b_2 = S_2 + r_2 + \rho b_1,$$

or

$$S_1 - S_2 = \rho(b_1 - b_2) = \rho(a_2 - a_1). \qquad \text{Eq. 66}$$

Thus we see, as previously indicated, that the difference between the resistance of the two coils S_1 and S_2 is equal to the resistance of that part of the bridge wire between the points at which the contact p is set to secure the balances for the two positions of the resistances.

It will be observed that our final relation above involves ρ, the resistance of unit length of the bridge wire. This constant must of course be known or determined in advance. In practice a Carey-Foster bridge has several wires of different resistance. The method to be followed in determining the value of ρ for each wire will depend to some extent upon the resistance of the particular wire being calibrated.

The last equation above may be written in the form

$$\rho = \frac{S_1 - S_2}{a_2 - a_1}. \qquad \text{Eq. 67}$$

Hence if the difference in the resistance of the two coils S_1 and S_2 is known, ρ can be found by determining two successive balances. If the total resistance of a given bridge wire be of the order of 1/10 ohm, S_1 may be one 1-ohm standard and S_2 may consist of a 10-ohm and a 1-ohm standard unit in parallel. This would give a value of 10/11 ohm for S_2; hence $S_1 - S_2$ would be 1/11 or 0.09091 ohm.

If the resistance of a bridge wire be of the order of 1/100 ohm a 1-ohm coil may be used on one side and a 1-ohm and 100-ohm

unit in parallel on the other. This would give for $S_1 - S_2$ a value of 0.009901 ohm.

If the entire bridge wire be considerably in excess of 1 ohm, then ρ may be found by the aid of a single standard resistance unit and a heavy copper bar or link, the resistance of the latter being negligible, in which case

$$\rho = \frac{1}{a_2 - a_1}.$$

Standard resistances are made of manganin wire, the coils being carefully "aged" after winding. The coil itself is inclosed in a metal case containing oil. Heavy copper leads are provided for making connections to the bridge.

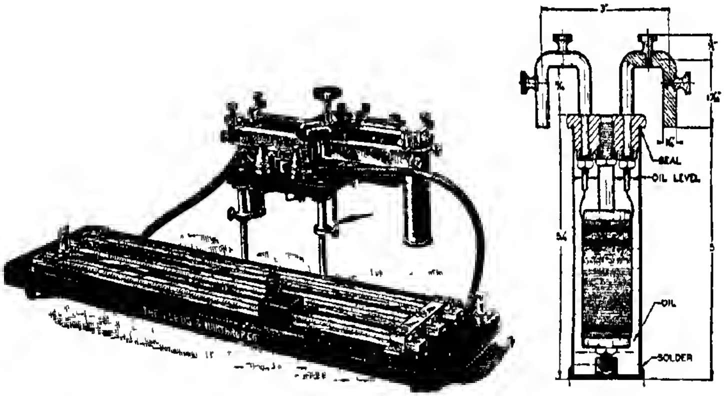


Fig. 55

Figure 55 shows one form of the Carey-Foster bridge and a typical standard resistance unit.

One important use of the Carey-Foster network is in the determination of the temperature coefficient of any conductor. If we take a reading of the resistance of a sample at a given temperature, t_1, we have (eq. 56)

$$R_1 = R_0(1 + at_1).$$

At some other temperature its resistance will be

$$R_2 = R_0(1 + at_2),$$

which leads to

$$\frac{R_1}{R_2} = \frac{(1 + at_1)}{(1 + at_2)}.$$

This reduces to

$$a = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}. \qquad \textbf{Eq. 68}$$

It is thus possible to find the temperature coefficient a without making one of the observations at the temperature of melting ice. An oil bath is usually employed in determining the resistance at the temperatures t_1 and t_2.

47. Kelvin Double Bridge.—For accurately measuring resistances of very low value (of the order of 0.0001 ohm) a method due to Lord Kelvin is superior to the Carey-Foster arrangement, and is widely used in commercial and research laboratory practice. It is particularly useful in the accurate determination of the resistance (or the conductivity) of samples of conductors of relatively large cross-section, such as heavy current conductors of copper and aluminum. The method also affords a convenient and accurate means of determining the temperature coefficient of such types of conductors.

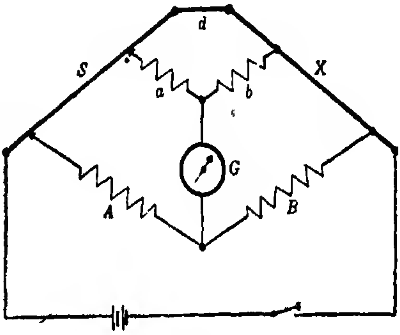


FIG. 56

Figure 56 is a diagrammatic sketch of the connections in the Kelvin bridge. It will be noted that the Kelvin network differs from the ordinary Wheatstone circuit in the addition of two auxiliary resistances a and b, which are in series and in shunt to the connector or "yoke" d. The purpose of these coils is to assist in eliminating the resistance of this connecting strip from the final considerations. All four of the resistances a, b, A and B are of relatively high value and are known as the ratio coils. Owing to the fact that there are two pairs of these ratio coils, the network is sometimes called a "double bridge."

When there is no current through the galvanometer the following relation obtains:

$$\frac{S}{X} = \frac{A}{B} + \frac{d}{X}\left(\frac{b}{a + b + d}\right)\left(\frac{A}{B} - \frac{a}{b}\right).$$

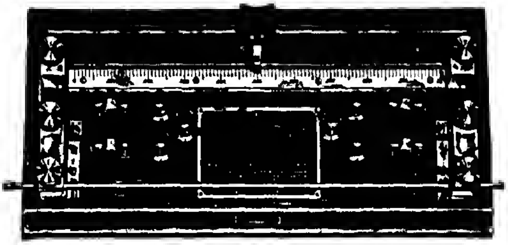
If conditions are so arranged that

$$\frac{A}{B} = \frac{a}{b},$$

the above equation reduces to

$$\frac{S}{X} = \frac{A}{B}, \qquad \text{Eq. 69}$$

and thus, if the values of A and B are made large, the resistance of d, as well as that of the lead wires and contacts, is eliminated.* Figure 57 shows a common laboratory form of this type of bridge.



(Courtesy Leeds and Northrup Co.)

FIG. 57.—KELVIN BRIDGE

48. Resistance Thermometry.—Reference to the table in Section 41 will disclose the fact that certain substances exhibit a relatively high temperature coefficient, and in one or two cases the change in resistance follows any change in temperature very accurately. This is particularly true of platinum and nickel, especially the former. Advantage is taken of this property in the construction of what are known as resistance thermometers.

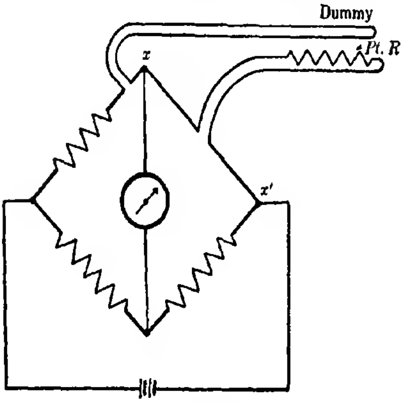


FIG. 58

Figure 58 shows the electrical network of such an organization. One of two plans may be followed in order to indicate the tem-

* For a clear and rigorous development of the fundamental equation of the Kelvin bridge the student is referred to *Methods of Measuring Electrical Resistance* by Northrup.

perature of the sensitive arm of the bridge. A variable resistance may form the conjugate arm of the bridge, and be used to restore the balance as the resistance of the arm xx' changes. This variable resistance may be calibrated in degrees of temperature. The device thus becomes direct reading.

Another possibility consists in having the resistance of the three non-sensitive arms of the bridge of fixed value and employing a deflection method. As the resistance of the arm xx' changes, the galvanometer will show a deflection as a result of the unbalance of the bridge. The galvanometer can thus be calibrated to read directly in degrees of temperature.

Dummy leads are inserted in the conjugate arm and of the same length as those which serve to connect the platinum resistance, and are housed in the same casing. This is to compensate for any change in the resistance of the thermometer leads.

The housing of the resistance wire takes on various forms, depending upon the use to which the equipment is to be put. For instance when the temperature of molten metal is to be observed the resistance element is inclosed in a porcelain case or jacket. The resistance type of thermometer is very widely used in industrial work as well as in research laboratories. Readings of temperature may be easily made by such means to one one-hundredth of a degree.

49. Measurement of E.M.F.—The values of electromotive forces and differences of potential are determined by comparison with a standard cell.* There are a number of possible methods of procedure, but the most accurate and widely used method is that originally devised by the German physicist Poggendorff and known as the *opposition* or *potentiometer method.* In this method the E.M.F. of a standard cell is balanced against an equal difference of potential due to the fall in potential along a conductor.

In its simplest form the potentiometer consists of a resistance wire of uniform cross-section through which flows a constant current. Means are provided for determining when the drop over a portion of this resistance is exactly equal to the E.M.F. of the standard cell. Figure 59 shows the circuit lay-out for this

* A standard cell giving rise to a definite and fixed E.M.F. is made up according to specifications laid down by the A.I.E.E. A detailed description of such a cell is given in Sec. 65.

simple form. B is a storage battery for supplying a steady current to the resistance wire AD; R is a rheostat used to control this current; E_s the standard cell (or unknown E.M.F.); G a galvanometer, and R' a protective resistance. With a current flowing in AD of such a value that the drop between A and some

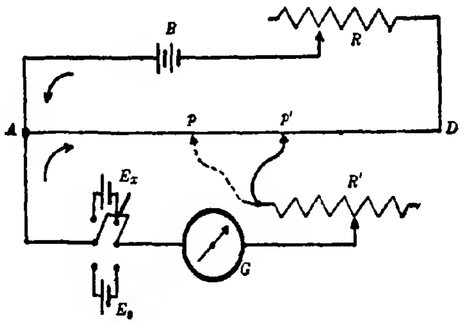


Fig. 59

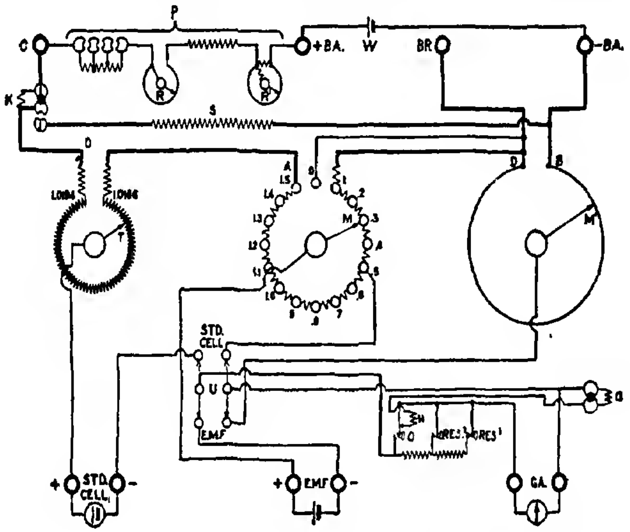
point p, near the middle of the resistance, will equal the E.M.F. of the standard cell, the contact p is adjusted so that the galvanometer shows no deflection. When this condition obtains the drop between A and p exactly equals the E.M.F. of the standard cell. As the balance is approached, R' is diminished in value. Its *presence prevents excessive current from the standard* cell. Having secured a balance with the standard cell, the unknown E.M.F. is then substituted for the standard cell and the system again balanced. In general the contact will fall at some point p'. Since the resistance of the potentiometer wire is proportional to length, and since the current was held constant, we may write

$$\frac{Ap}{Ap'} = \frac{E_s}{E_x}. \qquad \text{Eq. 70}$$

When convenient means are provided for quickly interchanging the sources of known and unknown E.M.F.'s the method is both elegant and rapid.

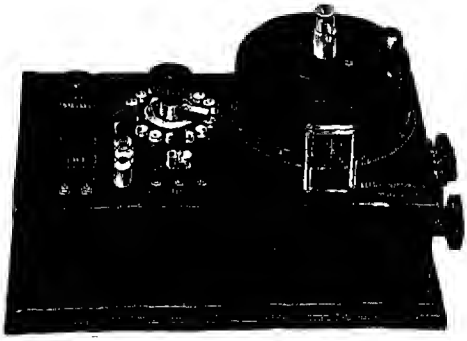
One of the several widely used practical forms of the potentiometer is that made by the Leeds and Northrup Company and commonly known as the Type K Potentiometer. The wiring diagram is shown in Fig. 60 and a view of the instrument itself is shown in Fig. 61.

In this form of the potentiometer, the resistance, corresponding to *AD* in the simple type, is made up of a series of resistance units differing in value by a fixed amount, and a slide wire resistance



(Courtesy Leeds and Northrup Co.)
FIG. 60

(*DB*) in series. When the current through the resistance units is adjusted to a certain value the drop over the several component resistance units is of such a value that the E.M.F. readings are

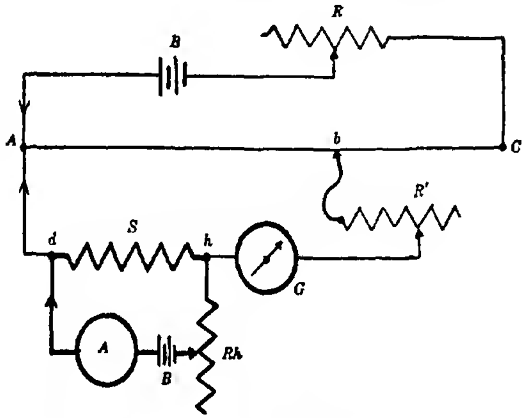


(Courtesy Leeds and Northrup Co.)
FIG. 61.—TYPE K POTENTIOMETER

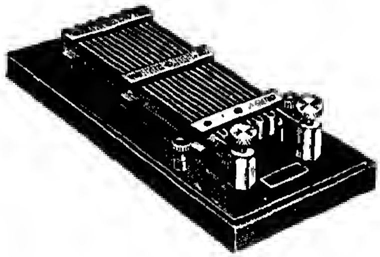
directly in volts. With this type of potentiometer it is possible to read directly to 0.0001 volt, and to estimate to 0.00001 volt. In measuring E.M.F. values greater than a few volts the E.M.F.

under test is accurately subdivided by means of auxiliary resistances and some convenient fraction of the total is determined by the potentiometer.

50. Measurement of Current.—Not only is the potentiometer used for measuring E.M.F. but it may also be utilized for the purpose of accurately measuring current strength. Referring to Fig. 62 *a* it will be noted that in the potentiometer circuit shown the source of the unknown E.M.F. is in this case a resistance S



(a)



(b)

(Courtesy Leeds and Northrup Co.)

FIG. 62

through which current is passing from a storage battery B. The fall of potential between the points d and h is the E.M.F. measured or compared with that given by the standard cell. Knowing the resistance of S and having determined the drop across this resistance we may by Ohm's law readily compute the value of the current through the resistance S. We thus have a method of accurately calibrating a current-measuring device such as the ammeter shown at A. The rheostat is inserted in the supply circuit

for the purpose of regulating the current through the resistance S and the ammeter A under test. By taking readings of the fall of potential across S for various current values the ammeter A can be accurately calibrated throughout its entire range. The resistance S is frequently spoken of as a "standard shunt" and is made in values ranging from 0.1 ohm to 0.001 ohm. The shunt having the highest resistance value may be used in a circuit having a maximum current of the order of 15 amperes, while the resistance of the lowest value is designed to carry a current as high as 500 amperes. Figure 62 *b* shows a typical standard shunt. In a later chapter reference will be made to an electrochemical and other methods of accurately measuring current strength.

CHAPTER VIII

THERMAL EFFECTS OF THE CURRENT

51. Joule's Law.—As the electrons move along a conductor in response to an E.M.F. energy appears in the form of heat, and a definite relation obtains between the heat developed and the electrical constants of the circuit. Let us discover this relation.

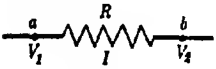


Fig. 63

Consider a conductor ab, Fig. 63, the terminals of which are at the potentials V_1 and V_2. In order to transfer q units of electricity from a to b work is done (eq. 18) which is measured by the product of the charge and the difference of potential, or

$$W \text{ (in ergs)} = q(V_1 - V_2).$$

Writing E for $V_1 - V_2$, we have

$$W \text{ (in ergs)} = q \text{ (in e.s.u.) } E \text{ (in e.s.u.)}.$$

If all of the energy expended in making this transfer is converted into heat it follows that

$$W \text{ (in ergs)} = JH \text{ (in calories)},$$

where J is the mechanical equivalent of heat, or 4.18×10^7 ergs/calorie. We may then write

$$W = JH = qE = ItE = I^2Rt, \qquad \text{Eq. 71}$$

or separately,

$$H \text{ (in calories)} = \frac{I^2Rt}{J}, \qquad \text{Eq. 72}$$

and

$$W \text{ (in ergs)} = I^2Rt. \qquad \text{Eq. 73}$$

In eqs. 71, 72 and 73 above, I and R are in electrostatic units. These equations are different expressions of what is known as *Joule's Law*, and indicate that the heating effect of the current is proportional to the square of the current, to the resistance and to the time during which the current acts.

Our equations must be modified somewhat if we are to employ

practical units of current and resistance. Bearing in mind (Sec. 39) that the ampere is equivalent to 3×10^9 e.s.u. of current and the ohm equal to $\frac{1}{9 \times 10^{11}}$ e.s.u., we get from eq. 72 that

$$H \text{ (in calories)} = \frac{I^2 \times 3^2 \times 10^{9\times2} \times R \times t}{4.18 \times 10^7 \times 9 \times 10^{11}} = 0.239I^2Rt, \quad \text{Eq. 74}$$

which for practical purposes may be written

$$H = 0.24I^2Rt,$$

where H is in calories, I in amperes, R in ohms, and t in seconds.

Equation 73 may also be changed in such a manner as to permit the use of practical units. The erg being a very small unit, it is more convenient to use the joule. Dividing, then, eq. 73 by 10^7, to reduce ergs to joules, we have

$$W \text{ (in joules)} = \frac{I^2Rt}{10^7}.$$

Changing I and R to amperes and ohms respectively our equation becomes

$$W \text{ (in joules)} = \frac{I^2 \times 3^2 \times 10^{9\times2} \times R \times t}{10^7 \times 9 \times 10^{11}},$$

which reduces to

$$W \text{ (in joules)} = I^2Rt, \quad \text{Eq. 75}$$

where I is in amperes, R in ohms, and t in seconds.

Example.—An electric stove takes 20 amperes at 110 volts. (*a*) How much heat is developed if the stove is operated for one hour? (*b*) How much energy is liberated as heat?

Solution: (*a*) Substituting $\frac{E}{I}$ for R in eq. 74, we have

$$\begin{aligned} H &= 0.24IEt \\ &= 0.24 \times 20 \times 110 \times 3600 \\ &= 1.9 \times 10^6 \text{ calories.} \end{aligned}$$

(*b*) Changing eq. 75 to fit this problem by substituting $\frac{E}{I}$ for R,

$$\begin{aligned} W &= IEt \\ &= 20 \times 110 \times 3600 \\ &= 7.92 \times 10^6 \text{ joules.} \end{aligned}$$

52. Electric Power.—It will be recalled that the term power, in general, means the time rate of doing work or expending energy. In the electrical field power may be thought of as the

electrical work done per second. If the electrical work is done at the rate of one joule per second, the power is said to be one watt.

Let us now derive a working relation which will enable us to compute the rate of energy expenditure (power) in an electrical circuit.

We have previously shown (eq. 75) that

$$W \text{ (in joules)} = I^2 \text{ (in amp.) } R \text{ (in ohms) } t \text{ (in sec.)},$$

or

$$W = IEt,$$

where E is in volts.

Divide this equation by t and we have

$$\frac{W}{t} = IE.$$

From our definition of the watt, we may write

$$P \text{ (in watts)} = E \text{ (in volts) } I \text{ (in amp.)}. \qquad \text{Eq. 76}$$

It should be noted that this relation holds only for those cases in which the current is constant in both value and direction. In alternating current practice another factor must be considered, which case will be discussed later.

Equation 75 may be written

$$P \text{ (in watts)} = I^2 \text{ (in amp.) } R \text{ (in ohms)}. \qquad \text{Eq. 77}$$

In certain cases it is convenient to express the power in terms of E.M.F. and resistance thus,

$$P = \frac{E^2}{R}. \qquad \text{Eq. 78}$$

Electrical power may be expressed in other units than watts. Remembering that 746 watts equal one HP, we have

$$1 \text{ Electrical HP} = \frac{\text{Electrical power in watts}}{746}. \qquad \text{Eq. 79}$$

In buying and selling electrical energy a unit known as the *watt-hour* is employed. It is *a unit of electrical energy* or work and *not* a unit of electrical power. A watt-hour is equivalent to the work done when energy is expended at the rate of one watt for

one hour. The kilowatt-hour is the usual basis of electrical merchandizing.

EXAMPLE.—A bank of electric lamps takes 50 amperes at 115 volts. What will it cost to operate these lamps for four hours if electrical energy costs 7¢/kilowatt-hour?

Solution: Employing the relation given as eq. 76,

$$\begin{aligned} P &= EI \\ &= 115 \times 50 \\ &= 5750 \text{ watts or } 5.75 \text{ kw.} \\ 5.75 \text{ kw. for 4 hrs} &= 23 \text{ kw.-hrs.} \end{aligned}$$

At 7¢/kw.-hr. this would amount to \$1.61.

CHAPTER IX

APPLICATIONS OF THERMAL EFFECTS

53. The Electric Furnace.—Furnaces which utilize the electric current as a heating agency are coming into very extensive commercial use. One reason for this is that they are, in general, more efficient than those of the fuel-fired type, and further, the temperature is under complete control. An additional feature is the freedom, when desired, from contaminating factors such as carbon, etc. Electric furnaces assume many forms, depending upon the particular use for which they are designed. As it would be beyond the scope of this volume to review them all, a description of only two or three typical units will be given.*

Electric furnaces may for convenience be classified into two general types: the *resistance furnace* and the *arc furnace.*

One of the first resistance furnaces to come into extensive use was that developed by Acheson in 1891 for producing the abrasive material known as carborundum. A cross-sectional view of the carborundum furnace is shown in Fig. 64.

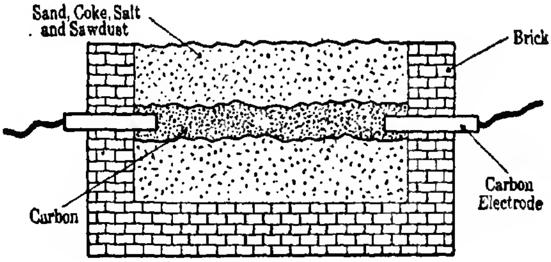


Fig. 64

A certain carborundum furnace consuming 2000 kilowatts has a length of 40 ft. and is 5 ft. in diameter. The outer walls of the furnace are made of loosely assembled bricks, and are temporary in nature, the side walls being taken down after each "firing" in order to remove the charge. The end walls are however permanent, and carry the large water-cooled carbon electrodes, two

* For a complete description of electric furnaces the student is referred to *Dictionary of Applied Physics* by Glozebrook, Vol. V, p. 293.

electrodes (3 ft. × 10 in.) being connected in parallel at each end.

In charging the furnace, the mixture, consisting of sand, coke, sawdust and salt, is filled in until the furnace is about half full. A trench-like path 2 ft. wide and $1\frac{1}{2}$ ft. deep is then made lengthwise in the mixture between the electrodes and into this is placed the "core." This core, consisting of carefully prepared coke, serves as a conductor and has a resistance of the order of 0.03 ohm.

When the core material has been laid in place, the remainder of the mixture is filled in and rounded over on top, the top not being covered. Owing to the relatively high resistance of the core when cold it is necessary to apply a voltage of 300 at the beginning. The operating E.M.F., after stable thermal conditions obtain, is about 200 volts, the voltage being regulated in order to maintain constant wattage. A stable working condition is established in from one to two hours.

Alternating current is used, the energy being supplied by means of step-down transformers. The high temperature developed (about 2000° C.) results in a chemical reaction between the sand and carbon as shown by the equation

$$SiO_2 + 3C \rightarrow 2CO + SiC,$$

the SiC being the carborundum. The function of the sawdust in the mixture is to make the mass more porous and thus facilitate the escape of the carbon monoxide gas. This gas as it escapes from the sides of the furnace is set on fire and burns during the time the furnace is in operation. It is interesting to note in passing that for every 1000 pounds of carborundum produced 1400 pounds of gas are given off. The salt reacts with any iron present to form the chloride.

The heating continues for 36 to 40 hours, after which the furnace is allowed to cool, then opened and the charge removed, by taking down a part of the brick walls. In a furnace of the size described from 5 to 8 tons of carborundum are secured from each heat. In one installation located near Niagara Falls twenty-seven such furnaces are in operation.

Incidentally it may be mentioned that graphite is produced in a furnace of the above described type. In fact if the temperature in a carborundum furnace be allowed to exceed 2240° C. decomposition of the silicon carbide begins, silicon being driven off as a vapor and graphitic carbon remaining. In a furnace designed

for the production of graphite, anthracite coal and sand constitute the mixture for the charge. Graphite is widely used as a constituent of lubricants and in the manufacture of electrodes for certain electrolytic processes, its electrical conductivity being about four times as great as that of the amorphous variety of carbon.

Comparatively small electric furnaces of the resistance type designed for laboratory use are constructed by winding a heat-resisting tube or vessel with resistance wire such as nichrome or molybdenum, and inclosing the unit in a jacket of thermal insulating material. Furnaces of this type are used, for example, for calibration purposes in connection with pyrometers and platinum thermometers.

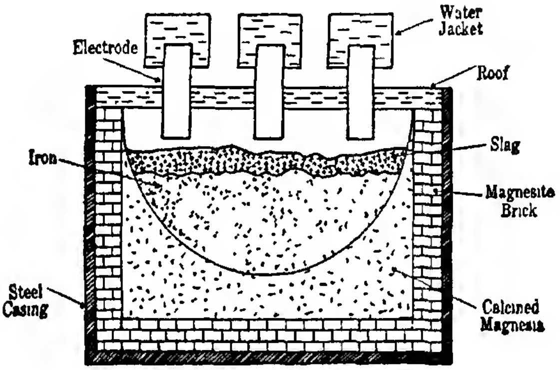


FIG. 65

As an example of the arc type of furnace we may take a unit which is coming into extensive use in the production of high grade steel. Figure 65 is a diagrammatic sketch of the Heroult steel furnace, the drawing being more or less self-explanatory. This unit is generally designed for use with a three-phased current (Sec. 104) and hence three electrodes are shown each of which is connected to the power supply. Furnaces of this type are designed to handle from 15 to 20 tons of steel at one charge. A 15-ton unit absorbs 2250 kilowatts at 110 volts between phases (Sec. 104). Such furnaces are frequently charged with molten metal from a Bessemer converter or open-hearth furnace. A mixture of lime and iron oxide is introduced on top of the molten metal and an arc is formed between the electrode and the slag, and thence to the steel beneath, the current leaving the furnace

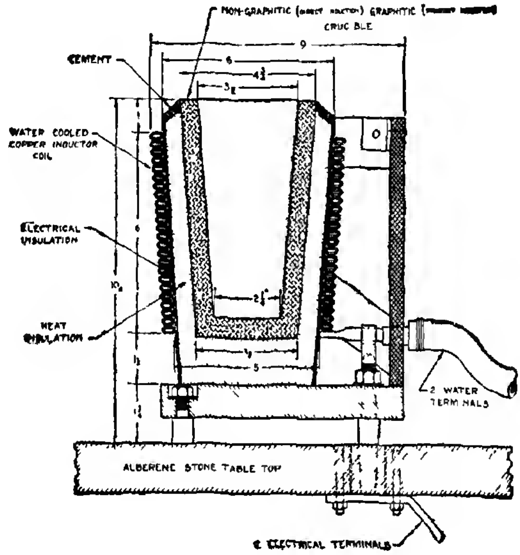
by arcing from the steel to the slag and in turn to the other electrode. The slag serves as an oxidizing agent for the purpose of removing phosphorus. When tests show that phosphorus has been sufficiently removed the furnace as a whole, which is so arranged mechanically that it may be rotated, is tilted and the slag removed. Other slag consisting of lime, fluorspar and coke dust is then added. The heat developed by the electric arc transforms a part of this slag into calcium carbide, which in turn is decomposed, liberating a certain amount of carbon which is absorbed by the steel. From time to time the steel is tested in order to determine when the desired grade is secured. By means of this furnace low-grade Bessemer steel is converted into a high-grade product relatively free from oxygen, sulphur and phosphorus and containing a definite and predetermined amount of carbon. A furnace of the type just described will yield a steel ingot of about 8 tons per charge. From $1\frac{1}{2}$ to 2 hours are required for a "heat."

In 1916 Dr. Edwin F. Northrup invented a special form of induction furnace in which a high frequency alternating current is utilized to develop the thermal effects. It will be shown in a subsequent discussion (Sec. 105) that electric currents will be developed in any conducting material which is subjected to a magnetic field which varies in intensity. Such "induced" currents may, under suitable conditions, assume large values and this is particularly true if the magnetic field varies (alternates) at a high frequency (several thousand cycles) per second.

In the high frequency induction furnace designed by Dr. Northrup the receptacle containing the material to be melted is surrounded by a so-called inductor coil (Fig. 66) from which it is electrically and thermally insulated. This inductor winding consists of a flattened copper tube through which water flows for the purpose of cooling the conductor. When a high frequency alternating current from a suitable source (Secs. 104 and 130) is passed through the helix large induced currents will be developed in any conducting material in the crucible and these induced currents will cause rapid and efficient heating of the material. If a non-conducting substance is to be heated, such as glass, the crucible is made of carbon and the induced currents are set up in the receptacle itself. Temperatures as high as 2800° C. may be produced, and the temperature is under complete control at all times.

Another interesting and valuable feature of the Northrup furnace is that the induced currents which cause the heating also cause a movement or circulation of the molten material, which, in many cases, is an important part of the melting process.

Aside from temperature control perhaps the most important advantage of this type of furnace is that the melting of any material may be carried out without chemical contamination of the substance being worked. In fact it is entirely practicable to melt a substance in vacuo or in the presence of some particular gas.



(Courtesy Ajax-Electrothermic Corp.)

FIG. 66.—HIGH FREQUENCY INDUCTION FURNACE

Furnaces of this type are made in sizes ranging in capacity from a few ounces to several tons. The frequency of the alternating current used depends upon the nature of the work for which a particular unit is designed, and ranges from 500 to 50,000 cycles per second. The power consumed ranges from 3 kw. in the smallest laboratory unit to 1500 kw. in the larger commercial installations. The Northrup high frequency induction furnace represents a distinct advance in electrical furnace practice.

54. Electric Welding.—The thermal effect of the electric current finds wide and increasing application in electric welding. Referring again to the relation expressing Joule's law (eq. 74)

we note that the heating effect of the current varies as the *square* of the current and the first power of the resistance in the circuit. If then arrangements are made to produce and handle very heavy currents we may produce intense heating effects, either throughout the entire body of the metal or locally on the surface. Further, the heat so produced, except for oxidation, is "clean," which is an important consideration in the process of welding.

In general there are three methods of electric welding: the Thompson process, arc welding, and pail welding.

In the Thompson process the two pieces of metal (as for example two street car rails) are placed end to end, and, by means of heavy leads clamped to them near their ends, a heavy current from a step-down transformer (Sec. 123) is sent across the junction. The resistance at the point of contact of the rails being relatively high, heating occurs at that point. When the end of the two members have been brought to the proper temperature great pressure is applied in order to force the ends into intimate contact and the current is turned off. Alternating current of 40 to 60 cycles is used, and the step-down transformer employed in the process has only one or two turns of heavy copper in the secondary. The current in the welding circuit is of the order of 25,000 amperes, depending upon the nature and size of the weld.

In arc welding the heat is developed locally and usually at or near the surface of the metal being worked. For example in "bonding" street car rails, which are not welded, heavy flexible copper conductors are welded to each rail near the end. This is accomplished by connecting the rail to the positive terminal of a D.C. supply and the negative lead to a portable carbon electrode. To effect a weld the carbon electrode is brought into contact with the metal to be welded and the resulting arc heats the material at the point where the weld is desired. Currents of 200 amperes are employed, at a voltage of the order of 50. Very extensive use is made of this method of welding, and for some classes of work it is replacing acetylene gas welding.

In the so-called pail welding process the heating of the metal to be worked is accomplished in a novel manner. A lead plate is immersed in a vessel of water in which has been dissolved some common borax or washing soda, and connected to the positive terminal of a D.C. supply. The metal to be heated is held by means of insulated tongs connected to the negative lead, and

plunged into the liquid. Intense heating occurs in that part of the metal which is submerged. When the desired temperature has been reached the metal may be withdrawn and worked on an anvil as in common blacksmith practice. The dissolved salt serves to make the liquid a good conductor, and also to prevent the formation of an oxid film. The high current density at the part of the metal submerged gives rise to a rapid evolution of hydrogen, some of which forms a gaseous film about the metal. Arcing takes place with the usual marked heating effects. The magnitude of the current employed varies from a few amperes to several hundred, depending upon the size of the metal being worked. This type of welding equipment is sometimes spoken of as an "electric forge."

CHAPTER X

ELECTRIC LIGHTING

55. Arc Lamps.—In 1881 Sir Humphrey Davy, the English scientist, discovered that a brilliant electric discharge took place when the electric current from a number of primary cells was caused to pass between two carbon terminals which were first in contact and then slightly separated. When the carbon electrodes were in a horizontal position the luminous vaporized carbon between the terminals assumed a bow-like form, due to the heated air, and hence the term "arc" was used by Davy to designate the phenomenon. After Faraday's later discoveries which served as a basis for the development of the dynamo, Davy's discovery led to the development of the modern electric arc.

At present there are several types of arcs which are employed for purposes of illumination. These will be described seriatim.

The *carbon arc* may be *open* or *enclosed*. The former consists of two carbon rods or electrodes about ½ inch in diameter, supplied by either alternating or direct current. Suitable mechanism is provided for "striking" the arc and for regulating the movement of the carbons with respect to one another. The characteristics of the carbon arc are such (decreasing drop in potential across the arc with increasing current through the arc) that it is necessary to provide a ballast or stabilizing resistance in series with the arc.

In any carbon arc the electrodes are in end contact when the current is off. Upon application of an E.M.F. the resulting current gives rise to heating at the point of contact. If now the electrodes are slightly separated, the space between will be filled with vaporized carbon which serves to conduct the current between the hot electrodes.

In a direct current arc carbon is carried off from the positive electrode but not from the negative. This results in a depression or "crater" in the end of the positive carbon. Negative ions move at high speeds from the negative to the positive electrode and the resulting impact causes the positive carbon to operate at a higher temperature than the negative. In fact the greater part

of the light from an arc originates at the heated ends of the electrodes, particularly at the crater, the carbon vapor of the arc proper being of comparatively low visible luminosity. The temperature of the arc is about 3500° C. In the alternating current arc both carbons show a crater, though to a lesser degree than in the direct current unit, and both terminals serve as a source of light. Carbon arc lamps may be burned either in parallel or series. In the former case they are operated from a constant potential line, and in the latter instance on a constant current supply (Sec. 125). The applied voltage is usually 40 to 60 in the case of D.C. arcs and 30 to 35 for the A.C. units. The current is of the order of 5 to 10 amperes, the drop across the arc being about 40 volts. The efficiency of the older forms of the open arc was of the order of 15 lumens * per watt.

It was found by Marks in 1893 that the carbons would give a much longer effective life and a steadier light if the arc is inclosed in a globe which excludes all but a limited supply of air. Inclosed arcs are operated chiefly from a constant voltage supply (110) and at something like half the current of the open arc. Owing however to the loss of energy in the current regulating circuit the efficiency of the inclosed arc is comparatively low, being only 4 to 6 lumens per watt. This type of arc is rich in ultra-violet radiation and hence is useful for photographic and therapeutic purposes.

Reference has already been made to the fact that the arc itself contributes little to the light emitted by the lamp. Efforts have from time to time been made to impregnate the electrodes with some substances such that the incandescent vapor will be strongly luminous. Hugo Bremer, a German investigator, in 1899 succeeded in producing what is known as the *flame arc lamp*. In this unit the carbons are "cored," that is, the centers of the carbon electrodes are hollowed out and the space is filled with a mixture of the alkaline earth compounds such as calcium, titanium, and strontium. The vaporizing of these compounds causes the arc proper to become a brilliant source of light, the color depending upon the particular elements used in the core. The first flame

* Any source of light is a source of energy, and we therefore have to do with what might be termed light flux, that is, the *rate at which energy is emitted by the source of light*. The lumen is the unit of luminous flux, and is numerically equal to the flux through a unit solid angle per unit of time when the source has an intensity of one international candle.

arc lamps employed carbons placed in a V-position with respect to one another. Later, a model having a vertical alignment of the electrodes and hence a greatly improved light distribution for street lighting was developed. The flame arc lamp has high efficiency, ranging from about 30 to 120 lumens per watt. Because of the nature of the electrodes the maintenance cost of flame arc lamps is comparatively high.

Still another form of arc lamp is one designed by the late Dr. Steinmetz and known as the *magnetite arc.* In this lamp the positive electrode is a rod of solid copper while the negative terminal consists of an iron tube packed with a mixture of magnetite (black oxide of iron), titanium oxide, and chromium oxide. This composite electrode has a comparatively long life, giving a service of from 100 to 350 hours, depending upon the current

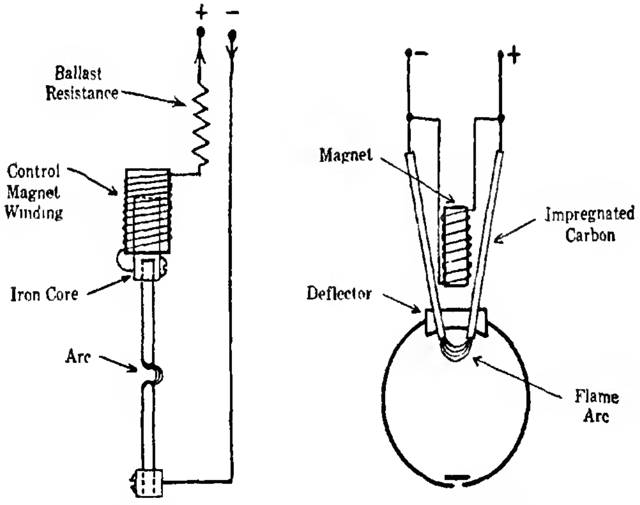


FIG. 67.—CARBON AND FLAME ARC LAMPS

value. This lamp is not adapted to interior lighting on constant voltage circuits but is quite extensively used for street lighting purposes on constant current supply mains. It operates at 75 volts with an efficiency of 11 to 18 lumens per watt. Figure 67 shows the relation of the elements in two common types of arc lamps.

A lamp which, in certain respects, might be classified as an arc lamp is the *mercury vapor* unit, commonly known as the *Cooper Hewitt* lamp, after its originators. This lamp consists of

a glass tube from one to four feet in length and about an inch in diameter. Mercury forms one electrode while a piece of metal or graphite serves as the other (Fig. 68). The arc is "struck" by tilting the tube (normally supported in an oblique position) until the mercury forms a thin thread connecting the two electrodes. When the tube is returned to its original position an arc is formed, thus vaporizing some of the mercury. The arc then continues to operate between the electrodes through the metallic vapor as a path. Both D.C. and A.C. models are in use. Means are provided for automatic starting. The lamp operates at an efficiency of about 12 lumens per watt and has a life of some 4000 hours.

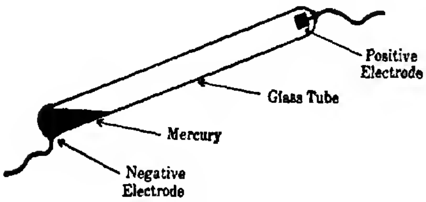


FIG. 68

The spectrum of mercury shows only a faint red line; hence this lamp yields a light in which greenish blue predominates, and consequently cannot be utilized where exact color values are important. The lamp however finds extensive use in warehouses, factories and other similar places. The light from the mercury lamp is rich in actinic rays and is therefore used as an artificial light in photographic work.

The efficiency of the mercury arc may be increased by operating the unit at a higher temperature and pressure. In practice this is made possible by making the inclosure of quartz instead of glass, a short unit being the result; and one giving a light more nearly white. Quartz however transmits ultra-violet light, and hence, when utilized for ordinary purposes, it becomes necessary to inclose the unit in glass, thus filtering out the short wave lengths. The fact that the unscreened quartz lamp may serve as a source of strong ultra-violet light is taken advantage of in the sterilization of water and for use in medical practice. When used as a sterilizing agent the water to be treated is caused to flow slowly over one or more such lamps and is thereby sterilized. The inclosed quartz mercury vapor lamp has an efficiency of 26 lumens per watt.

Another arc operating at reduced pressure is the *Moore tube lamp.* This consists of a long tube containing carbon dioxid,

nitrogen or neon at a low pressure. Energy is supplied to the tube in the form of alternating current from a suitable transformer. The high tension discharge heats the attenuated gas, with the result that we have a linear source of light. Various color effects may be produced by proper selection of gases to be utilized. Carbon dioxid yields a nearly white light, and is useful where accurate color matching is important. The Moore tube lamp gives about 12 candle power per lineal foot, and operates at an efficiency of 2 to 6 lumens per watt. The life of the tubes is from 3000 to 4000 hours. This unit has however not come into extended use.

However a lamp operating on the same general principle as the Moore lamp is coming into extensive use for special purposes, particularly in connection with the construction of signs. In this lamp neon gas is employed, the attenuated gas being rendered incandescent by the passage of the current and yielding a pinkish or reddish color. The efficiency is of the order of 17 lumens per watt.

In optical work a "point source" of light is extremely useful. A special form of inclosed arc operating in an atmosphere of inert gas has been devised which yields a highly concentrated source of light. This unit consists essentially of a positive electrode consisting of a globule of tungsten, and a negative electrode in the form of a tungsten spiral. Figure 69 shows the essential parts of this arc lamp.

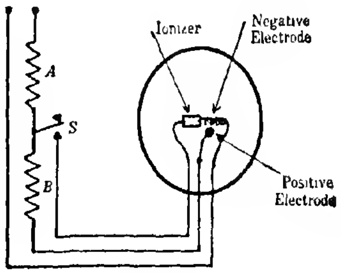


FIG. 69

Associated with the negative terminal is an "ionizer," as shown. This consists of a tube composed of a mixture of tungsten and thorium oxide, this tube surrounding a part of the negative spiral. Since this form of arc, in common with other types, has a "falling characteristic" (decreasing drop with increase in current) it is necessary to insert a ballast resistance, shown as A and B in the figure.

To put the lamp into operation the switch S is closed, thus causing current to flow through the ionizer, which when heated to incandescence ionizes the gas in its vicinity. When the switch

S is opened conditions are favorable for the formation of an arc between the tungsten bead and the spiral. The stem of the positive tungsten globule is mounted on a thermostat strip which causes the bead to move as the lamp becomes heated and assume such a position that the arc shifts from the spiral to the heavier part of the electrode, where it burns with little if any flickering. The incandescent globule of tungsten becomes the source of light. The ballast resistance and the switching mechanism are arranged in a separate control box. For the operation on A.C. circuit the lamp has two tungsten bead electrodes of about the same size, in addition to the ionizer for starting the arc.

Tungsarc is the trade name given by the Bausch and Lomb Company to the unit made in this country, while Pointolite is the designation employed by the Ediswan Company of England.

56. Incandescent Lamps.—The carbon lamp was the first successful unit of this type to be developed. It is said that such a lamp was made as early as 1845 but means were not available at that time for producing a satisfactory carbon filament or for attaining the necessary degree of vacuum. Platinum and other rare metals were tried as filaments but proved unsatisfactory. In 1879 Swan of England and Edison in this country began development work on a lamp of this type, with the result that in 1880 a practical incandescent lamp was available. The filament consisted of carbonized cellulose, on the outside of which, by a "flashing" process, was deposited a coating of graphitic carbon. For many years the 60-watt carbon lamp gave 16 c.p., thus operating at the very low efficiency of about 1.3 lumens per watt. Just before the advent of the metal filament lamps the efficiency was improved somewhat, though at its best it was always comparatively low.

The carbon lamp has in recent years been almost completely displaced by the high efficiency *metallic filament* unit. Owing to the high melting point of tungsten (3000° C.) a lamp filament made of this element may be operated at a relatively high temperature and hence at a comparatively high electrooptical efficiency. In 1911 Dr. Coolidge developed a process whereby tungsten could be made ductile, and in 1913 Dr. Langmuir found that if the bulb of a tungsten lamp were filled with an inert gas at or near atmospheric pressure it could be operated at a much higher temperature without excessive evaporation of the filament ma-

terial. This meant a correspondingly still higher electrooptical efficiency. As a result of these advances we now have an incandescent lamp having an efficiency ranging from about 10 lumens per watt in the smaller sizes to 20 lumens per watt in the larger units. With this increase in efficiency has also come a corresponding improvement in the visual quality of the radiation emitted. When one considers the physical properties of tungsten it is evident that the development of the modern metallic filament lamp is one of the outstanding achievements of applied science of the age in which we live.

CHAPTER XI

CHEMICAL EFFECTS OF THE ELECTRIC CURRENT

57. Electrolysis.—If some copper sulphate ($CuSO_4$) be dissolved in water and the resulting solution be connected to a source of electrical energy, as shown in Fig. 70, decomposition of the solution will take place. Assuming, for the moment, that the electrodes which dip into the solution are of some neutral material such as platinum or carbon, copper will be deposited on the negative terminal and the radical SO_4 will appear at the positive electrode. This process we know as electrolysis.

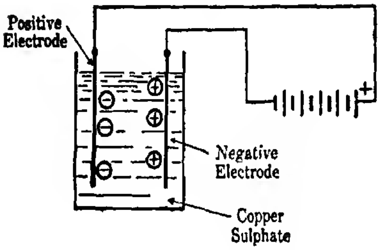


Fig. 70

This electrochemical process involves directly the question of the nature of solutions. While there are a number of points regarding the exact character of solutions yet to be settled, it has been reasonably well established that in an aqueous solution, for example of copper sulphate, at least some of the molecules of the salt have been dissociated or broken up into positively and negatively charged entities which we call *ions.* Symbolically we might set this fact down thus:

$$\text{Solution} = x\,Cu^{++} + x\,SO_4^{--} + m\,H_2O + (n - x)(CuSO_4).$$

If we had been considering the case of sodium sulphate the situation might be represented thus:

$$\text{Solution} = x\,Na^{+} + x\,Na^{+} + x\,SO_4^{--} + m\,H_2O + (n - x)(Na_2SO_4).$$

For sodium chloride we would have

$$\text{Solution} = x\,Na^{+} + x\,Cl^{-} + m\,H_2O + (n - x)(NaCl).$$

Under the electromechanical force action of the charged electrodes these charged atoms or groups of atoms will move through

the solution according to the laws which obtain between charged bodies under these circumstances.

It will be noted that in our symbolic representation of a solution two positive charges are shown associated with the copper atom. In the salt mentioned copper is divalent, that is, has a valence of two, and is found to carry two elemental charges. Likewise the radical SO_4 acts as a divalent group and hence carries two negative charges. The sodium atom being monovalent carries only a single charge. Trivalent elements carry a charge equal to three times that transported by a univalent element such for example as hydrogen. All univalent elements carry equal charges, and *this charge is the basic electrical quantity which* we have already referred to (Sec. 2) as the electron. The student is here cautioned against confusing the terms *ion* and *electron*. The former is the atom or group of atoms together with its associated charge; the latter refers to the natural unit quantity of electricity.

Referring again to the question of the nature of solutions, it may be said that in moderately dilute solutions probably a large percentage of the molecules are dissociated; * in concentrated solutions only a relatively small number of molecules are broken up, while in extremely dilute solutions it is probable that practically all have undergone dissociation. Only the dissociated molecules take part in the transfer of the current, though it is probable that undissociated (neutral) molecules sometimes become attached to ions and are dragged along as the latter move under the influence of the electric forces. As is to be expected, in the case of moderately dilute solutions the strength of the current increases with the concentration, but for highly concentrated or extremely dilute solutions the current is not strictly proportional to the concentration.

Only solutions of metallic salts, bases and acids conduct the current; in other words only these classes of substances mentioned are dissociated upon entering into solution. Such solutions are known as *electrolytes*. In general, non-aqueous solvents such as alcohol, ether, zyol, etc., do not produce electrolytes. Fused salts however serve to conduct the current, and, as we shall

* Some writers use the term "ionised" in this connection. The author prefers to reserve "ionisation" for use in connection with the discharge of electricity through gases.

presently see, important practical applications are made of this fact.

It is to Faraday that we owe the nomenclature connected with the process of electrolysis. We have already spoken of an electrolyte as an aqueous solution of a metallic salt, base or acid. The positive electrode is known as the *anode* and the negative terminal, the *cathode*. The ions liberated at the anode are spoken of as *anions* and those which appear at the cathode as *kations*.

The researches * carried out by Faraday in 1833–34 led him to formulate the following very useful generalizations or laws, the first of which may be stated thus:

The weight of the material liberated at either of the electrodes is proportional to the total quantity of electricity which passes through the electrolyte.

It will be recalled (eq. 51) that

$$Q = I \times t.$$

If then, as stated above, $M \propto Q$, we may write

$$M = \epsilon Q = \epsilon \times I \times t, \qquad \text{Eq. 80}$$

where M is the weight of an ion set free at the electrode, I the current, t the time, and ϵ a proportionality constant which is known as the *electrochemical equivalent*. By making Q unity in the last equation above, we may define the electrochemical equivalent to be the weight of an element in grams which is deposited by the transfer of unit quantity of electricity, that is, by one coulomb.

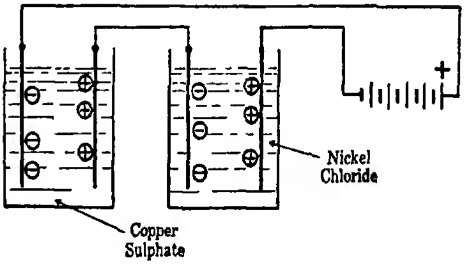


Fig. 71

* The extremely primitive experimental facilities available at the time of Faraday's epoch-making investigation makes this research one of the most remarkable in all scientific history. The student is urged to read Faraday's original account of this historical research.

Faraday's second law is to the effect that, *when a given quantity of electricity is passed through two or more electrolytic cells in series, the weight of the ions liberated at a given electrode will be proportional to the chemical equivalents of the particular elements involved.*

For example, if conditions are as illustrated in Fig. 71, and we pass, say, one coulomb through the two cells the relative amounts of copper and nickel deposited will be in proportion to the ratio between the chemical equivalents of the two elements.

Following is a table giving the electrochemical constants of a number of the elements encountered in the more common electrolytic processes.

Element	Atomic Weight	Valency	Chemical Equivalent	Electro-chemical Equivalent (ϵ) (Grams per Coulomb)
Aluminum....	27.0	3	8.96	0.00009355
Copper.......	63.57	1	63.57	0.0006588
Copper.......	63.57	2	31.78	0.0003294
Chromium....	52.01	3	17.50	0.0001796
Gold........	197.2	3	65.21	0.0006812
Hydrogen....	1.008	1	1.	0.000010459
Iron.........	55.84	2	27.92	0.0002895
Iron.........	55.84	3	18.61	0.0001930
Lead........	207.20	2	103.60	0.0010731
Nickel.......	58.68	2	29.34	0.0003041
Oxygen.....	16.0	2	8.	0.00008291
Platinum.....	195.23	2	97.6	0.0010115
Silver........	107.88	1	107.88	0.0011180
Tin..........	118.7	2	59.35	0.0006151
Tin..........	118.7	4	29.7	0.0003075
Zinc.........	65.37	2	32.68	0.0003387

Thus far we have confined our attention largely to what takes place at the cathode when metallic ions are liberated. It should be borne in mind that it frequently happens that secondary reactions occur during the electrolytic process. For example in the case of the separation of copper sulphate between carbon or platinum electrodes the radical SO_4 which is liberated at the anode, after delivering its charge to the electrode, immediately combines with water to form sulfuric acid. If the anode be made of copper the SO_4 group will combine with the copper to form copper sulfate and thus tend to maintain the concentration of the

solution. This is the basic process utilized in the electrolytic refining of the metals.

In the case where both electrodes are neutral, with respect to the constituents of the electrolyte, we have available a process whereby the electrolytic quantitative separation of the elements may be effected. Indeed this principle is widely used in electrochemical analysis. A platinum dish, after being carefully weighed, serves as cathode and a platinum spiral or piece of gauze functions as the anode. After the electrolyte has been entirely freed from the element being studied (as shown by the proper chemical test) the cathode is dried and again weighed. Figure 72 shows a typical arrangement of apparatus employed in electrochemical analysis.

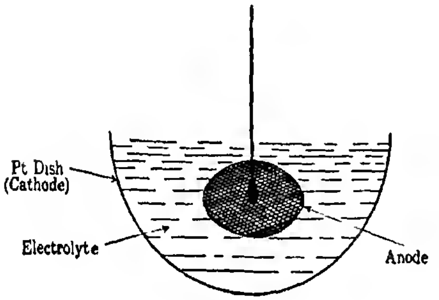


Fig. 72

In the determination of such elements as chlorine the anode may be made of a piece of silver gauze. When electrolysis takes place the chlorine, which is liberated at the anode, will immediately attack the silver to form silver chloride, and the quantitative determination of the chlorine content is made by weighing the chlorine as silver chloride.

In the case of the elements composing the alkaline earths, such as potassium, the cathode may consist of mercury. The potassium on being liberated at the cathode will form an amalgam with the mercury and may be weighed in that condition.

58. Industrial Applications of Electrolysis.—The extent to which the electric current is used in electrochemical processes in the manufacturing industries is very great and is increasing daily. These applications include the electroplating of copper, silver, nickel, chromium and gold, the refining of various metals, and the manufacture of a number of widely used chemical compounds. The plants which utilize electricity in this manner are located at or near points where cheap hydroelectric power is available. In this country many of them are situated near Niagara Falls where several hundred thousand kilowatts are

utilized for these purposes. We have already discussed the fundamental principles on which these various commercial applications rest. Hence it only remains to briefly describe a few of the typical commercial electrochemical processes.

In the plating of copper, silver, gold, nickel and chromium, the electrolyte usually consists of a complex salt. For example in nickel plating the double nickel-ammonium sulphate, $NiSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$, is used.

Copper plating is a somewhat more simple process than is the case in the deposition of gold and silver. An acid solution of copper sulphate may be utilized as an electrolyte. In the printing industry copper electrotype replicas are produced on an extensive scale. In making a copper electrotype of a printed page or photograph a wax or gelatine mold of the original is first prepared. This impression is then dusted over with finely powdered plumbago. Finely divided iron is then sprinkled over the carbon dust and the surface thus prepared is immersed in a solution of copper sulphate. As a result of the chemical reaction copper is deposited on the graphite. After washing, the prepared wax plate is placed in a solution of copper sulphate and made the cathode; a piece of pure copper serves as an anode. A current density of from 0.9 to 1.3 amperes per square foot is used. When copper has been deposited to the desired thickness the metallic shell is separated from the wax form and "backed" with type metal or other material. The production of master phonograph records involves essentially the same process.

The refining of copper has become an extensive industry. It consists of the electrolytic deposition of copper from an electrolyte of acidulated copper sulphate. Crude copper bars or plates are suspended in large vats and connected as anodes. Sheets of pure copper serve, at the beginning of the process, as cathodes. By the proper regulation of the current and voltage copper having a purity of 99.95 per cent is deposited on the cathodes. Copper of this purity is needed for use in the manufacture of wire which is to be utilized in the electrical industry. In order to carry on the electrolytic refining of copper on a large scale, immense D.C. generators (Sec. 104) are required, the vats being arranged in a series-parallel circuit. A total current of several thousand amperes may be employed in a given refining plant.

One of the most extensive commercial applications of the

electrolytic process is that connected with the production of the metal aluminum. In 1886 Hall in the United States and Heroult in France developed an economical method of utilizing the electric current for the separation of this important metal. Aluminum cannot be electrolytically deposited from an aqueous solution of its salts. However Hall and Heroult found that it is possible to deposit the metal from a molten mixture of cryolite and alumina (Al_2O_3). The mixture of bauxite and cryolite is put into carbon-lined iron pots, the carbon serving as a cathode. An E.M.F. of 5.5 volts is applied and a current of 10,000 amperes is passed through the mixture in each pot. The electric current serves the double purpose of maintaining the cryolite in a melted condition and in separating the aluminum oxide into the aluminum and oxygen. The metal appears at the cathode in a molten state, collecting at the bottom of the receptacle. The oxygen which is liberated at the anode combines with the anode, which is carbon, thus gradually consuming this electrode. The hot aluminum is drawn off from the pots and cast into ingots, after which it is worked into wires, sheets and rods. The principal plant utilizing the Hall process is located at Niagara Falls.

As an illustration of manufacture of chemical compounds by the electrolytic process mention may be made of the production of caustic soda. Various electrolytic cells or organizations have been devised whereby it is possible to economically produce this chemical compound by electrical means. Several of the commercially successful cells involve essentially the same basic principle, viz., the use of mercury as the cathode. One form of such cell is illustrated diagrammatically in Fig. 73, this particular organization being known as the Kellner-Solvay cell.

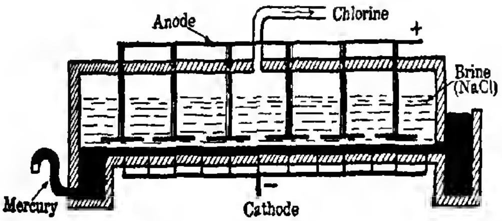


Fig. 73

In this unit a thin layer of mercury is caused to flow across the floor of a large cement trough. Above the mercury a stream of

brine (a solution of NaCl) is caused to move continually. A number of platinum wire-mesh anodes are supported just above the layer of mercury which serves as cathode. The application of an E.M.F. of 5 volts results in separation of the sodium at the cathode which reacts to form an amalgam with the mercury. Provision is made whereby this amalgam is separated by gravity from the general mercury stream and subsequently brought into contact with water, thereby forming sodium hydroxide (NaOH), the product sought. The chlorine which appears at the anode escapes into the chamber above the brine and may be drawn off and utilized as a by-product. The energy efficiency of the electrochemical process just outlined is about 45 per cent, one kilogram of NaOH being produced for each kilowatt-hour (Sec. 51) of electrical energy consumed.

CHAPTER XII

PRIMARY AND SECONDARY CELLS

59. The Nature and Theory of Primary Cells.—Intimately associated with the questions involved in electrolytic processes is the subject of the theory of the production of an E.M.F. by means of a primary cell.

The origin of the primary cell dates back to the well-known incident of the frog's legs hung by a copper wire on an iron support. Galvani, who chanced to observe the original happening, offered an explanation of the phenomena which was soon shown by Volta to be incorrect. Volta put forward what has come to be known as the *contact theory,* and he performed many classical experiments in support of his explanation. In brief, his contention was to the effect that when two dissimilar metals are brought into contact there is established a difference of potential between the two substances. Under certain conditions this is undoubtedly true, but such a theory does not completely explain all of the phenomena which take place in a primary cell.* Various other attempts have also been made to account for the E.M.F. developed in a cell but, like Volta's theory, they have all been found to have distinct limitations. However, as a result of the work of Arrhenius, Hittorf, Nernst and other investigators on the properties of solutions, particularly electrolytes, we have come to a fairly clear understanding of the origin of the E.M.F. in a primary cell.

Suppose we have a primary cell consisting of a piece of pure zinc immersed in a solution of zinc sulphate and a piece of pure copper immersed in a solution of copper sulphate, the two solutions being kept from rapid diffusion by means of a porous cup. This type of primary cell was originally designed by Professor Daniell in 1836 and bears his name.

As previously pointed out (Sec. 57) the salt solutions will consist of a mixture of neutral and dissociated molecules. In the zinc sulphate solution for instance we will have zincions and

* For our present purposes we may define a primary cell as consisting of two dissimilar metals immersed in an acid or other electrolyte.

sulphions, each moving about in a disorderly manner. Further, the metallic zinc exhibits a tendency to pass into solution in the ionic state, this tendency being commonly referred to as solution pressure. As soon as metallic zinc is brought into contact with the electrolyte it is probable that one or more atoms of zinc will unite with a chemically corresponding sulphion (SO_4^{-}) to momentarily form zinc sulphate. Dissociation, however, of the newly formed salt probably takes place at once.

The zinc atom in going into solution carries with it a positive charge, thus leaving the zinc electrode negatively charged. The solution, then, adjacent to the zinc electrode shows a positive charge, and, due to the resulting electrostatic forces, there is thus established what might be referred to as an electrical double layer.

There is also a third force involved in the situation, viz., a definite osmotic pressure, or tendency of the metalion to go out of solution. Electrochemical equilibrium will come to obtain when the sum of the electrical stress and the osmotic pressure equals the solution pressure. This condition is arrived at so quickly and involves so little material that no chemical change can be detected when a metal is immersed in one of its salts.

The electrochemical process just outlined has however resulted in establishing a difference of potential between the metallic electrode and the electrolyte at the surface of contact. In other words our electrode has acquired a definite potential with respect to the electrolyte, and this potential is spoken of as the *electrode potential.*

In the case cited (zinc in zinc sulphate) the solution pressure of the metal was greater than the osmotic pressure of the zincion. If, however, we have a combination, say copper in copper sulphate, where the osmotic pressure exceeds the solution pressure, there would be a tendency for the metallic ion to go out of solution and thus be deposited on the electrode. This would continue until the electrical stress resulting from the double layer just equalled the difference between the solution pressure and the osmotic pressure. In this case the copper electrode will become positively charged and the adjacent electrolyte will become negative.

In addition to the two sources of P.D. cited above there is, in the case of the Daniell and other similar cells, a third source of

P.D. If the ions of the two electrolytes do not move at the same velocity through their respective electrolytes there will exist a P.D. at the surface of contact between the two electrolytes. However with the electrolytes commonly employed this P.D. amounts to only a few millivolts and hence for most practical purposes may be neglected.

These several potential differences, established and maintained by electrochemical action, constitute the E.M.F. of the cell, and the numerical value of the E.M.F. of any particular cell is given by the algebraic difference of these potential differences. For instance in the case of zinc immersed in its normal * salt solution, the metal will have a potential of + 0.51 volt (Sec. 60) with respect to the electrolyte, and in the case of copper and its salt, − 0.60 volt; the algebraic difference would thus be 1.1 volts, which is the E.M.F. at the terminals of a Daniell cell.

To summarize then, it may be said that, in the case of two conductors immersed in one or more electrolytes, the principal E.M.F. resides, or has its origin, at the surfaces of separation of the solid and electrolytic conductors.

Having outlined the process by which the E.M.F. of a cell is established, we are now confronted with the problems involved in the energy relations of such a cell. If we connect the two electrodes of such a cell by means of a conductor the electrochemical equilibrium outlined above will be destroyed. This results from the movement of electrons along the circuit in response to the E.M.F. at the terminals of the cell. The electrical stress at each electrode will therefore be diminished, with the result that additional metal will go into solution from the negative electrode. This process will continue as long as the electrodes are electrically connected and any of the electrode material remains. Our problem is then to compute the magnitude of the E.M.F. of a given cell from the electrochemical energy relations which exist between the electrodes and the electrolyte.

Speaking broadly, it may be said that the energy represented by the electric current resulting from the E.M.F. of a cell is derived from the chemical changes which occur within the cell.

Referring again to a Daniell cell, let us examine the energy transformations which are involved in this typical case. Suppose

* A *normal* solution of a salt is one in which one liter of the solution contains an amount by weight of the salt equal to the molecular weight divided by the valence of the metal involved.

that we connect the electrodes of our cell through a resistance of such a magnitude that one ampere will flow in the circuit. Let this current flow for one second. A quantity of electricity equal to one coulomb will, under those conditions, have passed completely around the circuit. By reference to our table of electrochemical equivalents (Sec. 57) it will be noted that the passage of one coulomb will require that 0.0003387 gm. of zinc shall go into solution and that 0.0003294 gm. of copper shall be liberated from the electrolyte. From thermochemical relations it is known that when 1 gm. of zinc is *dissolved in sulphuric acid* approximately 1630 calories of energy in the form of heat are liberated. The electrochemical reaction at the zinc electrode would therefore involve an energy liberation given by the product of 1630×0.0003387 or 0.553 calorie. Since one calorie is equivalent to 4.2×10^7 ergs our reaction represents a liberation of 2.32×10^7 ergs.

When one gm. of copper is dissolved in sulphuric acid 881 calories of heat are liberated. Therefore the amount of energy involved in the *rejection of the copper from the solution* would be given by $881 \times 0.0003294 \times 4.2 \times 10^7 = 1.22 \times 10^7$ ergs. The difference between the energy liberated at the zinc electrode and the energy required to deposit the copper at the other electrode would be the energy available for driving the current around the circuit, or $(2.32 \times 10^7) - (1.22 \times 10^7) = 1.1 \times 10^7$ ergs.

In general, the work done in transferring a charge around the circuit would be given by $Q \times E$ joules, or $Q \times E \times 10^7$ ergs. In this case, $Q \times E \times 10^7 = 1.1 \times 10^7$. In our example we made Q = one coulomb; hence

$$E = 1.1 \text{ volts.}$$

As previously pointed out the known E.M.F. of the Daniell cell is approximately 1.1 volts.

The foregoing discussion is based on the assumption that the energy liberated as a result of the electrochemical reactions at the electrodes was completely converted into electrical energy. The close agreement of the computed and actual values indicates that such an assumption was warranted, at least in this particular case. There are however some cases in which all of the energy liberated by the reactions involved is not converted into electrical energy. In certain instances some of the liberated energy manifests itself

directly in the form of heat. Cells of this type become hotter during operation. In such cases *the E.M.F. decreases as the temperature of the cell rises.* In certain other cases energy in the form of heat is abstracted from the components of the cell and serves to augment the total energy available for maintaining the current in the circuit. Such a cell becomes colder when in operation and *its E.M.F. will show an increase with rise in temperature.* It is thus evident that some cells have what is known as a temperature coefficient, i.e., their E.M.F. is a function of their temperature.

By the application of the principles of thermodynamics to the reversible cell it is also possible to work out a relation giving *the E.M.F. of a cell in terms of the heat involved in the electrochemical reaction and the temperature change.*

The term "reversible" in this connection requires some explanation. Cells may be classified as *reversible* and *non-reversible* on the basis of the thermodynamic relations involved. These terms may be made clear by again considering our typical cell, the Daniell unit.

Suppose we apply to the terminals of such a cell an external and opposite E.M.F., slightly less in value than the E.M.F. of the cell being considered. Under these conditions the cell will cause a small current to pass through the external agent and through the cell itself. Zinc will be dissolved from the anode and copper will be deposited on the cathode. Suppose now that the externally applied E.M.F. be made larger than that of the cell being studied. The original process will now be reversed. Zinc will be deposited on the anode and copper will go into solution from the cathode. In short, the cell may be restored to its original condition. If we neglect one or two very slight losses in this process such a cell may be considered to be reversible. If our two electrodes had been immersed in a single electrolyte such as hydrochloric acid gas would have evolved when the cell was delivering current and the energy thus dissipated could not have been recovered; hence the process would have been non-reversible.

On the basis of the thermodynamic considerations involved Helmholtz developed a relation * which has the form

$$\text{E.M.F.} = H + T\frac{de}{dT}, \qquad \text{Eq. 81}$$

* A clear discussion of the deduction of this equation is to be found in *Electricity and Magnetism* by S. G. Starling, p. 187. See also Walker's *Introduction to Physical Chemistry*, p. 357.

where $\frac{de}{dT}$ is the rate of change of E.M.F. with absolute temperature, H the amount of heat measured in ergs, and T the absolute temperature. From this equation it is evident that if the temperature coefficient $\frac{de}{dT}$ is zero, $e = H$. In other words, under these conditions the energy liberated by the chemical reactions in the cell is just sufficient to account for the E.M.F. of the cell. Computed on the thermodynamic basis the E.M.F. of the Daniell cell turns out to be 1.112 volts, which is in fairly close agreement with the value obtained by other methods.

60. A Normal Electrode.—Reference has just been made to the nature of a reversible cell. If in such a cell we have an electrode such that the electrode potential is not changed as a result of the passage of a slight current through the cell in either direction, such an electrode is known as a *normal electrode.* There are a number of metals which, when immersed in solutions containing the corresponding ion, will fulfill the condition just indicated. The copper-copper sulphate combination of the Daniell cell forms such an electrode. This electrode is said to be reversible with respect to the kation. There are other electrodes which are reversible with respect to the anion. The most important example of this type is the *calomel electrode.*

If we use mercury as an electrode immersed in a solution of mercurous chloride (HgCl) and pass a current from the metal to the electrolyte mercury is changed to the chloride. Likewise if current passes from the electrolyte to the metal mercury is thrown out of solution and the resulting chlorine ion serves to neutralize the kation which took part in the movement of the current.

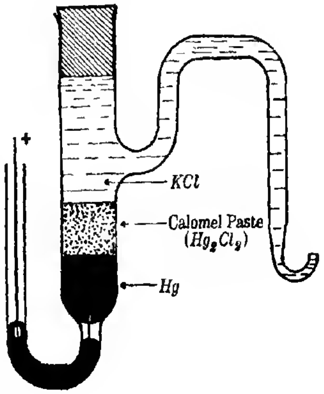


FIG. 74

It is possible to make up a so-called calomel electrode in a very convenient form and it has the important advantages over other possible normal electrodes in that it is readily reproducible and may be easily combined with other metallic electrodes in neutral solutions of their salts. There is

thus available a reference electrode of known potential value for purposes of comparison. Such an electrode is constructed as shown diagrammatically in Fig. 74.

The potassium chloride (KCl) in the connecting tube forms a "salt bridge" which serves to nullify the effect of any possible P.D. at the junction of the two electrolytes. The potential of the calomel electrode is $-$ 0.560 volt, the metal being positive.

By combining such an electrode with other electrodes immersed in their respective salts and measuring the E.M.F. of the cell as a whole by any convenient means, such as the potentiometer, it is thus possible to determine the potential of any electrode. As a result of measurements of this character the electrode potential values given in the following table have been determined.

The first column gives the potential difference between the element and a normal solution of the corresponding metallion. Such values are referred to as *absolute potentials*. The second column shows the potential when referred to the hydrogen electrode as zero. The values listed are taken from the *Standard Handbook for Electrical Engineers*, 4th edition.

Electrode Potential in Volts

Electrode	Absolute Potential	Potential Referred to Hydrogen Electrode
Bromine	−1.270	−0.993
Cadmium	+0.15	+0.420
Chlorine	−1.630	−1.353
Cobalt	−0.045	+0.232
Copper	−0.606	−0.329
Hydrogen	−0.277	0.000
Iodine..	−0.797	−0.520
Iron ..	+0 067	+0.344
Lead .	−0.126	+0.151
Manganese	+0.798	+1.075
Mercury	−1.030	−0.753
Nickel	−0.049	+0.228
Silver	−1.048	−0.771
Zinc.	+0.493	+0.777

An examination of the foregoing table will disclose the fact that if we were, for example, to use manganese as an electrode and immerse the metal in one of its normal salt solutions there

would be an E.M.F. of 0.8 volt developed at the surface of contact. The direction of this E.M.F. would be such as to drive the electrons from the electrolyte to the metal. If we were to form a complete cell by combining with the manganese a silver electrode also immersed in one of its normal salts there would be an E.M.F. of 1.04 volts developed at that contact, and the direction of the E.M.F. would tend to force the electrons from the metal to the electrolyte. Hence these two local sources of E.M.F. would be functioning in the same general direction with the result that their effect would be additive. It follows then that such a cell would have a terminal E.M.F. of 1.84 volts.

The question naturally arises as to whether the nature of the anion has any bearing on the magnitude of the electrode potential. Neumann has very thoroughly investigated this question. He found that the sulphates and chlorides when used as electrolytes with a given metal yielded slightly different values of electrode potentials. For instance, in the case of zinc immersed in a sulphate, the potential is 0.524; when the chloride forms the electrolyte the value is 0.503. Some metals however have the same electrode potential in different salt solutions. In any case, for the same degree of dissociation, the electrode potential is affected but slightly, if any, by the nature of the anion.

The degree of dissociation of the salt forming the electrolyte does however affect the magnitude of the electrode potential. In general the potential shows an increase with an increase in dissociation.

61. Concentration Cells.—If a cell be constructed of two like electrodes immersed in a common electrolyte of uniform concentration the P.D. between the electrolyte and the electrodes will be equal and oppositely directed with the result that the effective E.M.F. of the cell will be zero. It has however been found that electrode potential is a function of the ionic concentration. This is to be expected because of the fact that, as we have seen (Sec. 59), osmotic pressure is one of the factors which has a bearing on the electro-equilibrium conditions in a primary cell, and osmotic pressure is known to be directly proportional to the concentration of the dissolved substance.

Since electrode potential does depend upon ionic concentration it is possible to assemble a cell having like electrodes and a common electrolyte which will show a terminal E.M.F. which is not

zero. This is accomplished by arranging to have the electrolyte more concentrated in the region of one electrode than it is about the other. Such a cell is known as a concentration cell. The plan of a cell of this type may be represented thus:

copper	copper sulphate (concentrated)	copper sulphate (dilute)	copper.

The E.M.F. of such a cell is of small magnitude, being of the order of a few hundredths of a volt when the ionic concentration bears a ratio of one to ten. When such a cell is delivering energy the electrode in contact with the more dilute portion of the electrolyte goes into solution and metallic ion will be deposited on the other electrode. The latter terminal will therefore be positive and the former negative.

It may be shown * that the E.M.F. of such a cell is proportional to the absolute temperature and to the ratio of the concentration.

As a practical source of E.M.F. the concentration cell is of little value but the study of such a unit leads to several important observations. One obvious conclusion is that the E.M.F. of a primary cell of the Daniell type may be somewhat increased by *diminishing the concentration of the zinc sulphate solution.* It is also evident that the same end may also be attained by *increasing the concentration of the copper sulphate solution.*

The most important significance of the principles underlying the concentration cell lies, however, in the possibility of controlling the magnitude of electrode potential by means of variations in the ionic concentration of the electrolyte in which that electrode is immersed. We will note an important application of this principle in the next section.

62. Electrometric Analysis.—In the field of chemistry it is frequently necessary and important to know accurately the degree of ionic concentration of a given solution. Such information, for instance, makes it possible to determine the "acidity" or "alkalinity" of a solution. A knowledge of the ionic concentration of a solution also makes it possible to judge concerning the progress of a chemical reaction. The normal electrode described in Sec. 60 and the principles underlying the concentration cell

* For an analytical discussion of this type of cell see Starling's *Electricity and Magnetism*, p. 192.

(Sec. 61) find application in this connection. Because of the increasing use of these agents as scientific tools in both physical and analytical chemistry we will digress slightly to discuss what has come to be known in recent years as electrometric analysis.

This important process can perhaps be most easily made clear by outlining a typical application. Suppose it is desired to know the strength of a given acid. The older method consisted in adding a base until some "indicator" (frequently litmus) showed that neutralization had been effected. By the newer method we determine the "neutral point" by the electrical determination of the hydrogen-ion concentration. Figure 75 shows the electrochemical components involved in this process.

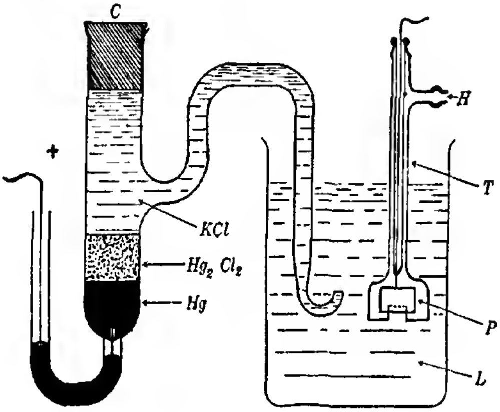


FIG. 75

The solution to be studied (acid in this case) is indicated by L. Into this dips the siphon connection of the standard calomel electrode C. Means (not shown) are provided for introducing a controllable amount of alkaline solution into the acid. That part of the apparatus designated by PT is known as a *hydrogen electrode*, a device first used for analytical purposes by Bottger * and later improved and simplified by Hildebrand.†

The hydrogen electrode consists of a piece of platinum foil P surrounded by a bell-shaped glass housing, this enclosure being for the double purpose of protecting the platinum and serving to confine the atmosphere of hydrogen admitted at H. A notch is cut in the bottom of the housing in order to allow the liquid under

* *Z. physik. Chem.*, 24, 253 (1897).

† *J. Am. Chem. Soc.*, 35, 849 (1913).

test to cover the lower part of the platinum foil. Before use the platinum foil is covered with platinum black; afterwards when hydrogen is admitted to the apparatus the gas is occluded by the platinum, and we have essentially a gas (hydrogen) electrode. When such an electrode is immersed in an electrolyte it shows a definite and reproducible potential with respect to the electrolyte. The combination of the calomel and hydrogen electrodes forms a primary cell whose E.M.F. is measured by means of a potentiometer * (Sec. 49).

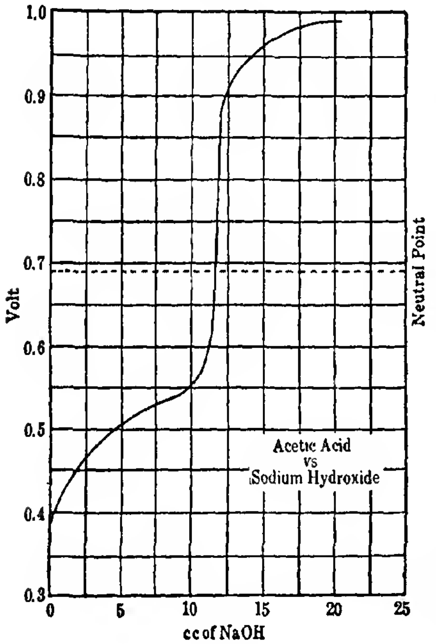


FIG. 76

As successive small additions of an alkaline solution, say sodium hydroxide, are made (a process called titration) the hydrogen-ion concentration will become less and hence the potential of the hydrogen electrode will become greater. The readings of the potentiometer will give the E.M.F. of the cell as a whole and from these values the varying potential of the hydrogen electrode may be determined. Thus we have an arrangement whereby the process of titration may be accurately followed. If we plot the amount of standard alkaline solution added to the acid against the E.M.F. of the hydrogen electrode a graph † such as that shown in Fig. 76 results. At the point on the curve designated as "neutral point" the acid was completely neutralized and thus the strength of the aci can be accurately determined.

It has been found ‡ that the absolute ionic concentration is

* Potentiometers are now available which are specially designed to facilitate the making of electrometric measurements.

† Hildebrand, *loc. cit.*

‡ Hildebrand, *loc. cit.*

given by the relation

$$E \text{ (in volts)} = 0.058 \log \frac{1}{C} + 0.28,$$

where C is the unknown ionic concentration. From this equation and a graph ionic concentration can be determined. A study of this and similar curves for other reactions yields highly important information concerning many chemical processes.

Experience with the electrometric method of observing chemical reactions has shown that this method possesses several important advantages over the older colorimetric procedure, among which may be mentioned: (*a*) color or turbidity of the solution does not affect the results; (*b*) the entire range of concentration, from normal acid to normal base, may be studied with one piece of apparatus; (*c*) titration to any particular point of acidity or alkalinity may be accurately carried out; (*d*) the "end points" of many precipitation reactions may be accurately determined.

The electrometric method is coming to be widely employed in the fields of biochemistry and agricultural chemistry and in the chemistry of foods. It is also finding a place in various manufacturing processes which involve chemical reactions.

For a more detailed discussion of this important subject the student interested in physical chemistry is referred to *The Determination of Hydrogen Ions* by W. H. Clark and to the section on "Electrometric Methods in Analytical Chemistry" in a work entitled *A Treatise on Physical Chemistry*, Vol. 2, edited by H. S. Taylor. The original paper by Hildebrand, *loc. cit.*, should also be consulted.

63. Polarization.—Returning again to the problems directly connected with the operation of primary cells, we encounter a fundamental phenomenon which manifests itself whenever certain types of primary cells are employed as a source of E.M.F.

We have already shown (Sec. 59) that the passage of current through a reversible cell does not change the chemical nature of the electrodes or the electrolyte. It has been pointed out, for instance, that, having a cell consisting of two copper electrodes immersed in copper sulphate, the application of the slightest E.M.F. will cause some current to pass through the cell. If however we have a combination consisting of carbon electrodes immersed in zinc sulphate and apply a definite external E.M.F. to

the terminal of such a cell it will be found that after a brief interval the current through the cell will diminish in value and unless the applied E.M.F. exceeds two volts the current will entirely cease. In the short time during which the current was passing through the cell a thin layer of zinc was deposited from the solution on the negative terminal (cathode), and one of our original carbon electrodes was therefore electrochemically transformed to another element. We now have a cell made up of carbon and zinc as electrodes with zinc sulphate as electrolyte. A test will show that the E.M.F. of this combination is about two volts, and that the direction of the E.M.F. is opposite to that of the original applied E.M.F., the carbon now being positive with respect to the zinc-plated rod.

The counter E.M.F. which has thus been established is known as the *E.M.F. of polarization,* and the phenomenon connected with the production of this opposing E.M.F. is known as *polarization.*

It is also possible to bring about polarization under somewhat different circumstances. If instead of using zinc sulphate as our electrolyte, in the cell just described, we had employed dilute sulphuric acid, decomposition of the solution would also have occurred but in this case hydrogen gas would have appeared at one of the electrodes (negative) and oxygen at the other. A counter E.M.F. would be manifest in this case also, having a value of about 1.7 volts. Here we have produced two dissimilar electrodes, one consisting of an extremely thin layer of hydrogen gas about the original cathode and the other being composed of molecules of oxygen clinging to the original anode. Thus we have established what amounts to a cell in which the electrodes are gaseous. In other words polarization has occurred, and in order to force a current through either this or the former cell we must apply an E.M.F. greater than the E.M.F. of polarization.

The process of polarization when brought about under circumstances similar to those outlined in the first case may be made use of in important ways, as we shall see shortly, but the type of polarization exemplified in the last instance becomes a troublesome phenomenon particularly in connection with the use of primary cells. It does not alter the essentials of the case if the gas is liberated as a result of the passage of the current through the cell due to its own E.M.F. rather than as a result of an ex-

ternally applied E.M.F. The liberation of hydrogen as a result of the operation of a cell itself, and its appearance at one of the electrodes, constitutes one of the serious defects of primary cells. Not only is a counter E.M.F. set up, but the existence of the layer of gas molecules introduces a high resistance into the internal circuit of the cell. As a result the effective E.M.F. of the cell is decidedly reduced. A number of plans have been devised to prevent the hydrogen from reaching the positive electrode, and thus prevent polarization. It is chiefly because of these various depolarization schemes that we have the several types of existing primary cells.

In designing a primary cell there are several important ends to be sought among which may be mentioned, (*a*) highest possible E.M.F., (*b*) rapid and complete depolarization, (*c*) low internal resistance, (*d*) low cost of materials, (*e*) absence of local action.

64. Local Action.—Because of its position in the electrochemical series (Sec. 60) and also due to its relative cheapness zinc is most commonly employed as the material for one of the electrodes in primary cells. Due to the impurities frequently present what is known as *local action* takes place. A bit of impurity, iron for instance, forms with the surrounding zinc and the electrolyte a tiny local primary cell, with the result that a local E.M.F. is developed which in turn gives rise to a locally circulating current. Thus the electrode tends to disintegrate even when the cell is not in use.

To prevent this recourse is had to what is known as amalgamation. This consists in mechanically or chemically * bringing mercury into contact with the surface of the zinc and thus forming a zinc-mercury amalgam. The impurities do not unite with the mercury and are covered up by the zinc-mercury amalgam formed on the surface of the electrode. As the zinc is dissolved the mercury remains and unites with the remaining zinc, the impurities falling to the bottom of the cell.

Local action may also be caused in some cells by concentration effects. We have seen (Sec. 61) that an E.M.F. may result from a difference in concentration between two points in an electrolyte. Near the surface of the electrolyte the density will tend to be less

* The most satisfactory method of amalgamating battery zincs is by immersion for a few minutes in a solution made by dissolving mercury in "aqua regia," convenient proportions being, mercury 15 c.c., nitric acid 170 c.c., hydrochloric acid 625 c.c.

than in the body of the solution. This gives rise to a local E.M.F. and the consequent local wasting of the electrode at a point near the surface of the solution.

65. Examples of Practical Primary Cells.—The modern "crowfoot" or "*gravity*" *cell* is essentially a Daniell cell (Fig. 77). It consists of a negative electrode composed of zinc, commonly in the form of a "crowfoot," and a number of copper strips riveted together serve as the positive terminal. A saturated solution of copper sulphate surrounds the copper electrode and the zinc electrode is immersed in a solution of zinc sulphate. Partial separation of the two electrolytic solutions is maintained as a result of the difference in density of the two electrolytes, hence the name, gravity cell.

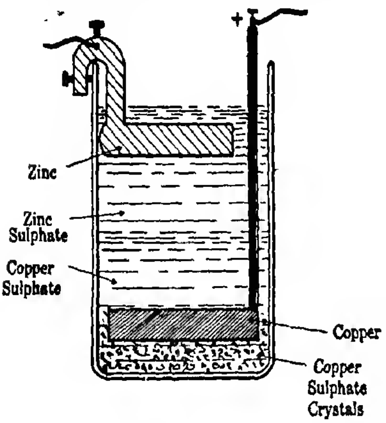


FIG. 77

Depolarization is rapid and complete in both this and the Daniell cell, and is accomplished as a result of the presence of the two electrolytes. The reactions involved may be represented as follows. Zinc passes into solution at the anode,

$$Zn^{++} + H_2SO_4 \rightarrow ZnSO_4 + 2H^+.$$

Simultaneously copper is deposited at the cathode,

$$CuSO_4 \rightarrow Cu^{++} + SO_4^{--}.$$

Where the two solutions are in contact the hydrogen ions and the SO_4 ions unite to form sulphuric acid thus,

$$2H^+ + SO_4^{--} \rightarrow H_2SO_4.$$

It will thus be seen that hydrogen does not reach the copper electrode and hence polarization does not occur. The gravity cell gives an E.M.F. of about 1.08 volts but its internal resistance is comparatively high, being of the order of one ohm; hence it can deliver only a relatively small current even on short circuit. Because of the fact that polarization is absent this cell may be

used in closed circuit work; it has been extensively employed in telegraphic circuits.

The modern *dry cell* is an outgrowth of a cell originally known as the *Leclanché cell.* The original consisted of a zinc and a carbon electrode in a single electrolyte, ammonium chloride (sal-ammoniac). In the Leclanché cell the positive electrode (carbon) was inclosed in a porous cup in which were packed manganese dioxide and granular carbon. Diffusion of the solution took place through the porous cup and its contents. In the modern portable form of this cell the porous cup is dispensed with and the zinc electrode forms the container. (See Fig. 78.) The positive electrode (a carbon rod) forms the center of the unit. This is surrounded by a mixture of manganese dioxide, granular carbon and zinc chloride. This in turn is surrounded by a layer of absorbent material such as sawdust, and a layer of blotting paper next to the zinc container. The sawdust and paper are saturated with a solution of ammonium chloride. The top of the cell is sealed to prevent evaporation.

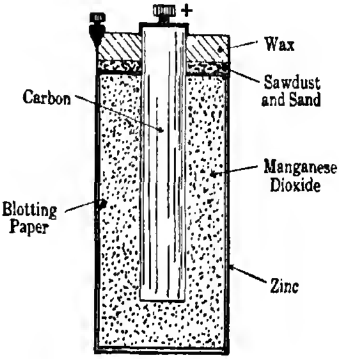


FIG. 78

When current is passing through the cell we have as a result of dissociation

$$NH_4Cl \rightarrow NH_3 + H^+ + Cl^-.$$

At the negative electrode (anode),

$$Zn^{++} + 2Cl^{--} \rightarrow ZnCl_2.$$

Simultaneously at the cathode the hydrogen undergoes oxidation thus,

$$2MnO_2 + 2H^+ \rightarrow Mn_2O_3 + H_2O.$$

Under the circumstances this reaction takes place slowly; hence if the circuit is closed for any appreciable length of time excess hydrogen will accumulate and polarization will occur. This type of cell is therefore not adapted for closed circuit work. Notwithstanding this limitation it is very extensively used for a wide variety of purposes. It is said that no less than fifty million dry

cells are made and used in the United States annually. The E.M.F. of the dry cell is about 1.5 volts and the internal resistance of the standard No. 6 size varies from 0.05 to 0.1 ohm.

Another form of single-electrolyte cell finds wide application in signal circuit work, particularly in connection with the operation of railroad block signals. This cell utilizes two zinc plates connected in parallel as the negative terminal, and a block of compressed copper oxide held in a copper band serves as the positive electrode (Fig. 79). The electrolyte is a concentrated solution of

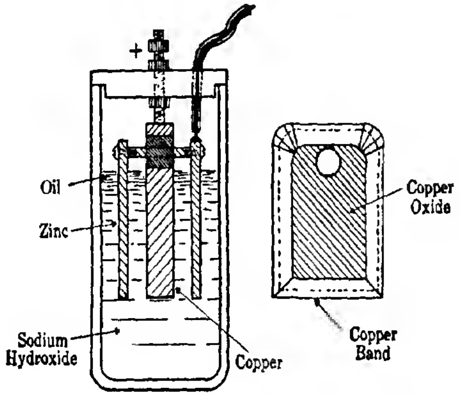


Fig. 79

sodium hydroxide (caustic soda). A layer of heavy mineral oil is placed on the top of the electrolyte, thus preventing evaporation and the "creeping" of the solution. The cell is ruggedly built and may be left for long periods without attention. It was originally devised by Dr. Lalande but the modern form is due largely to Edison and is now known as the *Edison primary cell.*

When this cell is delivering energy, we have at the negative electrode,

$$Zn^{++} + 2NaOH \rightarrow ZnNa_2O_2 + 2H^+.$$

At the cathode the hydrogen in reducing the copper oxide is itself oxidized thus,

$$2H^+ + CuO \rightarrow H_2O + Cu^+.$$

The last reaction takes place rapidly and hence polarization does not take place. Local action is also absent. The E.M.F. of the cell is about 0.75 volt. Owing to the area of the electrodes and the short distance of separation the internal resistance is low, varying from 0.02 to 0.1 ohm, depending on the size of the unit.

The cell is therefore capable of delivering large currents. The commercial form of the Edison primary cell is made in sizes ranging from 100 to 600 ampere-hours,* and the elements of a given cell are so designed that they all become exhausted at the same time, thus facilitating renewal.

The primary cell giving the most constant E.M.F., and which can be accurately reproduced, is known as the *Weston standard cell.* Figure 80 shows the general plan of the cell assembly. Mercury in contact with saturated mercurous sulphate forms the positive electrode while cadmium amalgam in contact with saturated cadmium sulphate serves as the negative element. This cell was designed by Dr. Edward Weston and has, because of its reliability, become the standard of E.M.F. throughout the world. The International Electrical Conference (London, 1908) set up specifications for the construction of the Weston cell, and when the unit is assembled according to those instructions the E.M.F. of different cells will not differ from one another by more than two or three parts in 100,000.

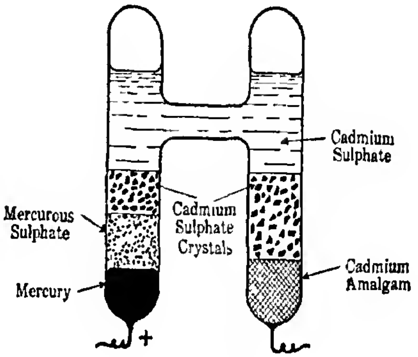


FIG. 80

The E.M.F. of the cell is 1.01830 volts at 22° C. The temperature coefficient is extremely low, being – 0.00005 volt per degree Centigrade. The Weston standard cell is the cornerstone, so to speak, of all electrical measurements.

66. Secondary Cells or Accumulators (Storage Batteries).—In our discussion of the theory of primary cells (Sec. 62) attention was directed to the fact that such a cell consists essentially of two electrodes of *unlike material*, and that at least one of these electrodes was, during the use of the cell, gradually *converted by electrochemical action into a different substance.* It was further noted that, in the case of certain primary cells (the reversible

* Batteries are rated on the basis of ampere-hours. For instance, a 150 ampere-hour cell will deliver 10 amperes for 15 hours. All batteries have a *normal discharge rate.* If this rate is exceeded the cell will in general not deliver its rated capacity.

type), the cell, after use, might be restored to its original condition by sending a current through the unit in the reverse direction. The Daniell cell was found to be of this type. Obviously then such a cell could be utilized for the conversion of electrical energy into chemical potential energy and *its re-conversion into electrical energy*. When used for such a purpose the cell would be called *a secondary cell or accumulator*. While such a process is theoretically possible in the case of any reversible cell certain practical considerations make it advisable to utilize a special combination of electrodes and electrolyte in the construction of secondary or storage cells.

From our previous study of the theory of cells it is evident that in order to produce a cell which will have a maximum E.M.F. we should select two elements which are far apart in the electrochemical series. It happens that if a *compound* consists of a metal and some very electronegative element such as oxygen the compound (in this case the oxide) will be electronegative with respect to the metal, and the greater the percentage of oxygen in the compound the more electronegative will it be. It is an interesting fact that the oxide of a comparatively cheap metal, viz., lead, is highly electronegative with respect to lead itself. Furthermore, lead and lead oxide (PbO_2) in sulphuric acid form a reversible cell.

In his work on the polarization of metals in electrolysis M. Gaston Planté in 1860 observed that an electrolytic cell showed a reverse E.M.F. after the applied E.M.F. was disconnected, and that this phenomenon was particularly marked when lead served as the electrodes. This was the beginning of the development of the lead accumulator.*

To better understand the theory of the secondary cell let us examine what happened in Planté's original experiment. If we immerse two pieces of clean lead in dilute sulphuric acid and connect the terminals to a source of E.M.F. exceeding 2.2 volts electrolysis will take place. Hydrogen will be liberated at the negative electrode (cathode) and oxygen at the positive terminal (anode). The metallic lead will be attacked by the oxygen forming a coating of brown lead oxide (PbO_2) on the anode. The hydrogen will not react with lead except to more effectively

* In passing it is interesting to note that the secondary cell was devised before dynamos came into use.

clean it. We began our hypothetical experiment with *two like plates.* As a result of electrolysis *one of these electrodes, in part at least, was changed to an entirely different substance,* so that we now have a lead electrode and a lead oxide electrode. In other words we have, by expending energy in the form of the electrical current, manufactured *a new electrode.*

If now we disconnect the source of outside current and test our cell it will be found that the electrode coated with lead oxide is positive with respect to the metallic lead plate, the terminal E.M.F. being about 2.2 volts. It will also be found that if we connect the terminals of our cell to some electrical device such as a door bell a current will flow for several minutes, gradually decreasing in value.

If we examine our cell after having used it *as a secondary source of electrical energy* it will be found that electrochemical reaction has resulted in the changing of the lead peroxide back to metallic lead, and the cycle of operations has thus been completed. During the first stage of our experiment, commonly but erroneously called "charging," electrical energy was converted into chemical potential energy. During the second stage of the process this chemical potential energy was made use of to produce an electric current exactly as is the case in a primary cell. After the cell was "charged" it did not contain one electron more of electricity than before the process started.

In the simple experiment just described only a comparatively small amount of oxide was formed and hence but little energy was "stored." Planté found that it was possible to materially increase the amount of peroxide formed by a given charging current if the plates were first put through a process known as "forming." This consisted in a series of reversals—charging first in one direction, discharging, allowing to stand for a time and then charging in the reverse direction. By this process * a superficial layer of the electrode is transformed into "active material" which results in a greatly increased efficiency. The process of forming is however quite expensive and as a result the Planté type of plate has to a great extent been replaced by one made by a more rapid and less expensive procedure.

* For a clear and comprehensive treatment of the chemistry and physics of the lead accumulator the student is referred to a small book entitled *Storage Batteries* by Dr. H. W. Morse, and to a larger volume, *Storage Batteries,* by M. A. Arendt.

M. Camille Faure introduced a type of electrode for secondary cells which is known as the *Faure or pasted plate.* This consists of a grid of lead (see Fig. 81) into the interstices of which is forced a paste consisting of a mixture of lead oxide and dilute sulphuric acid. Red lead (Pb_3O_4) is used in the positive grid and litharge (PbO) in the negative plate. As a result of the use of this

FIG. 81.—LEAD STORAGE BATTERY PLATES

"pasting" process a part of the work of forming the plates is done chemically and thus the time of preparing the plates for use is greatly lessened and the manufacturing cost materially reduced.

The Faure type of electrode is lighter in weight but is not so rugged as the Planté plate. Owing to its lighter weight and relative cheapness the Faure type of plate is used largely for portable units. It is also used to some extent in stationary installations, though the Planté type is still used to some extent in large fixed plants.

While the exact electrochemical reaction which takes place in a lead secondary cell is somewhat uncertain the following equations probably represent what takes place.

The dissociation of the electrolyte gives

$$H_2SO_4 \rightarrow H_2^+ + SO_4^{--}.$$

During "charge" $\begin{cases} \text{At the anode or "positive plate,"} \\ PbSO_4 + SO_4^{--} + 2H_2O \rightarrow PbO_2 + 2H_2SO_4. \\ \\ \text{At the cathode or "negative plate,"} \\ PbSO_4 + H_2^+ \rightarrow Pb^{++} + H_2SO_4\,. \end{cases}$

$$\text{During ``discharge''}\begin{cases}\text{At the anode,}\\ PbO_2 + H_2SO_4 + H_2^{+} \rightarrow PbSO_4 + 2H_2O.\\ \\ \text{At the cathode,}\\ Pb^{++} + SO_4^{--} \rightarrow PbSO_4.\end{cases}$$

These reactions show that during the charging process lead peroxide is formed on the positive grid and spongy lead on the negative plate. The amount of sulphuric acid in the electrolyte also increases and hence the specific gravity of the solution rises. While the cell is delivering energy in the form of electric current both the lead and the lead peroxide revert to lead sulphate, thus completing the electrochemical cycle. During these electrochemical reactions the active material in the plates undergoes considerable expansion and contraction. As the cell is repeatedly charged and discharged non-reversible reactions also tend to take place. Both of these factors operate to limit the "life" of the battery. The positive plates disintegrate much more rapidly than the negative grid.

The ampere-hour capacity of a secondary cell is proportional to the amount of active material available in the plates. To secure large capacity cells are usually made of a series of positive and negative plates (Fig. 82), like grids being connected in parallel. Such an arrangement not only serves to augment the capacity but also serves to reduce the internal resistance, which, in most cases, is only a few hundredths of an ohm. If a cell is to be used under conditions requiring a heavy discharge current the plates are made comparatively thin, thus allowing freer access of the electrolyte to the active material and hence a more rapid reaction.

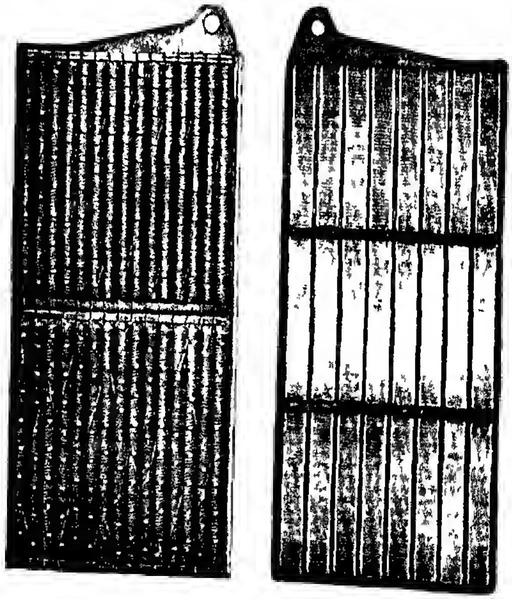
FIG. 82.—LEAD STORAGE BATTERY ASSEMBLY

In speaking of the ampere-hour capacity of a secondary cell it should be borne in mind that the rate of discharge is an important

factor in this connection. Any given cell is designed to be charged and discharged at a certain predetermined rate known as the normal charge and discharge rate. The ampere-hour rating of a cell is based on its normal rate of discharge.

A freshly charged secondary cell shows a terminal E.M.F. of about 2.2 volts which becomes less on discharge, and should never be carried below 1.8 volts. Further discharge brings about certain non-reversible reactions in the active material, thereby more or less permanently reducing the capacity of the cell. In practice it is found advisable to take the density or specific gravity of the electrolyte as an index of the condition of the battery. The concentration of the electrolyte used in a given cell depends somewhat on the type of plates used and the character of the service for which the battery is designed. In the case of stationary batteries the sp. gr. of a fully charged cell will be of the order of 1.23 and when completely discharged will show a reading of about 1.15. The electrolyte of cells used in connection with automobiles and for other similar purposes has a sp. gr. of 1.27 to 1.30 when charged and 1.15 to 1.18 when discharged.

By the efficiency of a storage battery is meant the ratio of the watt-hour output to the watt-hour input, and on this basis a good



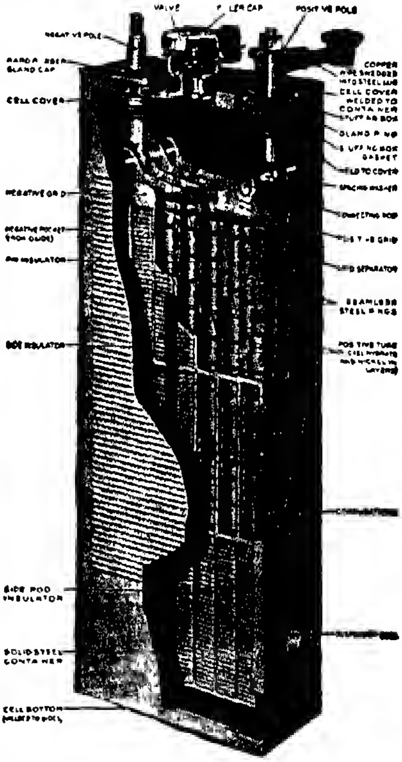
(Courtesy Edison Storage Battery Co.)
FIG. 83.—EDISON STORAGE BATTERY PLATES

secondary cell has an efficiency of about 75 per cent. The life of a good accumulator when adapted to the class of service it is called upon to render, and given proper care, should be not less than 200 cycles of charge and discharge.

While the lead secondary cell has a comparatively high efficiency and is extensively used for many purposes it has, nevertheless, certain inherent disadvantages chief among which are its weight, its tendency to lose capacity with use, the more or less undesirable character of the electrolyte (a strong acid), and the necessity of careful supervision at all times. Many attempts have been made to produce an accumulator unit in which these features would be absent or at least be present to a lesser degree. The *Edison storage battery* has, to some extent, accomplished this end.

In the secondary cell developed by Mr. Edison we have a unit which is in some respects radically different from the lead cell. The active material of the positive plate in the Edison cell is nickel hydrate [$Ni(OH)_2$], and that of the negative plate, iron oxide (FeO). The electrolyte is a 21% solution of potassium hydrate (KOH), to which is added a small amount of lithium hydrate.

The mechanical construction of the plates and the cell as a whole is unique. Figures 83 and 84 show the detailed construction and plan of the cell assembly. The positive grid consists of thirty small perforated nickeled steel tubes rigidly fastened to a nickeled steel frame. These tubes are packed with alternate layers of nickel hydrate and pure nickel flake, the latter



(Courtesy Edison Storage Battery Co.)

FIG. 84.—EDISON STORAGE BATTERY ASSEMBLY

constituent being introduced for the purpose of increasing the electrical conductivity of the active material. The negative grid is made up of a series of perforated nickeled steel pockets into which is packed the iron oxide, to which is added a small amount of mercury, the latter element serving to increase the conductivity. The metallic pockets are also rigidly fastened to a nickeled steel frame. The entire assembly of positive and negative grids is solidly bolted together and supported in a housing which consists of a corrugated nickel-plated steel container. It will thus be seen that the Edison cell is an extremely robust unit and therefore adapted to mechanically severe types of service.

The exact chemical reactions which take place in the Edison cell have not as yet been definitely determined. However it is probable that the following represent about what takes place.

By dissociation we have

$$KOH \rightarrow K^{+} + (OH^{-}).$$

$$\text{On charge} \begin{cases} \text{At positive grid,} \\ 2Ni(OH)_2 + 2(OH^{-}) \rightarrow 2Ni(OH)_3. \\ \\ \text{At negative grid,} \\ FeO + 2K^{+} + H_2O \rightarrow Fe^{++} + 2KOH. \end{cases}$$

$$\text{On discharge} \begin{cases} \text{At positive grid,} \\ 2Ni(OH)_3 + 2K^{+} \rightarrow 2Ni(OH)_2 + 2KOH. \\ \\ \text{At negative grid,} \\ Fe^{++} + 2(OH^{-}) \rightarrow FeO + H_2O. \end{cases}$$

From these reactions it is evident that the electrolyte remains unchanged during charge and discharge; hence there is no change in the sp. gr. of the solution during a cycle of operation. The only function performed by the electrolyte is to serve as a medium for the transfer of the hydroxyl ion from one plate to the other. It would appear that the nickel-iron-hydrate combination as arranged in the Edison cell makes possible *a group of completely reversible reactions*. In this respect this cell differs from the corresponding lead unit in which the reactions are not completely reversible and become less so as the age of the battery increases.

The capacity of the nickel-iron cell increases after it is put into commission. It is said that cells of this type which had been in

car lighting service for more than thirteen years developed 116 per cent rated capacity. The effective life of the Edison portable units is at least three times that of the lead cell and in some instances much greater than this. On the basis of watt-hours per pound the Edison cell is considerably lighter than the lead unit.

The electrical characteristics of the Edison cell are also different from those of the lead unit. Its terminal E.M.F. is about 1.4 volts when fully charged, falling to approximately 1 volt when discharged. Thus the average E.M.F. is about 1.2 volts. The internal resistance of the cell is somewhat higher than that of its rival, resulting in an energy efficiency of something like 60 per cent. The cell is not damaged by short-circuiting; can be completely discharged without injury; and may be left unused, either charged or discharged, for an indefinite time. Because of the fact that there is no change in sp. gr. of the electrolyte the hydrometer test would give no indication of the state of charge or discharge; hence the condition of the cell is judged by the terminal voltage. Because of the nature of the materials entering into the construction of the Edison cell its initial cost is about five times that of the lead unit, but when length of battery life is considered the cost is approximately the same for both types.

CHAPTER XIII

THERMOELECTRIC PHENOMENA

67. Thermoelectric Couples.—In 1822 T. J. Seebeck announced to the Berlin Academy of Sciences the discovery of an entirely new method of producing an electric current. Seebeck arranged a circuit of two metallic bars one of which was copper and the other bismuth, the two pieces of metal being soldered together at their ends, as illustrated in Fig. 85. A magnetic needle N was placed as shown. When one of the junctions was warmed Seebeck found that the magnetic needle was deflected and in such a direction as to indicate that a current flowed across the hot junction from bismuth to the copper. Further investigation showed that *the magnitude of the effect is a function of the difference in temperature between the two junctions*, and also that various combinations of metals give different results. It thus became evident that thermal energy may be utilized to establish an E.M.F. and thus be directly converted into electrical energy. A pair of metals so arranged that one junction of the pair may be maintained at a different temperature than the other and thus be used to develop an E.M.F. has come to be known as a thermo-element or thermocouple.

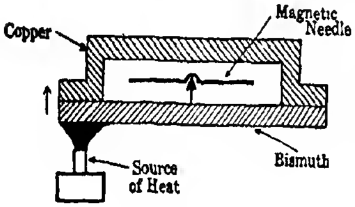


Fig. 85

On the basis of his observations Seebeck arranged a number of the metals in a series such that, when any pair of the metals are employed as a thermocouple, the current flows across the hot junction from the one occurring earlier in the series to the one appearing later in the list. Seebeck's list included, among others,

Bismuth	Mercury	Silver
Nickel	Lead	Zinc
Cobalt	Tin	Tungsten
Palladium	Chromium	Cadmium
Platinum	Molybdenum	Iron
Uranium	Rhodium	Arsenic
Copper	Iridium	Antimony
Manganese	Gold	Tellurium
Titanium		

At best the magnitude of the E.M.F. developed by a thermocouple is very small. For instance, using an antimony-bismuth pair, the E.M.F. developed per degree C. difference in temperature between 0° and 100° is 0.000057 volt. Hence if one of the junctions of such a couple were maintained at 0° and the other at 100° the E.M.F. produced would be less than 6 millivolts.

Notwithstanding the low E.M.F. produced by thermal means it was hoped, immediately following Seebeck's discovery, that the thermocouple might be utilized for the production of electrical energy directly from heat on a practical scale. Indeed Clamond designed a thermoelectric battery consisting of a large number of heavy metal bars so arranged that the E.M.F. developed would be additive when similar junctions of the entire series were simultaneously heated. Employing 120 pairs and heating the junctions by means of a gas flame, he secured an E.M.F. of 8 volts; the efficiency was however extremely low, being less than one per cent.

Though the thermojunction method of producing currents for power purposes has not proven feasible, nevertheless this device has found an important field of usefulness in connection with temperature and radiation measurements. As a result of the work of the Italian physicist, Meloni, and two American investigators, A. H. Pfund and W. W. Coblentz, the sensibility of the thermocouple has been marvelously increased and has thus become a research tool of great importance. We shall discuss certain uses of this agent in a later section.

68. Laws of Addition of Thermal E.M.F.'s.—Two simple but important laws in connection with the use of thermocouples have been experimentally established. In using a thermocouple it is obviously necessary to introduce some current or potential indicating device into the circuit in series with the metals forming the original junction. This naturally involves the presence of additional metallic contacts in the circuit which will, in general, constitute thermojunctions and which will therefore develop thermal E.M.F.'s. The question arises as to how these additional junctions will affect the E.M.F. developed in the circuit as a whole. Experiment has shown that the introduction of one or more pieces of metal into the circuit does not change the total E.M.F. *provided the junctions thus introduced are maintained at the same temperature as that of the point in the circuit where they were inserted.* This fact is known as the *law of intermediate metals.*

For instance, in the circuit shown in Fig. 86, the presence of the copper connecting wires will not alter the E.M.F. developed in the circuit if the temperature of the points x and x' is maintained at the same value, this value being what it was or would have been before the connecting wires were inserted. It is therefore also evident that the two metals M_1 and M_2 forming the couple may be soldered together at one of the junction points without changing the E.M.F. developed by the couple.

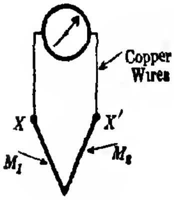


FIG. 86

The second law has to do with the relation which exists between temperature and the E.M.F. developed, and is referred to as *the law of successive temperatures.* It is to the effect that for any given thermocouple the E.M.F. developed when the junctions of the couple are maintained at any two specified temperatures is equal to the sum of the E.M.F.'s which would be produced if the couple were operated between successive temperature steps throughout the original range. Symbolically this fact may be represented thus:

$$E_{t_1}^{t_n} = E_{t_1}^{t_2} + E_{t_2}^{t_3} + \cdots E_{t_{n-1}}^{t_n},$$

where t_1, t_2, t_3, etc., are any successive temperature values between t_1 and t_n. We shall find this relation useful in the next part of our discussion.

69. Thermoelectric Curves.—Before proceeding to a description of practical applications of the thermocouple we may well examine the relation which obtains between the temperature factor and the E.M.F. developed.

If we set up a thermocouple circuit as sketched in Fig. 87 and apply heat as shown, gradually raising the temperature of the oil-immersed junction, the deflection of the galvanometer will increase for a time, but if the heating be continued beyond a certain temperature the current will gradually decrease to zero and will in fact reverse in direction if the heating be continued far enough. If we make a graph from the data obtained by such an

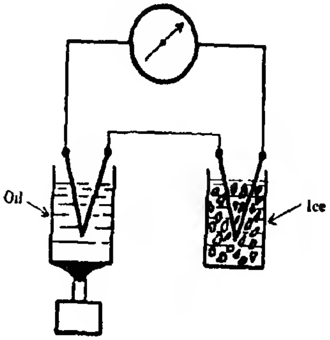


FIG. 87

experiment a curve of the form shown in Fig. 88*a* will result. It is found that a similar curve results for other thermocouple combinations.

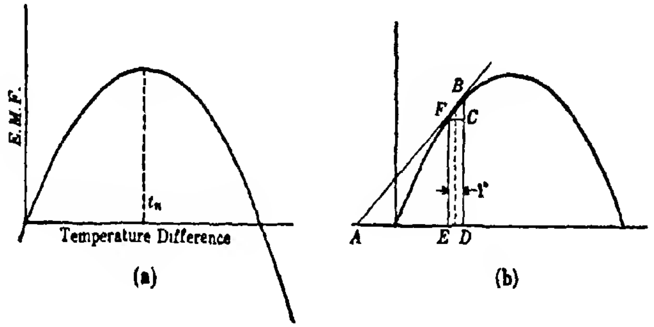


FIG. 88

An examination of our curve will disclose the fact that it is parabolic in form (axes parallel to E.M.F. axis), and consequently may be represented by the equation $y = ax + bx^2$. In our case this relation becomes

$$E_0{}^t = at + bt^2, \qquad \text{Eq. 82}$$

where $E_0{}^t$ represents the E.M.F.; t the temperature of the hot junction, the other being at 0°; a and b constants depending on the nature of the particular pair of metals used. If we differentiate eq. 82 with respect to t we get

$$\frac{dE}{dt} = a + 2bt, \qquad \text{Eq. 83}$$

in which $\frac{dE}{dt}$ will represent the slope of the curve, $\left(\frac{BD}{AD}, \text{Fig. 88}b\right)$, and hence the rate of change of E.M.F. with temperature. If we make the temperature interval one degree, represented by ED or FC, Fig. 88*b*, the corresponding change in E.M.F. will be represented by BC. Under these conditions $\frac{dE}{dt}$ is known as the *thermoelectric power* of that particular couple.

In practice it is frequently convenient to know the value of the E.M.F. developed by some given pair of metals when operated between any two known temperatures. Professor Tait has shown that it is possible to set up a simple diagram by the aid of

which one may readily compute the value of the E.M.F. produced in any given case.

Referring again to eq. 83 it will be noted that it is of the form $y = a + 2bx$, and that this represents a straight line. Further, the tangent which the line in question makes with the x-axis is given by $2b$.

Suppose then, in the typical case cited, we plot the thermoelectric power, $\frac{dE}{dt}$, against temperature, as shown in Fig. 89; the straight line AA' will result. In this graph OA represents the constant a

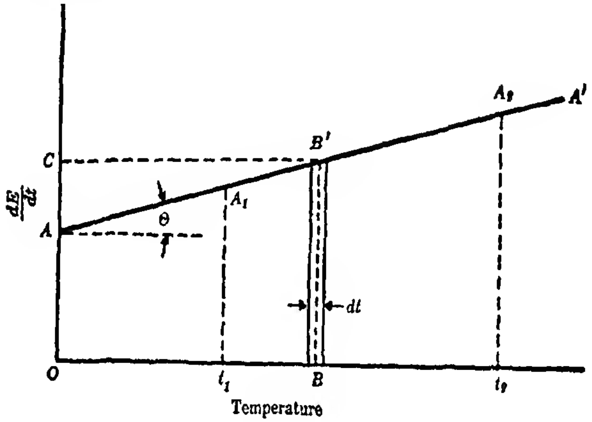


FIG. 89

and $\tan \theta = 2b$. At a temperature represented by t, the median point of the narrow temperature range dt, the thermoelectric power is represented by OC, the height of the elemental strip BB'. The area of the strip $= \frac{dE}{dt}dt = dE$. Therefore the elemental area BB' represents the small E.M.F., dE, developed by the thermocouple when its two junctions are at the infinitesimal difference of temperature represented by dt.

From the law of successive temperatures (Sec. 68) it follows that the total E.M.F. developed by the couple when its junctions are maintained at the temperatures t_1 and t_2 respectively will be represented by the area $A_1A_2t_1t_2$ in Fig. 89. Geometrically the area is equal to the product of one half the sum of the two thermoelectric power ordinates and the distance representing the difference in temperature $t_1 - t_2$. It may therefore be said that *the E.M.F. developed by a thermocouple whose junctions are maintained*

at two different temperatures is equal, numerically, to the product of the average thermoelectric power and the difference in temperature.

70. Thermoelectric Diagram.—Let us now set up and test a thermocouple using as one of the elements one of the metals which was utilized in the discussion of the last section. If we plot the thermoelectric power of our new couple against the temperature of the heated junction, as we did in the previous case, we will have a new thermoelectric line BB', Fig. 90, AA' being the corresponding line for our original couple.

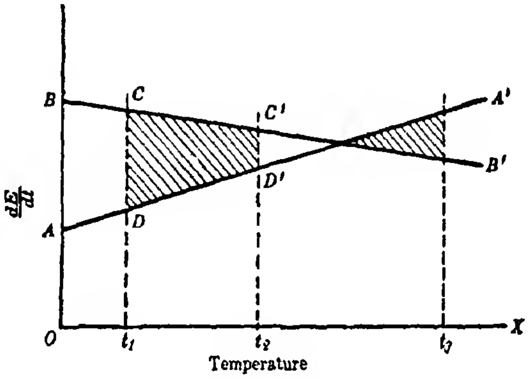


Fig. 90

The principles established in our last discussion (Sec. 69) may be extended to include the third metal utilized in our new couple. We may accordingly determine the E.M.F. developed by the second and third metals acting as a couple by finding the area $CC'DD'$.

It is thus evident that if some one metal be taken as a standard of reference we can extend our diagram to include as many metals as desired. For reasons to be pointed out later lead is usually taken as the reference metal, though platinum is also sometimes used as a basis. Such a thermoelectric diagram showing the lines for a number of metals is given as Fig. 91. The figure is based on one given by Noll in *Wiedemann's Annalen*, Vol. 53, p. 874. The point where two thermoelectric lines intersect represents the neutral temperature t_n in Fig. 88. If the temperature of one of the junctions falls to the right of the neutral temperature point, say at t_3, Fig. 90, the resultant E.M.F. of the couple under these circumstances would be given by the *difference* in the two shaded areas.

As an example of the use of the thermoelectric diagram suppose we take our original iron-copper combination and maintain one junction at 50° and the other at 150°. The mean difference in temperature will then be 200°/2 or 100°. From the diagram (Fig. 91) we see that the mean thermoelectric power for copper and iron corresponding to the temperatures 50° and 150° is

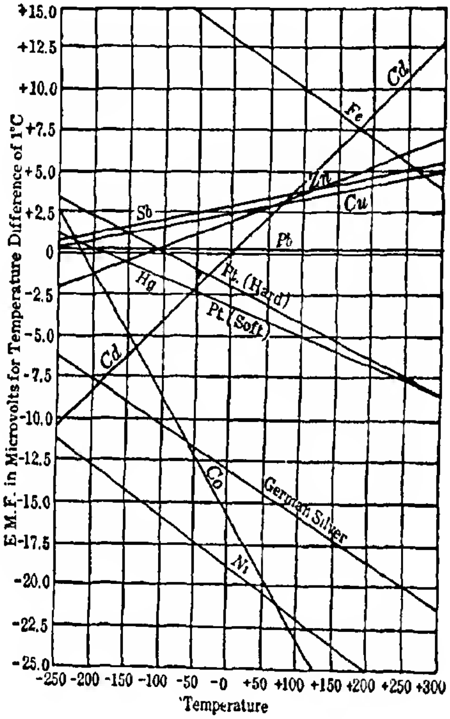


Fig. 91

about 6.5 microvolts. The total E.M.F. therefore which would be developed by the iron-copper couple under these conditions would be 6.5 × 100 = 650 microvolts.

The student will find a list of thermoelectric power values for various metals given in *Smithsonian Physical Tables*, 7th ed., p. 317 *et seq.*

71. Peltier Effect.—In 1834, twelve years after Seebeck (Sec. 67) made his discovery, J. C. Peltier of Paris discovered a phenomenon which is the converse of the Seebeck effect. He demonstrated that an electric current when passed across a thermojunction will, under certain circumstances, cause an in-

crease in temperature and under certain other conditions a lowering of the temperature. For instance when Peltier passed a current across an antimony-copper junction from the former to the latter element he secured a rise of 10° in temperature; when the current was reversed the temperature was diminished 5°. An antimony-bismuth couple gave even more marked effects. In fact it is said that Lenz, of whom we shall hear later in another connection, was able to freeze water by the Peltier effect.

It is not difficult to account for the Peltier effect. Our discussion of the Seebeck effect (Sec. 67) led to the conclusion that the application of heat to a metal changes its electrical properties, and that different elements are affected differently. (We are not referring here to changes in electrical resistance.) Furthermore the law of conservation of energy must hold in the case of thermocouples as well as elsewhere. If we refer again to Seebeck's original experiment as shown in Fig. 85, it is evident that an E.M.F. was developed at the hot junction, which E.M.F. caused a current to flow around the circuit. The energy represented by the current was supplied by the applied flame; in other words *heat disappeared at that junction.* In our study of the primary cell (Sec. 59) we saw that, if no energy were dissipated, the process which gave rise to an E.M.F. was reversible. In the case now under discussion the process proves to be, in part at least, a reversible one. When a bismuth-antimony junction is heated the direction of the current is as shown in Fig. 92. If the external heat source be removed and current from an outside source be passed across the junction heat will be absorbed and the junction cooled. Conversely, if current from an outside source be sent across the junction in a direction opposite to the normal thermoelectric current, as indicated in Fig. 92, the current will be converted into heat and the heat will be liberated. It is evident that these phenomena are entirely consonant with the law of the conservation of energy.

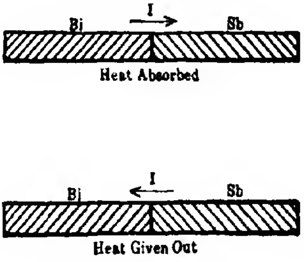


FIG. 92

It can also be shown that the Peltier effect is in conformity with the laws of thermodynamics, but it is beyond the scope of

this volume to enter further into this aspect of the case. In passing, however, it may be noted that the heat absorbed by the thermoelectric current at the heated junction is given out again at the colder junction; thus a thermocouple acts as a heat engine.

In closing our discussion of the Peltier effect the student is cautioned not to confuse this phenomenon with the thermal effect due to the resistance encountered by the current in passing through all conductors. In the latter case the heat produced varies as I^2R and is independent of the direction of the current. In the Peltier phenomenon the thermal effect varies as the first power of the current.

72. Thomson Effect.—In 1856 Lord Kelvin (then Professor William Thomson) was led to believe, from theoretical considerations, that an effect similar to the Peltier phenomenon might be expected to obtain throughout the body of any conductor when the conductor was *unequally heated.* Experiment has shown that there is a P.D. between different parts of a conductor when the parts are not at the same temperature. In any given case the direction of the E.M.F. depends upon the relative temperature and also upon the nature of the conductor. For instance in the case of copper the E.M.F. is directed from the colder to the hotter parts. In the case of iron, palladium, cobalt, nickel and platinum the reverse is true. Zinc, tin, silver and several other metals act similarly to copper. Lead does not show an appreciable Thomson effect and it is for this reason that lead is taken as the reference metal in the thermoelectric diagram. In the case of those metals which behave as does copper the Thomson effect is said to be positive, and in the conductors, such as iron, it is considered negative. On the thermoelectric diagram (Fig. 91) those metals whose lines slope downward to the right show a negative Thomson effect, and those which slope upward exhibit a positive Thomson effect.

73. Applications of Thermocouples.—Shortly after Seebeck's disclosure of the elementary principles of the thermocouple an Italian physicist, Leopoldo Nobili, a professor in Florence, devised what is known as the *thermopile.* This consisted of small alternate bars of bismuth and antimony connected in series, the bars being suitably insulated from one another except at the ends which were soldered together. There was thus formed a "battery" of thermocouples, the pairs being connected in series. The junc-

tions on one side of the "pile" were blackened in order to completely absorb any incident radiation, while the junctions at the other end of the group were covered by a metal cap. Provision was made for connecting the terminal elements to a galvanometer. Figure 93 is an illustration of a common form of this device.

Macedonio Melloni, a contemporary and fellow countryman of Nobili, greatly improved the thermopile and utilized it extensively in connection with his classical researches in the field of thermal radiation.

FIG. 93.—THERMOPILE

In recent years the thermocouple has been still further improved and now serves as the essential part of several research and engineering devices of remarkable sensitiveness.

One such instrument, known as a *radiomicrometer*, was devised by Professor C. V. Boys,* and used by him in the study of radiant energy. The essentials of this device are shown in Fig. 94. It consists of a delicate thermocouple and galvanometer combined in one instrument. A loop of silver wire W terminates in a bismuth-antimony pair J. The closed electrical circuit thus formed is suspended by a very fine quartz fiber between the pole pieces, N and S, of a strong magnet. Any rotation of the movable system is indicated by a beam of light reflected from the mirror M to a suitably positioned scale. In order to absorb completely all incident radiation a small piece of blackened copper C is attached to the active junction. Because of the fact that a magnetic field produces certain effects on antimony and bismuth (Sec. 95)

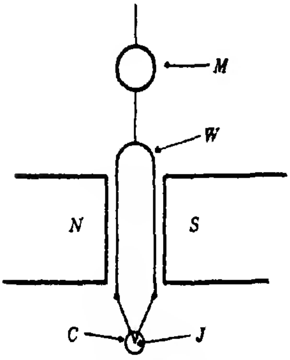


FIG. 94

* C. V. Boys, *Phil. Trans. Roy. Soc.*, 1889, CLXXX, 159.

and to avoid extraneous thermal effects, the thermocouple hangs in a thick-walled iron housing. An opening is provided in this inclosure through which the radiation is admitted to the thermojunction.

The slightest heating of the lower junction will give rise to a current around the loop which, due to the strong magnetic field, will result in a rotation of the movable system. Incredibly small quantities of radiant energy have been detected and measured by this instrument. It is said that it will give an appreciable deflection when actuated by the energy from a candle a quarter of a mile distant.

(Courtesy Cambridge Instrument Co.)

Fig. 95.—Duddell Thermogalvanometer

A development of Boys' radiomicrometer was made by W. Duddell in 1889 and is known as the Duddell thermogalvanometer. It is an instrument designed to measure small values of alternating current (Sec. 104). The only essential difference between the original Boys instrument and the Duddell modification consists in the manner in which heat is caused to reach the ther-

mocouple. In the Duddell thermogalvanometer a small resistance element is placed directly beneath the thermocouple. The current to be measured on being passed through the resistance heats the adjacent thermojunction, thus causing a deflection of the movable system due to the current in the loop. Figure 95 shows a convenient form of the Duddell galvanometer. The instrument is made with several heater units, thus adapting it to various classes of service. This type of galvanometer is quick acting, nearly aperiodic, and has practically no capacitance or self-inductance (Sec. 106). Because of these characteristics it is particularly adapted to the measurement of extremely small values of high frequency alternating current. Currents as low as twenty microamperes may be readily measured. Using a suitable resistor the current generated by a sustained tone spoken into a common telephone receiver is sufficient to give a full-scale deflection.

By modifying the mechanical details of the original suspension type of Duddell thermogalvanometer a portable Duddell thermoammeter has been designed which will handle currents up to 100 milliamperes.

Ammeters utilizing a thermocouple as a detecting agent but operating on a somewhat different plan from the Duddell instruments are now available for research and commercial purposes. The general design on which these instruments are based is shown schematically in Fig. 96. The current to be measured is passed through a resistance H which serves as a heater. The active junction of the thermocouple J is in direct contact with the resistor. Connections from the terminals of the thermocouple are made to a suitable direct current indicating instrument, such as a sensitive galvanometer or a milliammeter. By reducing the mass of the heater and thermojunction elements to a minimum and inclosing these parts in a vacuum the sensitivity of the organization is materially increased. Thermocouple ammeters may be accurately calibrated by means of direct current and are suitable for use in the measurement of alternating currents of any frequence and wave form.*

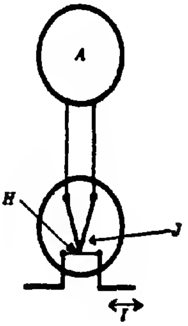


Fig. 96

* See chapter on alternating currents.

While the thermocouple is used to a considerable extent in the measurement of the electric current, it is as a temperature indicating agent that it finds its most extensive industrial application.

When one considers the possible use of the thermocouple as a temperature indicating device an interesting and important problem at once presents itself. On referring to Fig. 88*a* it will be seen that there are two possible temperature values corresponding to any given E.M.F. value. If then we take the E.M.F. developed by a couple as an index of temperature the result will in general be ambiguous. In order to avoid this a pair of metals must be selected whose neutral point is well above the maximum temperature it is desired to measure. Two metals whose thermoelectric lines (Fig. 91) are nearly parallel would fulfill this requirement, and such a pair would have an E.M.F.-temperature curve which would be almost straight. Fortunately there are several metals of this character available.

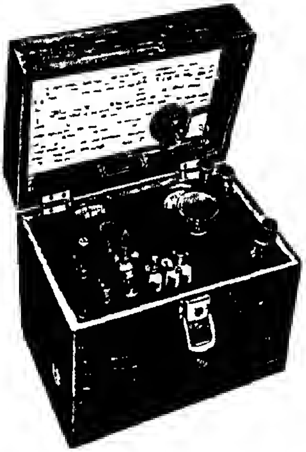
There are two general methods in use by means of which a suitable thermocouple may be utilized in temperature measurements. In one the couple is connected directly to a galvanometer or millivoltmeter, the indicating instrument being calibrated directly in degrees. Such an organization is referred to as a *thermoelectric pyrometer.*

In the plan just outlined the magnitude of the *thermoelectric current* is taken as an index of the temperature of the hot junction. In the second method, the *E.M.F. developed by the thermocouple* serves to indicate the temperature being observed. In order to conveniently and accurately measure the thermo-E.M.F. recourse is had to the potentiometer, a device which we have already studied in Chapter VII, Sec. 49. The thermocouple and the associated potentiometer are known as a *potentiometer pyrometer.*

A simple and compact form of the potentiometer is used in this connection, the readings being made directly in degrees of temperature instead of volts. Figure 97 is an illustration of a well-known portable potentiometer type of temperature indicator and Fig. 98 shows a wall form. On the wall type the small scale and pointer at the top of the instrument are that part of the galvanometer which indicates when the potentiometer balance is secured, and the larger scale is attached to the moving potentiometer contact and reads temperature directly. Recording

potentiometers are also extensively used in many industrial plants. A standard potentiometer recording unit is shown in Fig. 99.

In many manufacturing processes it is of the utmost importance to be able to note and control the temperature conditions. In



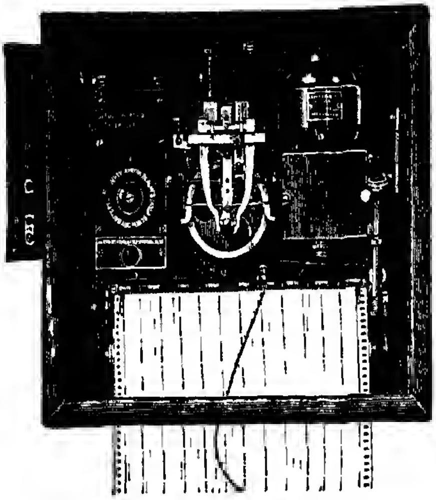
(Courtesy Leeds and Northrup Co.)
FIG. 97.—PORTABLE POTENTIOMETER PYROMETER INDICATOR

(Courtesy Leeds and Northrup Co.)
FIG. 98.—WALL TYPE POTENTIOMETER INDICATOR

many industrial plants suitable thermocouples are placed at different points and electrical connections made to a single recording potentiometer which is arranged to automatically select and record the temperature given by each individual thermocouple. The extent to which thermoelectric pyrometers are employed in industrial plants is well shown by the battery of recording potentiometers illustrated in Fig. 100.

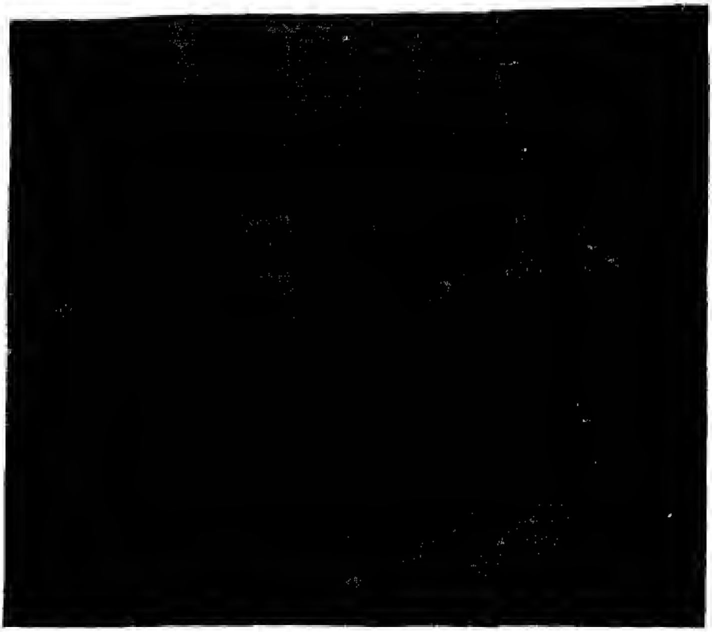
The thermocouple unit itself when used in temperature measurement work is usually housed in a metal, porcelain or quartz tube, depending on the type of service for which the couple is intended. Two forms of housings are shown in Fig. 101. One of the units illustrated is designed to be built into a furnace or oven wall, while the other couple is intended to be thrust by the operator into molten metal.

Reference was made above to the inherent limitations surrounding the choice of metals for use in thermoelectric pyrometers. There are also other factors which have an important



(Courtesy Leeds and Northrup Co)

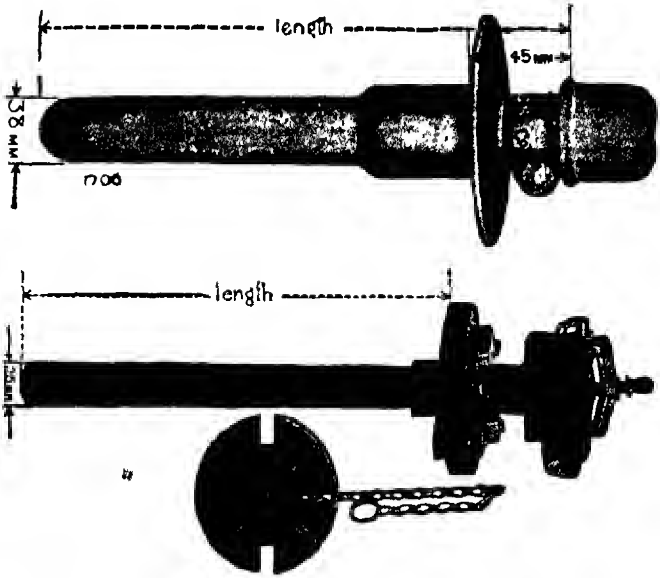
FIG 99.—CURVE DRAWING POTENTIOMETER PYROMETER



(Courtesy Leeds and Northrup Co)

FIG. 100.—BATTERY OF AUTOMATIC POTENTIOMETER RECORDERS

bearing on the selection of the materials which go to make up the pair. Aside from the magnitude of the E.M.F. per degree which a given pair will develop the next most important consideration is the effect of high temperatures on the physical and electrical properties of the metals. A vast amount of research has been carried out along this line.



(Courtesy Cambridge Instrument Co.)

FIG. 101.—THERMOCOUPLES FOR USE AT HIGH TEMPERATURES

At present there are two general types of thermoelements employed and they are designated as *base metal couples* and *noble metal couples.* In the first mentioned class we have the copper-constantan * combination which though it shows a high E.M.F. per degree (40 to 60 microvolts) tends to deteriorate at temperatures above 300° C. A couple of these metals can therefore be used only for the lower range of temperatures.

In the same general class may be mentioned the iron-constantan couple. This combination has a straighter E.M.F.-temperature curve than the copper-constantan unit but has the defect that the iron may rust in a humid atmosphere. The iron-constantan couple is used for observations up to about 800° C.

As a result of an effort to find a substitute for iron in thermo-

* Constantan is an alloy consisting of 60 per cent copper and 40 per cent nickel.

couples Hoskins developed two alloys which have proven to be quite satisfactory particularly for use at higher temperatures. One of these alloys is composed of 90 per cent nickel and 10 per cent chromium. The other metal consists of 98 per cent alumi-num, 2 per cent nickel and a trace of silicon and manganese. This combination is known under the trade name of the chromel-alumel thermocouple. The chromel-alumel pair may be used in measuring temperatures up to 1100° C. continuously, and will function satisfactorily for short periods up to 1300° C.

For still higher temperatures, and particularly where greater accuracy is essential, noble metal thermocouples consisting of *platinum, platinum-10 per cent rodium* are employed. These couples were introduced by Le Chatelier and are reliable up to 1500° C. (2732° F.). For the measurement of higher tempera-tures optical methods are used.

74. Pyro-Electricity.—The thermocouple is not the only agent by means of which one may obtain electrical effects directly from thermal energy. When certain crystals are heated they manifest an electrical charge and this phenomenon is referred to as the pyro-electric effect. Among the crystals which exhibit this property may be mentioned tourmaline, quartz, fluor and bora-cite. If a crystal of tourmaline, for instance, be heated the ends will, during the process of heating, manifest opposite charges, and when being cooled the sign of the charges is reversed. The initial temperature of the crystal does not affect the signs of the charges. If the crystal be broken into fragments or even reduced to pow-dered form each part will exhibit the same characteristics.

75. Piezo-Electricity.—Intimately related to the pyro-electric effect is another phenomenon discovered in 1880 by F. and P. Curie, and known as the *piezo electric* effect. Certain crystals, notably quartz, tourmaline, and crystals of rochelle salt,* exhibit electric charges on certain parts of the crystal when the crystal as a whole is subjected to mechanical pressure. It was found by the Curies that the magnitude of the charge thus made manifest is proportional to the pressure, and may be represented by the relation

$$Q = KP, \qquad \text{Eq. 84}$$

where Q is the charge, P the *total* force applied to the crystal, and K a constant known as the piezo-electric constant.

* Rochelle salt is sodium potassium tartrate, $NaKC_4H_4O_6\ 4H_2O$.

Of the bodies thus far studied rochelle salt crystals exhibit this phenomenon most strongly. Nicholson * has very thoroughly investigated the piezo-electric effect in the case of this crystal, particularly in connection with the possible use of this phenomenon in the field of electroacoustics. A salt crystal may be easily arranged to show the piezo-electric effect by compressing the crystal between two metal plates applied to two opposite faces and arranging a metallic band around the middle of the crystal. If the two pressure plates are electrically connected to form one electrode and the central band serves as the other connection and the system thus formed be connected to a potential measuring device it will be found that very slight variations in pressure will develop a variable E.M.F. of considerable magnitude. Indeed the pressure due to sound waves, under suitable conditions, may be made to produce a terminal E.M.F. of as much as several volts.

It is also a remarkable fact that the piezo-electric effect is reversible. If a crystal of rochelle salt is subjected to a variable E.M.F. its crystalline structure will oscillate mechanically with sufficient amplitude to produce audible sounds. Certain at least of the bodies which exhibit the piezo-electric effect have a natural period of mechanical vibration, this being particularly marked in the case of quartz. We shall have occasion to refer to quartz in this connection later.

It is an interesting and perhaps significant fact that all piezo-electric substances also show the pyro-electric effect. It is possible that the stresses developed by heating give rise to what amounts to a piezo-electric effect. In fact the existence of a true pyro-electric effect has been seriously questioned by several investigators, notably by the late Professor Röntgen † and Lindman.‡

* *Am. Inst. Elec. Eng. Proc.*, 1919.
† *Ann. d. Phys.*, 1914, XLV.
‡ *Ann. d. Phys.*, 1920, LXII.

CHAPTER XIV

MAGNETISM

76. Natural and Artificial Magnets.—Certain aspects of the phenomenon which we broadly designate by the term magnetism were known to the ancients. The mining of iron was carried out at a very early date, and the first knowledge concerning magnetism undoubtedly had its origin in connection with the handling of the iron ore now known as magnetite * (Fe_2O_3). This particular ore was, so tradition goes, found in or near Magnesia in Asia Minor, and Lucretius records that the term "magnet" was derived from "Magnesia." Natural magnets, consisting of pieces of iron ore, were originally called by various names, among which was the designation "lodestone." Several of the ancient writers, including Lucretius, Pliny and Socrates, mention some of the more obvious facts in connection with these natural magnets, but magnetic polarity and the concomitant phenomenon of attraction and repulsion between poles were apparently unknown to Greek antiquity. The lodestone itself appears to have been employed as a crude compass before the beginning of the Christian era, but it was probably not until the Middle Ages that it became known that permanent artificial magnets could be made by bringing pieces of steel into proximity with natural magnets. In 1269 the writings of Peter de Maricourt indicated that he was acquainted with magnetic polarity, the law of attraction, and with other important properties of magnets. Indeed it is to this investigator that we owe the term magnetic poles.

Magnetism as a definite branch of science had its beginning with the publication in 1600 of an epoch-making treatise entitled *De Magnete* by Dr. William Gilbert, an eminent English physician. In this classical volume Dr. Gilbert reviews the then known *facts* concerning magnetism and in addition presents many original and valuable observations of his own. In fact it may be said that the nomenclature of magnetism is largely due to Gilbert.

* Magnetite is widely distributed, being found in the Ural Mountains, Scandinavia, Finland, Canada and in the States of New Jersey, Pennsylvania, and New York.

77. Laws of Magnetism.—Passing now to a consideration of a few of the more important laws connected with magnetism we may note first the well-known fact that *like poles repel and unlike attract,* meaning by "like poles" those that tend to point in a like direction if the magnet be free to rotate about a vertical axis.

The second basic relation may be stated symbolically thus,

$$F = k\frac{m_1m_2}{d^2}, \qquad \text{Eq. 85}$$

where F is the force of attraction or repulsion exerted between two magnetic poles whose pole strength is m_1 and m_2, respectively, d the distance separating the poles in question, and k a constant depending for its magnitude upon the units in which the other terms are expressed. It will thus be seen that we have in magnetism an inverse square law similar in form to that which obtains in connection with electrostatics (eq. 2, Sec. 4).

The term "pole strength" requires some explanation. If we are dealing, for instance, with two magnetic poles of equal strength and of such a magnitude that m_1 and m_2 equal unity when the force F and the distance d are both unity the constant k will also become unity. Under these circumstances eq. 85 would become

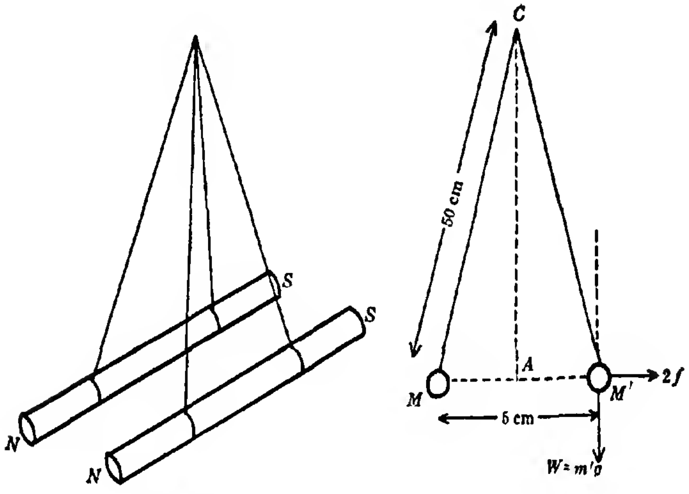
$$F = \frac{m^2}{d^2}. \qquad \text{Eq. 86}$$

We may then use this relation as a basis for a definition of pole strength, and express this quantity in terms of force and distance as follows: *Unit magnetic pole is one whose pole strength is such that when it is placed at a distance of one cm. from a pole of like strength it will experience a force of one dyne.*

The concept of pole strength can be made more real by applying eq. 86 to a simple problem Suppose we have two long and slender magnets each weighing 10 gms. suspended from a common support by bifilar suspensions as shown in Fig. 102*a*. The force of repulsion will cause them to assume a position in which they will be slightly separated from one another. If we view, end on, the system thus created it will appear as in Fig. 102*b* and by applying the simple laws of mechanics we may readily determine the magnitude of the force holding them apart, and from that

their *pole strength*. Applying eq. 86 we have as the force of repulsion due to each pair of like poles

$$F = \frac{m^2}{5^2},$$



(a) (b)

FIG. 102

and since there are two pairs of like poles the total force will be

$$2F = \frac{2m^2}{5^2}.$$

Each magnet is in mechanical equilibrium under the action of two forces, $2f$ and $m'g$, which have moments about the axis of suspension C. The equation of equilibrium would then be

$$\frac{2m^2}{5^2} \times AC = m'g \times AM',$$

or

$$\frac{2m^2}{5^2} \times 49.95 = 10 \times 980 \times 2.5.$$

Solving, we find the pole strength $m = 78.3$ c.g.s. units.

78. Field Strength.—Waiving for the time being any attempt to discuss the nature of a magnetic field it may, for present purposes, be said that *a magnetic field is any region in which a magnetic effect may be detected.*

If a magnet be placed in a magnetic field it will experience a mechanical force and the magnitude of that force will depend upon the pole strength of the magnet and also upon a factor known as *field strength* or *field intensity*. We may express the relation between the factors thus,

$$F = mH, \qquad \text{Eq. 87}$$

where F is the force in dynes, m the pole strength in c.g.s. units, H the field intensity. If our magnet has a pole strength of unity *field strength at a given point might be defined, numerically, as the mechanical force in dynes experienced by unit pole when placed at the point in question.* The unit of field intensity is known as the *gauss*. For instance, if a test pole when placed at a certain point in a magnetic field were found to experience a mechanical force of 10 dynes we would say that the field strength or field intensity was 10 gausses.

While it is of course not physically possible to establish an isolated magnetic pole yet it is analytically convenient to assume that such a test pole may exist, and such a hypothetical test pole is usually assumed to be north-seeking in character.

While it is convenient to express field strength in terms of a mechanical force it should be clearly understood that what we have referred to as the *intensity of a magnetic field at a point is in reality a property of that field which gives rise to a mechanical force* if a magnet be located at that point. As Professor Guthe * has well put it, "it" (field intensity) "is independent of the presence of a magnet or the existence of a force at the point." From our numerical definition it will be evident that field intensity is a vector quantity.

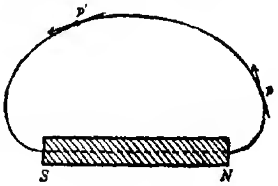


Fig. 103

79. Lines of Force.—The properties of a magnetic field may be conveniently represented by a convention introduced by Faraday. Since field intensity is a vector quantity we may draw lines in a magnetic field to indicate the direction of the force action. Thus in the case of the magnet shown in Fig. 103, a north-seeking test pole if placed at the point p would tend to move in the direction indicated by the arrow. If located at p' the direction of the acting force would

* *Definitions in Physics*, by Dr. Karl E. Guthe.

be as shown. If we were to connect a large number of such points we should have the path along which our test pole might move under the force action of the two poles. Such a trajectory is known as a line of force.

In addition to indicating direction of force action it is also customary to give to these lines another significance. In the preceding section reference was made to magnetic field strength, and a definition in terms of units of force was set up. In dealing with magnetic fields it is convenient to adopt the convention that *the magnitude of the field intensity at a point shall be represented by the number of lines per unit area,* this area to include the point in question and also be normal to the direction of the lines of force. If then we have a magnetic field whose intensity is 10 gausses we would have 10 lines per cm.^{2}.

By applying Gauss' Theorem, as was done in electrostatics (Sec. 7), it may be shown that there are $4\pi m$ lines of force emanating from a magnetic pole whose strength is m.

80. Magnetic Moment.—Due to the fact that it is not easy to measure pole strength, and also because it is difficult to specify what particular point in the pole of a magnet shall be taken as the force center, the solution of many of the problems encountered in the study of magnetism is greatly facilitated by the use of a quantity known as magnetic moment.*

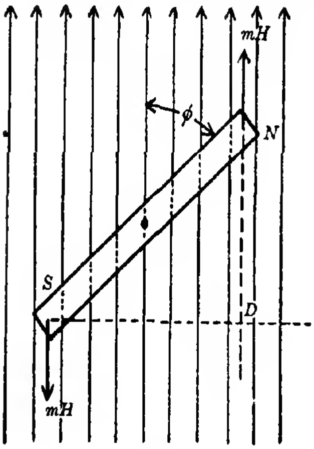


FIG. 104

If we suspend a magnet of pole strength m in a uniform magnetic field H as shown in Fig. 104, it will be acted upon by a couple tending to turn it into a position where its axis will be parallel to the direction of the field. The magnitude of this magneto-mechanical couple will be given by the expression

$$C = mH \times SD = mH \times L \sin \phi,$$

where L is the distance between the poles N and S. Rearranging

* It is important that the student have a clear understanding of "moments" and "couples" and to this end it is advisable to review this subject as presented in mechanics.

terms,

$$C = mL \times H \sin \phi.$$

If an outside mechanical couple were applied to the system in such a manner that the magnet were held in a position perpendicular to the lines of force $\sin \phi = 1$, and if the field strength were unity the couple would, under these conditions, be given by

$$C = mL.$$

The product mL is the magnetic moment, and is usually written

$$M = mL. \qquad \text{Eq. 88}$$

It may therefore be stated that *the magnetic moment of any magnet is the product of the strength of either pole and the distance between the two poles*, and is numerically equal to the mechanical couple which must be applied in order to hold the magnet in a position perpendicular to the direction of a field of unit intensity.

It is possible to easily and accurately determine by experiment the value of M for any magnet. The experimental procedure involved in such a measurement will be found described in *Electrical and Magnetic Measurements* by C. M. Smith.

81. Intensity of Magnetism.—Another concept which is sometimes useful in connection with magnetism is that of *intensity of magnetization.* Assuming that we have a body which is uniformly magnetized, intensity of magnetization may be defined as the ratio of the magnetic moment to the volume. This may be expressed as

$$I = \frac{M}{V} = \frac{mL}{LA} = \frac{m}{A}, \qquad \text{Eq. 89}$$

where A is the cross-sectional area. It will thus be seen that this quantity may also be defined as the pole strength per unit area.

Actual permanent magnets do not exhibit uniform magnetization throughout their entire volume. In such cases it is customary to deal with the intensity of magnetization *at a point*, and to express the quantity in the form

$$I = \frac{dM}{dV}\cdot \qquad \text{Eq. 90}$$

82. Magnetic Shell.—Among the various other concepts in connection with the theory of magnetism which might be dis-

cussed if space permitted there is one which, because of its importance in electromagnetism (Chapter XVI), will be briefly touched upon here. Reference is made to what is spoken of as a *magnetic shell.*

If we imagine a thin sheet of metal so magnetized that all of one face exhibits polarity of one sign and all of the other face polarity of the opposite sign we have a magnetic shell. Such a magnetized sheet or lamina may be thought of as made up of a large number of very short parallel magnets all having like poles facing in one direction. In dealing with such a lamellar magnet it is considered that the material constituting the shell is magnetized at each point in a direction normal to the face at that point.

What is known as the *strength* of such a magnetic shell, at any point, is given by the product of the intensity of magnetization and the thickness of the shell taken normally at that particular point, thus,

$$\text{Strength of shell} = It. \qquad \text{Eq. 91}$$

By eq. 89 this becomes

$$\text{Strength of shell} = \frac{M}{V}t = \frac{M}{At}t = \frac{M}{A}, \qquad \text{Eq. 92}$$

where A is the total area of one face of the shell. This means that *the strength of the shell is numerically equal to the magnetic moment per unit area.*

PROBLEMS

1. Making use of the fact that 4π lines emanate from unit pole, prove that the field intensity at a point near a large sheet of uniformly magnetized material is equal to 2π times the intensity of magnetization.

2. Prove that the field strength in the space between two magnetic poles placed near together is given by the product $4\pi I$, where I is the intensity of magnetization.

3. Show that the magneto-mechanical force holding together two poles which are in contact is given by $2\pi I^2$, where I has the same significance as in the two preceding problems.

4. Prove that the field strength due to a bar magnet at a point on a line including both poles and distant d from the center of the magnet is given by

$$\frac{2Md}{(d^2 - x^2)^2},$$

where M is the magnetic moment and x half the length of the magnet.

5. Derive an expression for the field strength due to a bar magnet at a point on a line perpendicular to the magnet at its center.

CHAPTER XV

TERRESTRIAL MAGNETISM

83. The Earth's Field.—Though the compass was probably in use as early as 1200 the fact that the earth acts as a magnet appears not to have been realized until the idea was advanced by Gilbert in 1600. In recent years the position of the earth's poles has been more or less definitely located. The north magnetic pole is in the vicinity of Boothia Felix in Northern Canada, lat. 70° 50′ N., long. 96° 00′ W. The south magnetic pole is known to be at lat. 70° 10′ S., long. 150° 45′ E.

At the north and south poles a magnet, if supported so that it may rotate about a horizontal axis, will assume a vertical position. At other points on the earth's surface it will come to rest at varicus other angular positions in a vertical plane. We have seen (Sec. 79) that a test magnet will assume a position tangential to the direction of the lines of force at the point of observation. It follows then that the lines of force due to the earth's field are in general not parallel to the earth's surface. In dealing with the earth's field it is therefore necessary to resolve the actual field intensity into vertical and horizontal components as shown in Fig. 105. The horizontal component H_h acts to move a compass needle about a vertical axis and the vertical component H_v tends to deflect the magnet about a horizontal axis. Both the horizontal and vertical components of the earth's field must be taken into account in connection with the operation of various electrical measuring devices.

Fig. 105

Since the geographical and magnetic poles do not coincide the compass does not, in general, point to the geographical north. The angle between the bearing of the needle and the geographical meridian is referred to as *declination*. The angular departure of the needle from the horizontal is known as the *inclination or dip*.

There are various means by which the value of the magnetic elements may be accurately determined. One modern method

of determining these quantities makes use of a piece of equipment known as the *earth inductor*. This device consists essentially of a coil of wire so arranged that it may be rotated at constant speed. When the coil of the inductor is connected to a galvanometer and caused to rotate in the earth's magnetic field it will generate an electric current. The magnitude of the current thus produced will depend upon the strength of the earth's field and also upon the position of the plane of the coil with respect to the direction of the earth's field. If the axis of rotation of the coil be set in the magnetic meridian and inclined so as to coincide with the direction of the earth's field, the sensitive galvanometer indicates no current and the vertical circle of the instrument will give the angle of magnetic inclination or dip. In actual practice either on land or at sea it is not necessary to make the axis of rotation coincide exactly with the direction of the earth's field, but the exact direction may be determined by readings first on one side and then on the other, the true direction of inclination being computed from the relative magnitudes of the small currents. The intensity of the earth's field is determined by the strength of the current induced by rotating the coil of the inductor at a constant speed where the axis of rotation

(Courtesy Carnegie Institution of Washington)

FIG. 106.—MARINE EARTH-INDUCTOR AND REVERSIBLE GIMBAL-STAND USED ON THE *Carnegie* FOR THE DETERMINATION OF MAGNETIC INCLINATION

is perpendicular to the particular component for which the determination is required. Figure 106 gives a view of the latest type of marine earth-inductor designed and constructed by the Department of Terrestrial Magnetism of the Carnegie Institution of Washington and used on board the non-magnetic ship *Carnegie* in the course of her cruises to determine the distribution of the earth's magnetism over the oceans. This same type of instrument may also be used for observations at land stations. As used on ship, a special gimbal-mounting is provided (see Fig. 106) to maintain an average mean position of equilibrium and to permit complete reversal of the gimbal-rings and bearings in order to eliminate errors of level.

84. Variation in the Earth's Field.—The value of the earth's magnetic field at any given point is not a fixed quantity but is continually changing. Not only does the magnitude of the earth's field change but its direction is also subject to variations. The variation in declination appears to have been known to the Italians as early as 1436, and the experience of Columbus on his memorable voyage of discovery clearly established the fact that the declination is radically different at different places on the earth's surface.

The first authentic record of magnetic declination was made in the vicinity of London in 1580, the value being 11° 15′ E. In 1669 the declination there had become zero. Later it swung to the west, reaching a value of $24\frac{1}{2}$° W. in 1823, and is now decreasing. From the data secured at the famous Kew Magnetic Observatory and at other magnetic stations throughout the world it appears that there is a cyclic change in the magnetic declination which has a period of something like 960 years. This type of change is known as a *secular variation.*

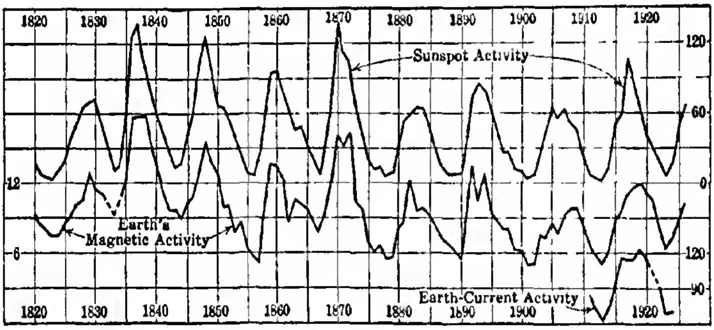
In addition to the change just mentioned the declination is subject to a small *annual variation.* It is an interesting and perhaps significant fact that the yearly variation is in opposite directions north and south of the geographical equator.

There is also a slight daily or *diurnal variation* in all of the magnetic elements which can be detected by delicate magnetic instruments.

It has also been established that there is a cyclic change in the magnitude of the diurnal variations. This superimposed cyclic change has a period of something like eleven years and is fre-

quently referred to as the eleven-year period. It happens that there is a maximum of sun-spots at regular intervals of eleven to twelve years also. Whether sun-spots are the *cause* of these variations has yet to be established.

In addition to the more or less regular variations already noted there are irregular and sometimes violent disturbances of the earth's field. These erratic variations in the earth's magnetic elements are known as *magnetic storms*. Strangely enough these disturbances also are frequently concomitant with sun-spots. At the time of writing (August, 1929) a large sun-spot has just crossed the sun's surface and magnetic observatories in various parts of the world have just reported the existence of magnetic storms. In fact observations extending over a period of one hundred years appear to indicate that sun-spot activity and the earth's magnetic activity are in some way related. This relation is strikingly depicted by the graphs shown as Fig. 107. In this



(Courtesy Carnegie Institution of Washington)

FIG. 107.—EARTH'S MAGNETIC ACTIVITY COMPARED WITH SUNSPOT ACTIVITY

connection it is interesting to note that under ordinary circumstances the solar magnetic field has a value of the order of 50 gausses as compared with a value of something like a half a gauss for the earth's field. The region of the sun covered by a spot may have a field intensity of 3500 gausses. The observers at Mt. Wilson Observatory report that the present sun-spot area shows an unusually high field intensity, reaching a value of 4000 gausses.

In the appendix are given the approximate values for the annual rates of secular change in the magnetic elements. We are indebted to the Department of Research in Terrestrial Magnetism of the Carnegie Institution of Washington for this table of valuable data.

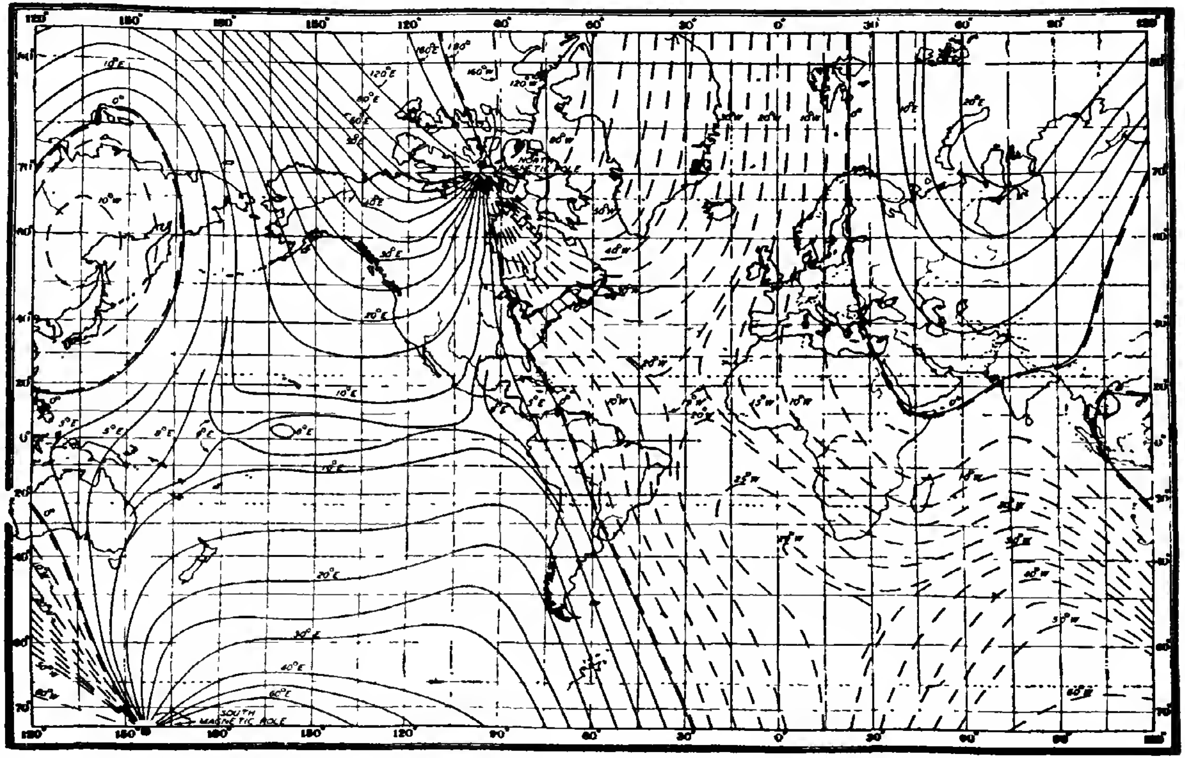
85. Magnetic Maps.—The importance of the study of the earth's magnetic behavior cannot be overemphasized. Through the cooperation of the U. S. Department of Terrestrial Magnetism and the Carnegie Institution of Washington a ship constructed entirely of non-magnetic materials and named the *Carnegie* has made several extended voyages for the sole purpose of carrying out magnetic surveys. The U. S. Coast and Geodetic Survey has also collected much valuable magnetic data. Scattered throughout the world there are a number of magnetic observatories where continuous observations have been made for many years. Among the older observing stations may be mentioned the one at Kew, already referred to, one at Potsdam, and another at Bombay. By the cooperation of these several agencies it is possible to prepare magnetic maps showing the value of the magnetic elements throughout the world.

If lines are drawn connecting points on the earth's surface having the same declination value we have a chart such as that shown as Fig. 108. Lines of this character are known as *isogonic lines*. It will be noted that the isogonic lines are more or less irregular, and in one instance form a closed loop, the "Siberian oval." There are other isogonals, one passing through North and South America, one through Europe and another through western Australia, along which the declination is also zero. Lines of zero declination are designated as *agonic lines*.

If lines are drawn connecting points showing the same angle of dip we have a map of the character shown as Fig. 109. These lines are referred to as *isoclinic lines*. It will be observed that the isoclinals are considerably more regular than the isogonals and that they roughly parallel the geographical lines of equal latitude.

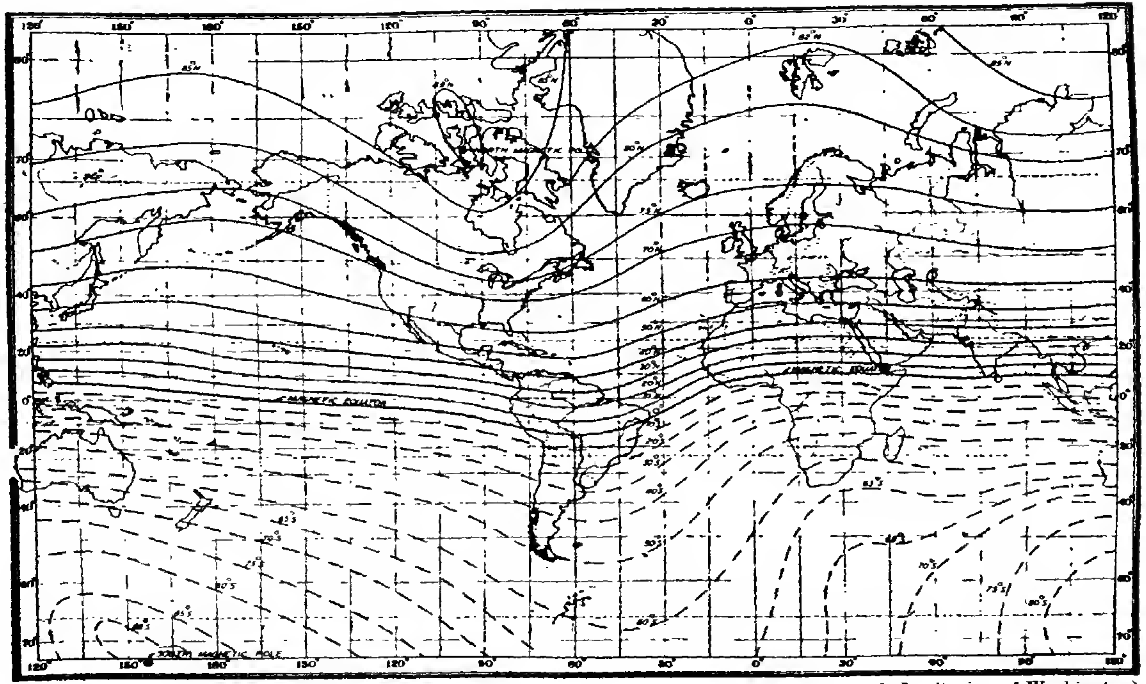
Charts are also made (Fig. 110) showing lines which pass through points having the same value for the horizontal component of the earth's magnetic field. Such tracings are referred to as *isodynamic lines*.

The isomagnetic world-charts of Figures 108, 109, and 110 are based upon the latest compilations of data largely the result of observations by the Department of Terrestrial Magnetism of the Carnegie Institution of Washington on field expeditions and at sea on the non-magnetic ship *Carnegie* which have been compiled in coöperation with the United States Hydrographic Office.



(Courtesy Carnegie Institution of Washington)

FIG. 108.—LINES OF EQUAL MAGNETIC DECLINATION (ISOGONICS) FOR EPOCH 1930



(Courtesy Carnegie Institution of Washington)

Fig. 109.—Lines of Equal Magnetic Inclination (Isoclinics) for Epoch 1930

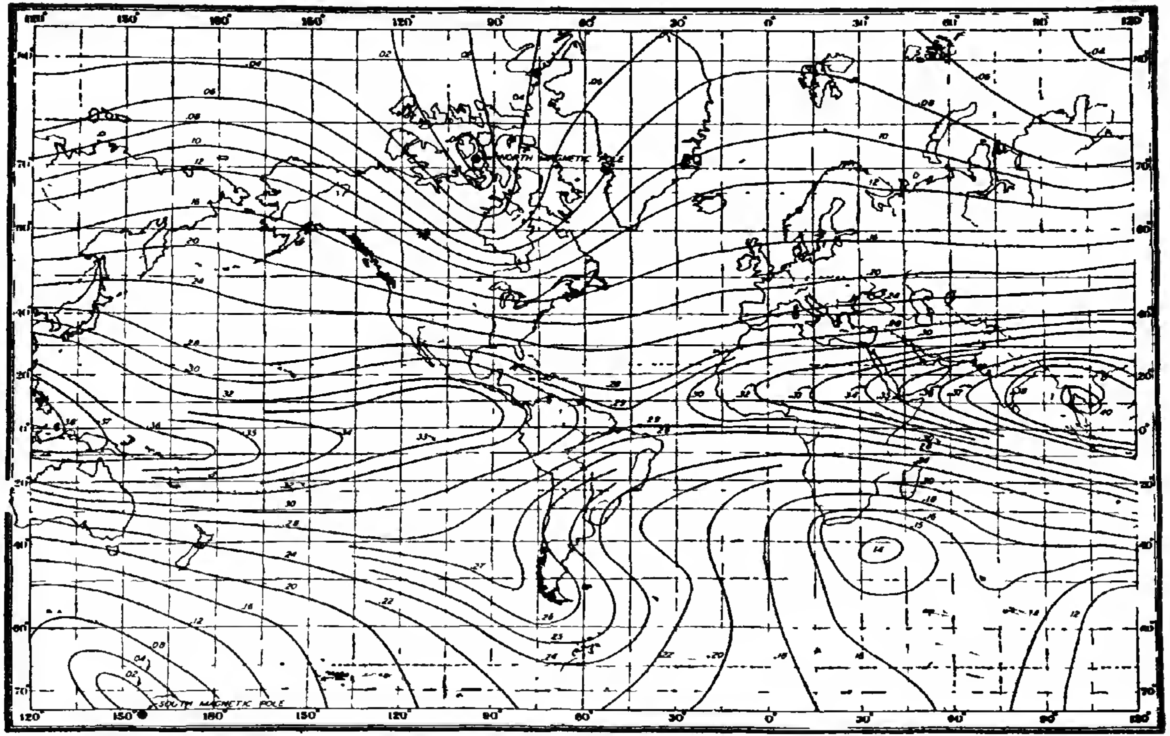


FIG. 110.—LINES OF EQUAL MAGNETIC HORIZONTAL INTENSITY (ISODYNAMICS) FOR EPOCH 1930

(Courtesy Carnegie Institution of Washington)

86. Theory of the Earth's Magnetic Field.—From time to time various theories have been suggested to account for the earth's magnetic field. Anything like a complete explanation of this phenomenon has not yet been offered. Speaking broadly it may be said that the earth, in some respects, behaves magnetically as if a small magnet existed at its center whose longitudinal axis made an angle of about 17° with the axis of the earth. The long time changes in declination would appear to indicate that the axis of such a hypothetical magnet slowly rotates about a fixed point somewhat as does the geographical axis.

Quite recently it has been shown that the rapid rotation of a magnetizable body will bring about the magnetic state. Since the earth is known to contain a considerable amount of material that is magnetizable there is a possibility that the rotation of the earth on its axis may give rise to the magnetism exhibited by the earth as a whole.

It is also possible that the cause of the earth's magnetism has its origin quite outside the earth itself. In any event the variations in the earth's field are, in some way, intimately related with solar activity as already implied. There is reason to believe that the sun emits electrons. Some of these may possibly reach our atmosphere and thus ionize (Sec. 139) the gases constituting the atmosphere. Such a process would probably give rise to electric currents in the region immediately surrounding the earth, which in turn (Sec. 87) would cause magnetic disturbances.

We know that magnetic disturbances always accompany auroral displays, and a study of the spectrum of the aurora would appear to indicate that they are caused by the passage of an electrical discharge through a rarified gas.

In considering the factors involved in any theory as to the earth's field, a study of the data embodied in the world-charts shown as Figures 108, 109, and 110 brings out several significant facts. Dr. J. A. Fleming of the Department of Research in Terrestrial Magnetism of the Carnegie Institution of Washington, in a recent communication to the author, makes the following important observations concerning these data.

"Mathematical analyses have been made from data scaled from world-charts applying for different epochs. The results of the latest of these analyses show that the magnetic field may be ascribed to the combined effect of three agencies, one within the

earth possessing a potential and which accounts for 94 per cent of the earth's total field, another around the earth outside also possessing a potential, and a third having no potential. The coefficients of the series of solid spherical harmonics expressing the potentials are known as Gaussian constants. There are three of the first order which have a simple physical meaning. The terms containing them represent for the inner field the potential arising from the uniform magnetization of a sphere (the earth) parallel to a fixed axis. The magnetic moment corresponding is 8.04×10^{25} c.g.s. units, and according to the analyses this has been decreasing during the last 80 years by 1/1500th part annually. For the outer field the moment is 0.14×10^{25} c.g.s. units. The point where the fixed axis pierces the earth's surface in the northern hemisphere is in latitude 78° 32′ north and longitude 69° 08′ west of Greenwich. For the outer field the fixed axis pierces the surface in the northern hemisphere in latitude 75° 48′ north and longitude 121° 24′ west of Greenwich. The other ends would pierce their antipodes. Neither axis passes through the magnetic poles, that is, the magnetic poles are not quite diametrically opposite but each is about 2,300 kilometers from the antipode of the other. The axis of the inner field is moving slowly, the north end towards the west and the equator. All the data available at present are insufficient to determine whether a complete rotation will be made around the earth's rotation axis.

"Besides the constants of higher orders in the analyses of the two fields having a potential, there are others awaiting confirmation and physical interpretations. Especially problematical is the field having no potential, which at present can only be explained by the existence of vertical currents of electricity passing through the earth's surface of about 10^{-11} amp/cm², whereas the observed currents of atmosphere are too small to be detected by the methods of terrestrial magnetism."

The whole subject of terrestrial magnetism is a most fascinating and important field, made more so because magnetic history is still in the making. The student who is interested in this subject will find a fund of valuable material in papers which appear on this subject in the journal *Terrestrial Magnetism and Atmospheric Electricity*, edited by Dr. J. A. Fleming of the Department of Terrestrial Magnetism, Carnegie Institution of Washington.

CHAPTER XVI

MAGNETIC EFFECTS OF THE ELECTRIC CURRENT

87. The Oersted Effect.—Having made a brief survey of the general subject of magnetism we are prepared to consider the magnetic effects which result from the movement of electrons. From time to time in the history of science investigations have been undertaken which have led to epoch-making discoveries. Such an one was the famous experiment of Professor Hans Christian Oersted as first performed by him in 1819, and announced the following year. By holding a wire, through which a current was flowing, parallel to a magnetic needle Oersted found that the needle was deflected, thus clearly establishing the enormously important fact that an electric current has about it a magnetic field. His experiment also showed that the magnetic field at any given point, due to the current in the wire, is perpendicular to the direction of the current. It may truly be said that the science of what we shall call electromagnetism had its beginning in Professor Oersted's classical experiment. Oersted's discovery united the fields of magnetism and electricity and thus opened a vast domain of untouched possibilities.

Oersted's announcement excited widespread interest and resulted in energetic experimentation on the part of other investigators. Within a few months important advances were made in this new field. M. Andre Ampere conceived the plan of forming the conducting wire into a spiral or helix in order to increase the magnetic effect due to a given current. He also discovered that two conductors each of which is carrying a current will react magnetically upon one another without the presence of a magnetic material, and further that such parallel currents when flowing in the same direction attract one another and when in opposite direction exhibit repulsion. We shall see that these observations of Ampere have served as the basis of the science of electrodynamics.

We have dwelt somewhat at length upon the early history of electromagnetism in order that the reader might have a background against which to examine the important quantitative rela-

tions which exist between the magnitude of the current, in any given case, and the resulting magnetic effects.

Before, however, proceeding to examine these relations it will be well to note that an electromagnetic effect obtains only when the electrons, which constitute the current, are in motion. A single electron in motion gives rise to a magnetic field, but a thousand coulombs *at rest* would produce absolutely no magnetic effect. It should also be observed that the magnetic effects due to electrons in motion are entirely independent of the nature of the conductor. For instance an isolated electron whether being driven along a copper wire or through the highest possible vacuum will give rise to a perfectly definite magnetic field. The magnitude of the magnetic effect produced in either case will depend upon certain commensurable factors, but the character of the conductor is not one of them.

One other fact, noted soon after Oersted's discovery, must be mentioned, viz., that the lines representing the magnetic field about a conductor form concentric circles about the direction of the electronic movement as an axis. For example, if the stream of electrons constituting the current is moving in a direction normal to this page and toward the reader, as shown in Fig. 111*a*, the lines of force will be as shown by the concentric circles

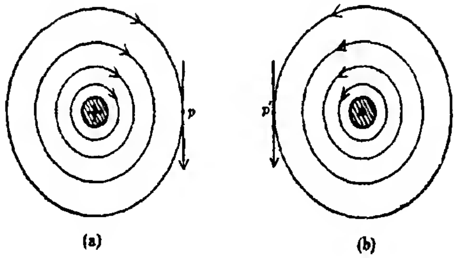


Fig. 111

and the general direction of the field will be clockwise as shown by the arrows. If the current is away from the reader the field will be as shown in Fig. 111*b*. At any particular point *p* the direction of the magnetic field will be as indicated by the straight arrows.

88. Laplace's Rule.—Within two months after the announcement of Oersted's discovery, two French physicists, J. B. Biot and F. Savart, reported to the French Academy that they had discovered the relation which exists between the magnetic field in-

tensity produced by a current flowing in a long straight wire and the distance from the conductor. Biot and Savart found that the field intensity varied directly as the current strength and inversely as the distance from the conductor carrying the current. Laplace, an eminent French analyst and astronomer, in discussing the law established by Biot and Savart for the special case examined by them, showed that a generalization might be formulated which would take the form

$$dH = \frac{Idl \sin \phi}{r^2}, \quad \text{Eq. 93}$$

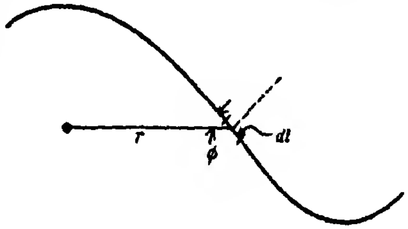


Fig. 112

where dl (Fig. 112) is the element of current (length), r the distance of the point from the middle of the element, and ϕ the angle which the element dl makes with the line r. Further, he assumed the direction of the field to be normal to the element dl and to r. This generalization has come to be known as *Laplace's Rule;* it will serve as a basis for important deductions to follow.

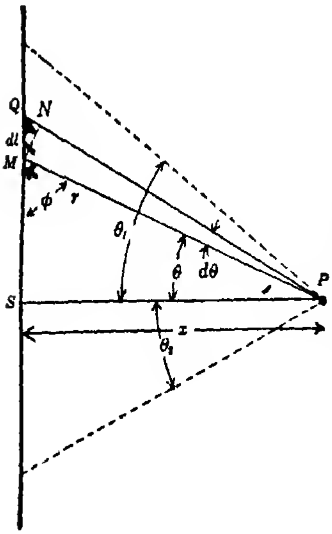


Fig. 113

89. Field Due to a Linear Current.—The results enunciated by Biot and Savart may be readily deduced by employing Laplace's general relation. Suppose we have a conductor of finite length through which a current I is flowing; let us find an expression for the field intensity at any point P distant x from the nearest point S on the conductor, as set forth in Fig. 113. In making use of Laplace's equation it will be in our case more convenient to deal with $\angle\theta$ than with $\angle\phi$. Since $\sin \phi = \cos \theta$ we may then write Laplace's equation in the form

$$dH = \frac{Idl \cos \theta}{r^2}.$$

We must eliminate dl, and also express r in terms of x. From the similar triangles QMN and MPS we may write

$$\frac{dl}{MN} = \frac{r}{x}.$$

But

$$MN = rd\theta.$$

Hence

$$\frac{dl}{rd\theta} = \frac{r}{x},$$

or

$$dl = \frac{r^2 d\theta}{x}.$$

Substituting in our form of Laplace's equation and simplifying we have

$$dH = \frac{Id\theta \cos\theta}{x}$$

as the contribution of the current element dl to the field intensity at P. The total field at P will be given by

$$\int dH = \frac{I}{x}\int_{\theta=-\theta_2}^{\theta=\theta_1} \cos\theta d\theta = \frac{I}{x}[\sin\theta]_{-\theta_2}^{\theta_1}$$

$$H = \frac{I}{x}[\sin\theta_1 - \sin(-\theta_2)]$$

$$= \frac{I}{x}[\sin\theta_1 + \sin\theta_2].$$

For an infinitely long wire

$$\theta_1 = \theta_2 = \frac{\pi}{2},$$

and hence

$$\sin\theta_1 = \sin\theta_2 = 1.$$

Therefore the total field at P due to the current in an infinitely long straight conductor will be given by

$$H = \frac{2I}{x}, \qquad \text{Eq. 94}$$

which is in conformity with the observations of Biot and Savart. This means that if a unit test pole be placed at a distance x from an infinitely long wire carrying a current it will experience a mechanical force given by $\frac{2I}{x}$. Obviously this expression will

also give the field strength at a point *very near* to a wire of finite length.

90. Field at Any Point on the Axis of a Circular Loop.—Assume a circular loop carrying a current I, as shown in Fig. 114. Our problem is to find the field intensity at any point P on the axis of the loop. Laplace's equation may be used to advantage in this case also. Consider first the contribution made by the element dl to the field at P. Since the line S is normal to dl, Laplace's relation becomes, in this case,

$$dH = \frac{Idl}{S^2}.$$

dH will have the direction of PQ. We however are interested in the field parallel to the axis OP. The field in the direction PQ may be resolved into components along PN and PR. If the entire loop be considered the elemental components normal to OP will cancel out in pairs leaving the vector sum of the dH components (along PR) as the resultant field.

The component parallel to OP due to dl will be given by

$$dH = \frac{Idl}{S^2}\sin\theta$$

$$= I\frac{dl}{S^2}\cdot\frac{r}{S}.$$

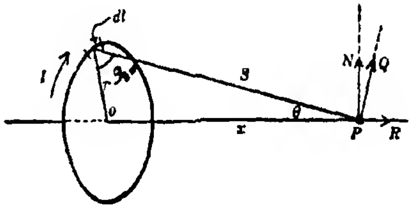


Fig. 114

The total field will be

$$\int dH = \frac{Ir}{S^3}\int_0^{2\pi r} dl,$$

which leads to

$$H = \frac{2\pi I r^2}{S^3}. \qquad \text{Eq. 95}$$

We may express S in terms of x and r and get

$$H = \frac{2\pi r^2 I}{(x^2 + r^2)^{3/2}}. \qquad \text{Eq. 96}$$

If P is at the center of the loop, eq. 96 becomes

$$H = \frac{2\pi I}{r}, \qquad \text{Eq. 97}$$

which gives the field intensity at the center of a loop of one turn.

Further, if the loop consists of several turns instead of one, the turns being connected in series and occupying a space small compared with the radius, the field at the center will be given to a close approximation by

$$H = \frac{2\pi In}{r}, \qquad \text{Eq. 98}$$

where n is the number of turns making up the coil.

91. Electromagnetic Unit of Current.—In applying eqs. 94 and 97 to actual cases, we are at once confronted with the question of units. These equations are different expressions for field intensity *in terms of force action on unit magnetic pole.* We have already seen (Sec. 77) that when dealing with numerical values the c.g.s. unit of field intensity is expressed in dynes. In the equations referred to distances will of course be expressed in centimeters. If the other factors in these relations are expressed, then, in c.g.s. units, the current I must be expressed in like units, and will accordingly be referred to unit magnetic pole. It therefore becomes necessary to set up a definition of the c.g.s. unit of current.

Following the usual custom in such cases we may make all the independent variables in the defining equation unity. In deriving eq. 97 we integrated over the entire loop. If instead of doing thus we had made the radius of the loop unity and had considered only unit length of the conductor the field intensity H would numerically equal the current I. It may therefore be said that the c.g.s. unit of current is defined as a current of such magnitude that when flowing in a conductor in the form of an arc whose radius is one cm. and whose length is one cm. it will exert a force of one dyne on unit magnetic pole placed at the center of the arc. Since this unit is based on unit magnetic pole it is known as the c.g.s. electromagnetic (e.m.) unit of current.

The practical unit of current is the *ampere* (named after the physicist mentioned in Sec. 87) and is equivalent to one-tenth * of the electromagnetic unit referred to above. It follows therefore that if I in eqs. 94 to 98, inclusive, is to be expressed in amperes the factor 10 must appear in the denominator of each of these equations.

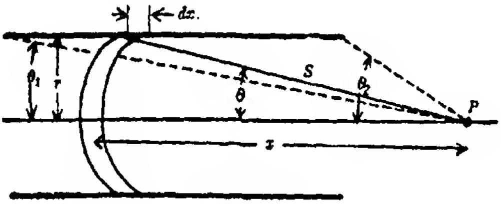
* In Chapter XXI the reasons for this and other unit ratios will be discussed.

EXAMPLE.—A closely wound coil consisting of 100 turns has a mean diameter of 10 cm. If a current of 2 amperes flows through the wire, what is the field strength at the center of the coil?

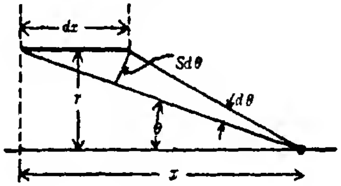
Substituting in eq. 98 we have

$$H = \frac{2\pi \times 2 \times 100}{10r} = 8\pi \text{ gausses.}$$

92. Field Intensity within a Helix.—We may extend the results of the last section to include the case of the solenoid. An helixal winding is a form of circuit frequently met with in practical electrical equipment; its magnetic properties are therefore of peculiar interest. Let us suppose that we have a solenoid the wire of which is wound in a single layer and having n turns per unit length. Our problem consists in finding an expression for the field intensity at any point P on the axis as shown in Fig. 115a.



(a)



(b)

FIG. 115

Consider a very small section of the winding dx. This section will contain ndx turns. The field at P due to the element dx is (Sec. 90, eq. 95)

$$dH = \frac{2\pi I r^2 n dx}{S^3}. \tag{i}$$

It will facilitate the evaluation of this relation if we express dx in terms of θ. Referring to Fig. 115b we may write

$$\frac{Sd\theta}{dx} = \sin \theta,$$

or

$$dx = \frac{Sd\theta}{\sin\theta}. \tag{ii}$$

Further,

$$\sin\theta = \frac{r}{s}. \tag{iii}$$

Substituting the values from (ii) and (iii) in (i) we get

$$dH = 2\pi In \sin\theta d\theta.$$

The field at P due to the complete helix will be

$$\int dH = 2\pi nI \int_{\theta=\theta_2}^{\theta=\theta_1} \sin\theta d\theta,$$

where θ_1 and θ_2 are the values of θ at the ends of the solenoid. This gives

$$\begin{aligned} H &= 2\pi nI \left[\cos\theta\right]_{\theta_2}^{\theta_1}. \\ &= 2\pi nI \left[\cos\theta_1 - \cos\theta_2\right]. \end{aligned} \qquad \text{Eq. 99}$$

If the point P is moved to the nearest end of the helix, $\theta_2 = 90°$ and hence $\cos\theta_2 = 0$, thus giving for eq. 99

$$H = 2\pi nI \cos\theta_1.$$

Expressing the angle θ_1 in terms of the length of the solenoid l and its radius r the above equation becomes

$$H = 2\pi nI \frac{l}{\sqrt{(r^2 + l^2)}}, \qquad \text{Eq. 100}$$

which gives the magnetic field intensity at the end of a solenoid.

If the length of the helix is great in comparison with its radius eq. 100 becomes

$$H = 2\pi nI. \qquad \text{Eq. 101}$$

If P is at the center of the solenoid the angles θ_1 and θ_2 are equal and for a very long slender coil approach zero as a limit. Hence $(\cos\theta_1 - \cos\theta_2)$ becomes sensibly equal to 2, and eq. 99 reduces to

$$H = 4\pi nI, \qquad \text{Eq. 102}$$

which is the field intensity at the center of a long slender helix. A comparison of eqs. 101 and 102 shows that the field strength at the ends of the winding is only half what it is at the center.

If the current I is in amperes eqs. 101 and 102 become

$$H = \frac{\pi n I}{5}, \qquad \text{Eq. 103}$$

and

$$H = \frac{2\pi n I}{5}, \qquad \text{Eq. 104}$$

respectively.

In the case of a toroidal winding, Fig. 116, the field is uniform throughout the interior of the coil and hence eq. 102 applies at any central point within the turns.

If the solenoid is made up of a multiple layer winding, as is commonly the case in practice, and if the thickness of the winding is small compared with the radius of the coil, eq. 102 will give the field intensity to a first approximation. In such a case the mean radius is used in the computation.

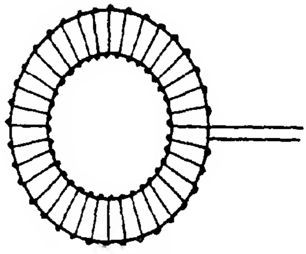


FIG. 116

EXAMPLE.—A solenoid consisting of 500 turns is 20 cm. in length. What will be the field intensity near the center of the coil if it carries a current of 5 amperes?

Since the number of turns is 500 and the length 20 cm. the number of turns per cm. will be 25. Employing eq. 104 we have

$$H = \frac{2\pi \times 25 \times 5}{5} = 50\pi \text{ gausses.}$$

93. Mechanical Force on a Current in a Magnetic Field.—We have seen that a current gives rise to a definite magnetic field and that the intensity of that field may be easily calculated (Sec. 89) in any given case. We have defined the magnitude of the field in terms of the *mechanical force experienced on unit test pole at the point in question.* Newton's fundamental laws of dynamics apply here as well as in mechanics. If the current exerts a force on a magnetic pole, *the pole also exerts a force on the current.* The force action is mutual, as we would expect from Newton's third law. Since this mutual force action between currents and magnetic fields lies at the basis of the operation of many electrical meters, motors and other similar electrical equipment it is important that we examine the basic relations which obtain between the several

factors involved. Fortunately the relations are simple in form and direct in their application.

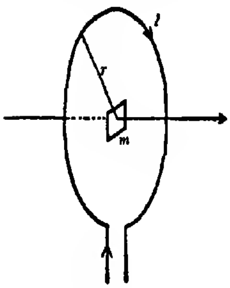


FIG. 117

Let us derive first an expression for the mechanical force experienced by a conductor of unit length carrying a current and located in a magnetic field due to some agency outside the conductor itself. Referring again to a simple loop (Fig. 117), the field at the center as given by eq. 97 is

$$H = \frac{2\pi I}{r}.$$

If a unit magnetic pole is placed at the center of the loop it will experience a mechanical force of $\frac{2\pi I}{r}$ dynes. If a pole whose strength is m be similarly placed the force will be

$$F = \frac{2\pi I m}{r} \text{ dynes.}$$

As implied above, such a magnetic pole will likewise exert an equal force on the current and hence on the conductor forming the loop. This force is exerted by the magnet *on the loop as a whole.* The force experienced by *unit length* of the loop would be

$$\frac{2\pi I m}{r} \times \frac{1}{2\pi r} = I\frac{m}{r^2},$$

because the total length of the conductor is $2\pi r$. It follows from the fundamental law of magnetism (Sec. 77) that the field intensity H at the loop *due to the magnet at the center of the loop* will be given by $\frac{m}{r^2}$. Hence the *mechanical force experienced by unit length* of the conductor* may be expressed in the form

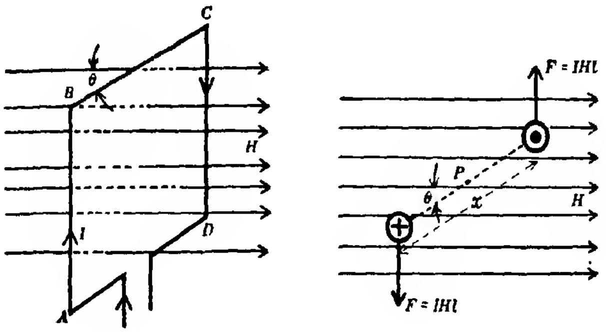
$$F = IH \text{ dynes per cm. length.} \qquad \text{Eq. 105}$$

This equation represents one of the basic laws in electrodynamics.

The results just derived may be extended to include a form of circuit frequently met with in practice, namely, a *rectangular coil.*

* In the case of a loop any section of the conductor however small will not be straight, but as r increases unit length will approach a straight line, and since eq. 97 applies to a loop of any size eq. 105 will give the force on unit length of a straight conductor.

Let the conditions be as illustrated in Fig. 118*a*. The side AD will experience a force $IH(AD)$ tending to move it upward, and the side BC will be acted upon by an equal force $IH(BC)$ directed downward; hence the resultant force on these two sides will be zero. The force action on the sides AB and CD will be as diagrammed in Fig. 118*b*, when viewed from the side AD. If the



(a) (b)

FIG. 118

length of AB and CD equals l, the force F operating on each side will be IHl dynes. It is evident that we have a couple acting on the coil tending to produce rotation about an axis through P. The value of this couple will be given by

$$L = IHl(x \cos \theta), \qquad \text{Eq. 106}$$

where L represents the couple in dyne-cm., θ the angle which the coil makes with the direction of the field, and x the width of the coil, in this case AD or BC. It will be noted that the product xl gives the area of the coil; hence we may write

$$L = IHA \cos \theta, \qquad \text{Eq. 107}$$

where A is the area of the coil. If the coil consists of a number of turns in series each loop would contribute its part to the total torque action, and a general expression for the couple on a coil in a magnetic field will therefore take the form

$$L = IHAN \cos \theta, \qquad \text{Eq. 108}$$

where N is the number of turns in the coil and A the average effective area of the individual turns. When $\theta = 0$, L will have its maximum value. This condition obtains when the plane of

the coil is normal to the direction of the field. It is therefore evident that the coil will tend to turn so that the area within the coil will include the greatest possible number of lines of force.

94. Galvanometers.—A typical application of the relation just deduced is to be found in the *moving coil type of galvanometer* frequently referred to as the D'Arsonval* type of instrument. In this form of current indicating device a rectangular coil of many turns is suspended in the field of a permanent magnet by means of a delicate phosphor bronze or steel strip. (See Fig. 119.) The suspension strip connects electrically with one terminal of the coil winding and serves as one connection to the source of current. A coiled metallic strip connects the other terminal of the coil to the source. A mirror or pointer attached to the coil serves to indicate any rotation of the moving system.

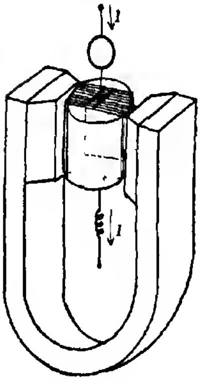


FIG. 119

The relation between the deflection of the coil and the current passing through it is easily deduced When current is passing through the coil an electromagnetic couple will be developed as given by eq. 108. This couple will be opposed by a second couple due to the twisting of the suspension. The coil will come to rest when this mechanical or restoring couple just equals the electromagnetic couple. From the principles of mechanics we know that the couple due to the torsion of the suspension is proportional to the angular displacement of the end attached to the coil, i.e.,

$$\text{Mechanical couple} = k\theta,$$

where θ is the angular displacement in radians, and k a constant depending on the physical characteristics of the suspension. Therefore we may write

$$k\theta = IHAN \cos\theta,$$

or

$$I = \frac{k\theta}{HAN\cos\theta}, \qquad \text{Eq. 109}$$

* The suspended coil type of instrument was first used by Sir William Thomson in 1870. However this general form of galvanometer did not come into extended use until D'Arsonval introduced the design of moving coil instrument which bears his name.

which is the fundamental equation of the D'Arsonval type of galvanometer.

Since $\sin\theta$ appears in the above expression it is evident that the relation between the deflection and the current will not be linear. In many cases it is convenient to have a current indicating instrument available in which a linear relation does obtain between the rotation of the moving system and the current passing through the coil. In order to accomplish this the magnetic system of the galvanometer is designed as shown diagrammatically in Fig. 120. It will be noted that the pole pieces are curved and that the coil moves about a centrally located soft iron "core." This iron within the coil, together with the shape of the pole pieces, serves to make the direction of the magnetic lines of force radial in the space through which the coil is free to move. Hence $\sin\theta$ always equals unity. With this arrangement the current is strictly proportional to the deflection of the coil. Figure 121 is an illustration of a well-known make of high sensitivity D'Arsonval galvanometer. Instruments of this type are made which will give a readable deflection when 0.00004 microampere passes through the winding.

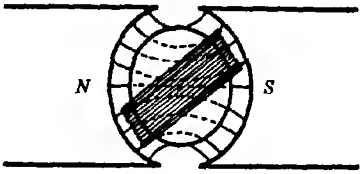
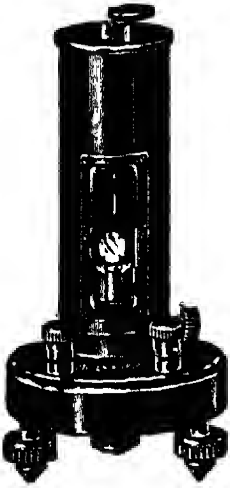


Fig. 120



(Courtesy Leeds and Northrup Co.)

Fig. 121

Another type of current indicating instrument having restricted but important fields of application is known as the *Einthoven galvanometer*. This instrument is constructed as sketched in Fig. 122. Between the poles of an electromagnet is stretched a very fine fiber usually made of quartz and which is made conducting by means of an extremely thin coating of silver, gold or platinum. The current to be detected or measured is caused to pass through the fiber element. The magnetic field due to this current reacts with the intense field due to the pole pieces of the electromagnet with the result that the fiber is

slightly deflected at its middle point in a direction at right angles to the field and to the direction of the current. The movement of the fiber is observed through a hole in the pole pieces by means

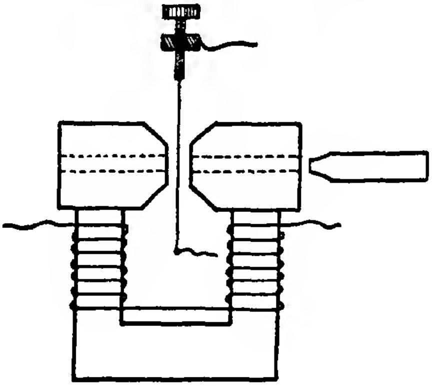
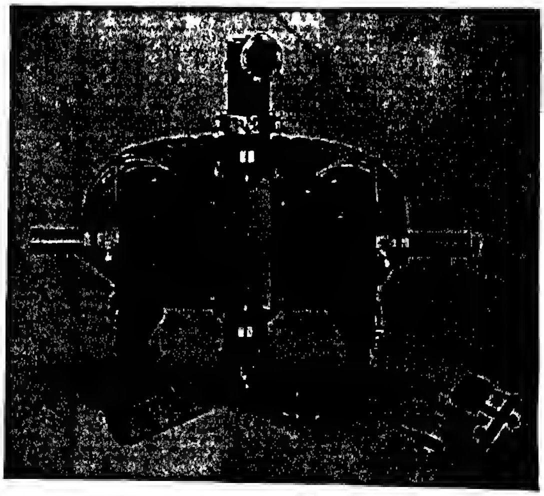


Fig. 122

of a microscope equipped with a micrometer eyepiece. The sensitivity of the Einthoven type of galvanometer is of the same order as that of the D'Arsonval. In some applications of this type of instrument the image of the fiber is projected, by means of



(Courtesy Cambridge Instrument Co.)

Fig. 123.—Einthoven Galvanometer

a suitable optical system, on to a screen or photographic film. When observed by the latter means this type of galvanometer may be used in the study of variable and alternating currents of

extremely small magnitude. Figure 123 is an illustration of a practical form of the Einthoven instrument.

95. The Magnetic Circuit and Magnetic Flux.—In dealing with magnetism it should be observed that we have what may be called a magnetic circuit, corresponding roughly to the electric circuit. In the magnetic case the lines of force take the place of the paths of the electrons in the electric analogue. There is however one marked difference between an electric and a magnetic circuit; an electric circuit may be "open," so that the electrons will not flow, but *a magnetic circuit is always closed.* Lines of force always form closed paths. For instance in the case illustrated in Fig. 124 the magnetic circuit consists of the horse-shoe magnet NMS, the two air gaps G and G', and the iron bar D. If the bar D were not there the lines of force would pass through the air between the N and S poles.

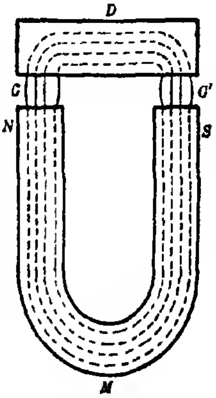


FIG. 124

The total number of magnetic lines which exist in a magnetic circuit is referred to as the magnetic flux. In the example cited above the magnetic flux would be the number of lines which exist in the area defined by the cross-section of the magnetic circuit at any point. It is customary to designate magnetic flux by the Greek letter Φ. The unit of magnetic flux is a single line of force and is called the maxwell. For instance a magnetic circuit in the cross-section of which there existed 10,000 lines would be spoken of as having a flux of 10,000 maxwells.

It is frequently convenient to deal with the magnetic flux per unit area, rather than with the total flux, and this leads to another concept known as *flux density. Flux density is accordingly defined as the number of lines passing normally through unit area.* Flux density is sometimes spoken of as *normal induction* or more simply as *induction.* The relation which exists between the total flux and flux density may be expressed thus,

$$B = \frac{\Phi}{A}, \qquad \text{Eq. 110}$$

where B is the flux density, Φ the total flux, and A the cross-

section of the magnetic circuit at the point being considered. The unit of flux density is the same as for field intensity, viz., the gauss. A flux density of 3000 gausses would mean that 3000 lines of force existed in each square centimeter cross-sectioned area of the magnetic circuit.

In dealing with the electric circuit we had to do with the concept of conductance (Sec. 41). In the magnetic circuit there is a corresponding factor which is termed *permeability*, and designated by μ. Certain materials permit the building up of magnetic flux within the region occupied by the material more readily than do others. If, for instance, the region within a solenoid be occupied by air or glass or wood a certain current in the winding will give rise to a definite field intensity and hence a certain total flux in that region. If however we introduce into the solenoid instead of the materials mentioned a bar of soft iron the number of lines will be greatly increased. In other words many more magnetic lines are established in the iron than existed in the same region when occupied by any one of the other materials. We say that the permeability of iron is greater than that of the other substances and since the permeability of the air is usually taken as unity, *the permeability of a medium may be defined as the conducting effectiveness for lines of magnetic flux as compared with air.*

It should, however, be carefully noted that the permeability of a material is not a constant factor; its value depends upon the strength of the magnetic field in which the material is. Thus, when we speak of the permeability of a material, we should specify the magnetic conditions under which the specimen is being held. In general it may be said that the permeability varies with the magnetizing force,* being less for the higher values of field intensity. Temperature is also a factor. The relation which obtains between permeability, flux density, and magnetic force may be expressed thus,

$$B = \mu H, \qquad \text{Eq. 111}$$

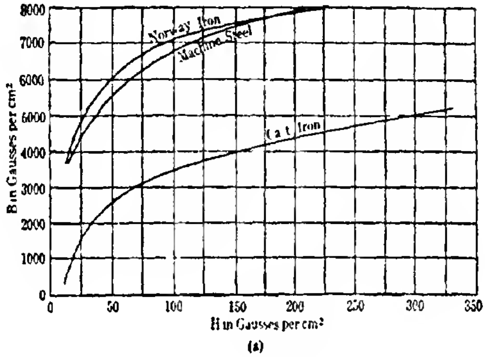
or

$$\mu = \frac{B}{H}. \qquad \text{Eq. 112}$$

It will thus be noted that permeability may be defined as the

* The flux density in air is frequently referred to as "magnetizing force." If the field intensity within a solenoid is, say, 10 e.m.u.'s the flux density in air would be 10 gausses and this would be the magnetizing force.

ratio between the flux density and the magnetizing force. Because of the fact that the permeability is a function of the field intensity, it becomes impracticable to attempt to magnetize iron or other magnetic material beyond a certain flux density. Thus, for example, it does not contribute to the total magnetic flux in the pole pieces of a dynamo to increase the field current beyond a certain value. When the maximum flux obtains in any given



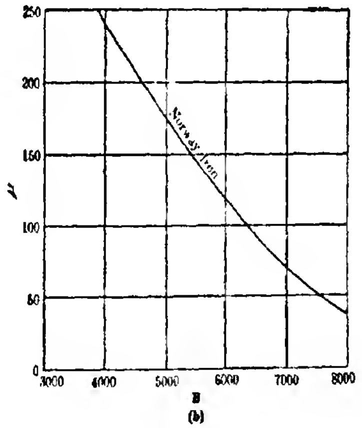


Fig. 125

magnetic material it is said to be *saturated.* An examination of the magnetization curves (Fig. 125*a*) of various samples of iron discloses the significance of permeability as a factor in connection with the magnetic circuit. Permeability as a function of flux density for a given specimen is set forth in the curve shown in Fig. 125*b*.

It is the aim of research to produce a material for use in magnetic circuits which has a high permeability particularly

when worked in comparatively weak fields. T. D. Yensen, by subjecting pure iron to a special heat treatment in vacuo, has developed a technique whereby there may be produced an iron having a permeability higher than that exhibited by the best iron heretofore obtainable.

Recently a special magnetic alloy, known as *permalloy*,* has been produced which, in very weak fields (a fraction of a gauss), is said to have a permeability 200 times as great as that of the best iron. The new alloy consists of about 73.5 per cent nickel and 21.5 per cent iron. Permalloy is useful in magnetic circuits employed in communication engineering, for it is in this work that extremely weak fields obtain. Figure 126 shows the B–H curves for a specimen of this new alloy as compared with a sample of high grade silicon steel.

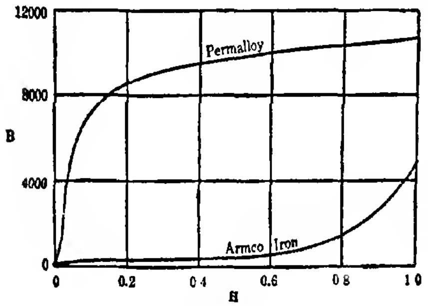


Fig. 126

Another group of magnetic alloys called perminvar,† composed of iron, nickel and cobalt, has also recently been announced by Mr. G. W. Elmen. It is stated that these latter alloys "have unusual constancy of permeability" and possess other magnetic properties which adapt them to uses in connection with the telephone and other similar equipment.

In passing it may be noted that there is an alloy containing no iron which possesses magnetic properties. Dr. Heusler in 1901 discovered an alloy which bears his name and which is composed of 14.3 per cent aluminum, 28.6 per cent manganese, 57.1 per cent copper. In this alloy the permeability increases with the addition of manganese up to the point where the proportions of manganese and aluminum are in proportion to their atomic weights. The maximum magnetization which may be obtained with this alloy is about one-third of that of the best iron. The copper which enters into its composition apparently does not

* H. D. Arnold and G. W. Elmen, *Journal Franklin Institute*, May 1923, pp. 621–632.

† G. W. Elmen, *Journal Franklin Institute*, Vol. 206, Sept. 1928.

affect its magnetic properties but serves only to make the alloy soft enough to be mechanically worked.

In dealing with iron, it may be said that the practical saturation point for wrought iron, soft annealed sheet iron and cast steel is between 17,000 and 20,000 lines per cm.^{2}. In the case of grey iron the saturation limit varies from 60,000 to 70,000 lines per square inch. Permalloy may become saturated in the earth's field.

Corresponding to resistance in the electric circuit we have what is known as *reluctance* in the magnetic circuit. By reluctance is meant the resistance which any substance offers to the setting up of magnetic flux. The magnitude of the reluctance is a function of the length of the magnetic path, the cross-sectional area, and the permeability. These factors are related thus,

$$\text{Reluctance} = \frac{l}{\mu a}\,. \qquad \text{Eq. 113}$$

If a magnetic circuit is made up of several component parts, as shown in Fig. 124, the total reluctance of the circuit would be given by the relation,

$$\text{Reluctance} = \frac{l_1}{\mu_1 a_1} + \frac{l_2}{\mu_2 a_2} + \frac{l_3}{\mu_3 a_3} + \frac{l_4}{\mu_4 a_4},$$

where l_1 = length of the magnet NMS, l_2 = length of air gap G, l_3 = length of bar D, l_4 = length of air gap G'. The several μ's and a's represent the permeability and sectional area of the corresponding parts of the path.

96. Law of the Magnetic Circuit.—Magnetic flux and reluctance are related by a law which lies at the basis of the design of all electromagnetic machinery. The law of the magnetic circuit has been known in a general way for some two hundred years, possibly dating back to the time of Euler in 1761, but Rowland in 1873 was the first to give definite form to this relation. In 1883 Bosanquet introduced the term *magnetomotive force* (M.M.F.), corresponding to E.M.F. in the electric circuit. *Magnetomotive force is that which gives rise to magnetic flux.* The law usually takes the form,

$$\text{Magnetic flux} = \frac{\text{magnetomotive force}}{\text{reluctance}}\,.$$

Expressing this in symbols we have

$$\Phi = \frac{\text{M.M.F.}}{\mathcal{R}}. \qquad \text{Eq. 114}$$

The resemblance of this relation to Ohm's law is obvious. It will be recalled that the E.M.F. in any circuit is measured by the work required to carry unit quantity of electricity completely around the circuit. Likewise M.M.F. is *numerically* equal to the work done in carrying a unit test pole once around the magnetic circuit. The unit of M.M.F. is the gilbert.

Suppose we have a toroidal winding as shown in Fig. 116, the length of the magnetic path being L. It has been shown (Sec. 92, eq. 102) that the field intensity in such a case would be

$$H = 4\pi nI,$$

where n is the number of turns per unit length and I the current in e.m.u.'s. Multiplying the above equation by L we have

$$HL = 4\pi nIL.$$

But $H \cdot L$ would be the work done in transporting unit test pole around the magnetic circuit. Therefore, by definition,

$$\text{M.M.F.} = 4\pi nIL.$$

For convenience in evaluating we may let $N = n \cdot L$, where N is the total number of turns in the winding. Then

$$\text{M.M.F.} = 4\pi NI. \qquad \text{Eq. 115}$$

Combining eqs. 113, 114 and 115, we have as the equation for any magnetic circuit

$$\Phi = \frac{4\pi NI}{l/\mu a}. \qquad \text{Eq. 116}$$

If I is to be expressed in amperes, eq. 116 becomes

$$\Phi = \frac{2\pi NI}{5l/\mu a} \text{ maxwells.} \qquad \text{Eq. 117}$$

The term NI is frequently spoken of as "ampere-turns." It is evident that a given value of M.M.F. may be produced by making the number of turns in the magnetizing coil relatively large and

the current small, or by utilizing a comparatively large current and few turns in the winding. In any given case, circumstances will determine the relative values of N and I.

The utility of the law of magnetic circuits may be illustrated by the application of eq. 117 to a specific problem. Suppose we have a winding arranged to produce magnetic flux in an iron core having an air gap as shown in Fig. 127. Let us assume the mean length of the magnetic circuit in iron to be 60 cm. and that the air gap is 2 cm. in length. Let the cross-section of the core be 5 × 5 cm. What ampere-turns will be required to develop a flux density of 15,000 gausses/cm.^{2} in the iron if the permeability of the iron at that flux density is 400?

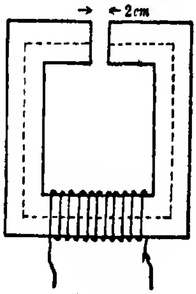


FIG. 127

Combining eqs. 110 and 117 we have

$$BA = \frac{2\pi NI}{5l/\mu a}.$$

Solving for NI we get

$$NI = \frac{5BA}{2\pi} \cdot \frac{l}{\mu a}.$$

Substituting and solving,

$$NI = \frac{5 \times 15{,}000 \times 25}{2\pi}\left(\frac{60}{400 \times 25} + \frac{2}{25}\right)$$
$$= 25{,}664 \text{ ampere-turns.}$$

If, for instance, the available current were 0.5 ampere the total number of actual turns of wire in the winding would be 3080.

97. Hysteresis.—If a magnetic substance such as iron be subjected to a gradually increasing magnetizing force the flux in the material will increase quite rapidly at first and then more slowly, at length reaching a point beyond which an increase in the magnetizing field will not cause any increase in the flux. This is shown by the curve CD in Fig. 128a, the data for which were taken from an actual test. At something over 6000 gausses this particular iron became saturated. If after reaching the point D in the curve the magnetizing current is gradually reduced to zero the curve DE will result. It is evident that, though the magnetiz-

ing field has become zero, a flux density of about 4500 lines still exists in the iron.

Referring now to Fig. 128*b*, which is a complete graph, including that part already shown in Fig. 128*a*, if the magnetizing current be reversed and gradually increased in value *the flux* will continue to diminish, becoming zero when the magnetizing field has attained a reverse value represented by *CQ*. This magnetizing force which must be applied in the opposite direction in order to demagnetize a material after the initial magnetizing field has been reduced to zero is known as the *coercive force.*

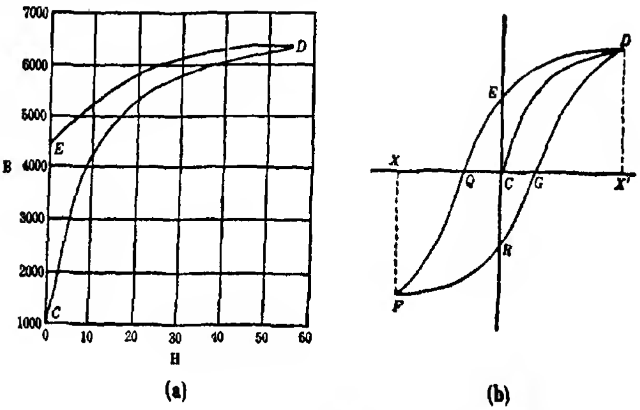


Fig. 128

If the magnetizing field be increased beyond *Q* the flux will build up in the reverse direction as shown by *QF* on the curve, reaching saturation in the region of *F*. Reducing the magnetizing current from *x* to zero causes the flux to decrease in value, leaving a residual flux represented by *CR*. As a final step in the cycle of magnetization the field may be increased from zero, in the original direction, with the result that the flux will vary as shown by that part of the curve marked *RGD*. When *D* has been reached the magnetic cycle has been completed, and the resulting closed figure is referred to as the "hysteresis loop."

An examination of the curve just traced discloses the fact that the magnetic flux lags the magnetizing force throughout the entire cycle. This phenomenon is known as *hysteresis*. If our specimen of magnetic material were to be taken through subsequent magnetic cycles the flux would repeat the changes as shown by the original loop *DQFGD*. Owing to hysteresis the branch *CD*

will not be described except when the specimen is entirely without magnetism at the beginning of the magnetic cycle. Figure 129 is reproduced from a photographically traced hysteresis curve for a sample of annealed iron wire.

If we accept, for the time being, the molecular theory of magnetism, it would appear that the physical particles of the iron suffer actual changes in orientation, and that this change in position cannot take place as rapidly as the changes in the magnetizing field. This movement of the particles gives rise to thermal effects which results in heating of the iron and the consequent dissipation of energy. There is, then, what is known as a "hysteresis loss." The nature of the iron being used, as well as the frequency of the cyclic changes, governs the magnitude of the heating effect, and hence the loss due to hysteresis. The magnitude of the loss due to hysteresis in a given case is proportional to the area of the hysteresis loop. Knowing the scale of plotting one may, by determining the area inclosed by the curve, compute the hysteresis loss per cycle.

Fig. 129

Steinmetz developed an empirical formula for use in computing the loss due to hysteresis. It is

$$P = aVfB^{1.6}, \qquad \text{Eq. 118}$$

where P is the loss in watts, V the volume of the magnetic material in cubic centimeters, f the frequency in cycles per sec., B the maximum flux density, and a is a constant the value of which depends upon the material being used for a core. On the following page is a table showing values of the hysteresis coefficient for different materials, as given by Mr. M. G. Lloyd.*

It is interesting to note that, in general, the effect of impurities is to decrease the permeability and to increase the hysteresis loss, although there are certain exceptions to this statement, and these exceptions are of considerable commercial importance. For example the presence of a small percentage of silicon or aluminum increases the permeability and decreases the hysteresis loss. As

* Lloyd, M. G., *Journal of the Franklin Institute*, July 1910, pp. 1–25.

a result of this silicon steel is extensively used in certain types of electromagnetic equipment. Reference has already been made (Sec. 95) to the iron produced by Yensen. In this particular iron the hysteresis loss is only about one third of that of standard transformer steel. At low magnetization values the new magnetic material perminvar (Sec. 95) is said to be almost free from hysteresis, and to show only a slight coercive force. If a material is to be used for permanent magnets it is important that the co-

Material	a	Authority
Annealed cast steel	0.008	Foster
Hard tungsten steel	0.058	Steinmetz
Hard cast steel	0.025	Steinmetz
Forged steel	0.020	Steinmetz
Cast steel	0.012	Steinmetz
Soft machine steel	0.009	Foster
Silicon steel	0.0009	Gumlich
Best silicon steel	0.0006	Lloyd
Hard nickel	0.039	Steinmetz
Soft nickel	0.013	Steinmetz
Two-mil steel wire	0.016	Lloyd
No. 36 iron wire	0.005	Lloyd
Soft iron wire	0.002	Foster
Magnetic iron ore	0.020	Steinmetz
Cast iron	0.013	Steinmetz
Pure iron	0.003	Gumlich
Ingot iron	0.0016	Lloyd
Cobalt	0.012	Steinmetz
Heusler alloy I	0.012	Gumlich
Heusler alloy II	0.0024	Gumlich
Ordinary sheet iron	0.004	Lloyd
Annealed iron sheet	0.002	Lloyd
Silicon steel sheet	0.0010	Lloyd
Best annealed sheet	0.0010	Lloyd
Electrolytic iron	0.009	Schild

ercive force be high. K. Honda, a Japanese metallurgist, has produced a cobalt-steel alloy which contains in addition to the steel about 35 per cent of cobalt, 8 per cent of tungsten and 3 per cent of chromium. This special steel is very resistant to demagnetization, its coercive force being three or four times that of tungsten steel. The Honda alloy is utilized to some extent as the material for magnets which are to be used in devices requiring great permanency of the magnetic field.

98. Magnetostriction.—It was shown by S. Bidwell in 1886 that changes in the dimensions of bodies occur when they are magnetized. A wire or rod of magnetic material, for instance,

shows a measurable change in length when subjected to a magnetic field. If the applied field is of a variable nature the magnetic material will expand and contract in response to the changing magnetizing force. The phenomenon of a change in one or more dimensions of a body when it undergoes magnetization is termed magnetostriction. Pure iron and steel exhibit only very feeble magnetostrictive effects. Pure nickel however shows a very marked magnetostrictive response. Certain alloys also exhibit this phenomenon. While the absolute change in dimensions due to magnetostriction is small yet this effect has recently been utilized in certain special electrical equipment for use in connection with high frequency alternating currents.

99. Magnetic Testing.—From what has been said in the preceding section it is obviously of great importance in connection with the design of all equipment which involves the magnetic circuit to have available accurate data bearing on the magnetic properties of the material it is proposed to use. Hence the testing of magnetic materials forms an important part of electrical measurements. Magnetization and permeability curves corresponding to those shown in Figs. 125*a* and 125*b* are extensively used as aids in design calculations.

For example if one is designing a transformer which is to be used in a telephone circuit, it is important to so arrange conditions that the iron will not be worked at or near the saturation point; otherwise speech distortion will result. In such a case as this the volume of iron and the number of turns in the windings would be so laid out that the core would be worked on the comparatively straight part of the B–H curve. In order to secure data from which to plot the B–H and μ–H curves, together with the complete hysteresis loop, various methods are followed. It is without the scope of this volume to describe these methods in detail. However it may be said that the general process consists in first completely demagnetizing the sample of material and afterwards subjecting it to a magnetizing field the value of which is changed by definite steps. Means are provided whereby the resulting flux in the material can be directly or indirectly measured as the magnetizing force is changed. For a complete treatment of the methods used in magnetic testing the student is referred to the two following publications of the U. S. Bureau of Standards: *Bulletin*, Vol. 6, p. 31, and *Circular No. 17*, "Magnetic Testing."

100. Theories of Magnetism.—In discussing the magnetism of the earth (Sec. 86) it was pointed out that a satisfactory explanation of terrestrial magnetism has not yet been worked out. It is also true that a complete theory of magnetism in general has yet to be proposed and verified. This however does not mean that we do not have tentative explanations to account for many of the known magnetic phenomena. All substances which exhibit any magnetic properties whatever may be classified under three types, that is,

1. Diamagnetic substances,
2. Paramagnetic substances,
3. Ferromagnetic substances.

Faraday (1845) investigated the behavior of a large number of substances when placed in the field of a powerful electromagnet. He found that many substances set themselves *across* the magnetic field. Among these substances are to be found copper, aluminum, lead, bismuth,* gold, silver, glass, carbon, wood, paper. Such substances tend *to move toward the weaker part of the field.* Faraday called such substances *diamagnetic.* Many other substances such as iron, cobalt, nickel, platinum, titanium, manganese, liquid oxygen and others set themselves along the lines of force. In other words they tend to move *toward* the stronger parts of a field. Substances which behave in this manner are said to be *paramagnetic.* Those substances which exhibit the phenomenon of hysteresis such as iron, cobalt, nickel and certain alloys are called *ferromagnetic.* Any comprehensive theory of magnetism must account for the possibility of the three classes of substances just mentioned, and in addition must also furnish an explanation of the following magnetic phenomena:

(*a*) The fact that a magnet may be physically subdivided into the smallest possible parts and yet each minute part will exhibit polarity.
(*b*) The effect of mechanical jarring on the magnetic intensity.
(*c*) That a magnetic substance when raised to what is known as its *critical temperature* is rendered non-magnetizable.
(*d*) Hysteresis, with the attendant heating.
(*e*) Saturation.

* Brugmans of Leyden in 1778 discovered that bismuth is repelled by both poles of a magnet.

(*f*) Magnetostriction.
(*g*) The effect of heat treatment in the production of the special magnetic alloys referred to in Sec. 95.

Many if not all of the phenomena of magnetism can be explained on the basis that the molecules, or possibly the atoms, of the magnetic substance act as minute magnets. According to the molecular theory of magnetism these molecular magnets, in the case of an unmagnetized substance such as soft iron, arrange themselves in magnetically closed configurations leaving no free or uncompensated poles. When the material is subjected to a magnetizing force the molecular magnets are thought to rearrange themselves in such a manner that their magnetic axes are parallel to the lines of the magnetizing force. Their north poles would thus all point in one direction and their south poles in the opposite direction. There would therefore be a group of like and unneutralized molecular poles at one point in the body and a corresponding group of like but opposite uncompensated poles at some other point in the material. While such an explanation of magnetic phenomena is very useful, *it does not account for the magnetism of the atom or molecule itself.*

As a result of his study of the interaction of magnetic fields and electric currents following Oersted's discovery, Ampere was led to suggest that magnetism is due to currents circulating about the atoms or molecules. Ampere's theory however does not extend to the *cause* of these hypothetical molecular currents. Later studies in this extremely interesting and important field by P. Curie (1895), P. Langevin (1905), Weiss (1907) and others have led to a theory which is in fact but an extension of the Amperean current idea.

We know that a single electron in motion constitutes an electric current, the resulting field amounting to a magnetic shell. Each electron which rotates about the nucleus of an atom gives rise to a definite magnetic field. In a diamagnetic substance it is probable that some electrons are rotating in clockwise direction while others rotate counter-clockwise, and the planes of the orbital motions are so positioned with respect to each other that the resultant moment of the electronic magnetic shells is zero; hence no external magnetic effect obtains. If an external field be applied the relative orientation of some of the orbital planes

probably shifts with the result that the equilibrium of the internal magnetic system is destroyed. In diamagnetic substances the resultant magnetic moment is of such a sense that the substance is driven to the weaker part of the applied field.

In the case of paramagnetic bodies it is supposed that the resultant moment of the electronic magnetic shells *is not zero*, and hence such atoms are equivalent to minute magnets which will, under the action of an externally applied field, align themselves in such a manner that their magnetic axes are parallel to the direction of the applied field.

In ferromagnetic substances there appears to be a more or less pronounced magnetic interaction between the electronic magnetic fields of adjacent atoms, thus giving rise to the phenomenon of hysteresis.

In conclusion it may be said that the latest data available appear to indicate that magnetization is not accompanied by a movement of the atomic or molecular systems, but that the planes of the electronic motion are rotated about their orbital axes. The electrons are, however, not moved from their position relative to the nucleus. The electronic theory of magnetism is still in the formative stage but it serves to correlate many of the known facts and is proving useful in magnetic research.

PROBLEMS

1. What will be the field intensity at the center of an helixal coil having 500 turns in the winding, the length of solenoid being 10 cm. and its diameter 4 cm.? Assume the current to be 5 amperes.

2. What will be the flux density if a bar of iron occupies the space within the winding of Problem 1, the permeability being 300?

3. Find the ampere-turns necessary to produce a flux density of 10,000 gausses/cm.^{2} in a closed ring of iron having the following specifications:

Mean diameter of ring = 12.5 cm.,
Diameter of iron = 2.5 cm.,
Permeability = 400.

4. What ampere-turns would be required to produce the same flux density in the iron of Problem 3 if the core had an air gap 1 cm. in length?

5. Show that the force exerted upon one another by two parallel conductors d cm. apart will be $2I_1I_2/d$ dynes/unit length.

CHAPTER XVII

ELECTROMAGNETIC INDUCTION

101. Induced Electromotive Force.—On November 24, 1831, eleven years after Oersted's basic discovery, Faraday read a paper before the Royal Society of London in which he disclosed his discovery that whenever the flux in the region of any conductor is caused to vary in intensity that conductor becomes the seat of a temporary E.M.F. If the conductor in which the E.M.F. is developed forms a closed circuit a current will result and this current is referred to as an *induced* current. This discovery of Faraday's was of transcendent importance. Together with Oersted's findings Faraday's disclosures form the basis of all modern electrical power generation and distribution.

Faraday however was not alone in the early work done in connection with what has come to be spoken of as electromagnetic induction. The American, Joseph Henry, also made valuable contributions to our knowledge of the fundamental facts in this field. Indeed it is probable that the possibility of obtaining an electric current by means of a magnet was recognized independently by Henry at practically the same time at which Faraday was carrying on his induction experiments in England.

The facts discovered by Faraday and other early investigators in connection with the production of an induced E.M.F. and the resulting currents led to the formulation of laws which tied together the various observed phenomena. Emil Lenz presented a paper before the Academy of Sciences at St. Petersburg in 1833 in which he gave a very simple law by which the direction of the induced E.M.F. may always be predicted. This rule, known as Lenz's law, is to the effect that "the direction of an induced current is always such that by its electromagnetic action it tends to oppose the motion which gives rise to it." The law is of great importance in many of the applications of electromagnetic induction as we shall see in later sections.

In 1845 F. E. Neumann developed a quantitative expression embodying Faraday's experimental findings. Expressed in

terms of the calculus Neumann's description of the relation existing between the cause and effect in electromagnetic induction takes the form

$$E = -\frac{d\Phi}{dt}, \qquad \text{Eq. 119}$$

where E represents the induced E.M.F. and $\frac{d\Phi}{dt}$ the time rate of change of magnetic flux. In other words, the magnitude of the induced E.M.F. depends upon the rate at which the magnetic flux changes in value in the region of the conductor in which the induction is taking place. The negative sign indicates that the induction takes place in conformity with Lenz's law. *The magnitude of the induced E.M.F. is independent of the particular method by which the magnetic field is caused to vary in intensity.* In dealing with electromagnetic induction it should also be emphasized that *the E.M.F. is what is induced.* If the circuit in which the induction takes place is not closed no current will flow, but an induced E.M.F. may however exist.

The relation $E = -\frac{d\Phi}{dt}$ (eq. 119) gives the *instantaneous* value of the induced E.M.F. The *average* value is given by the ratio of the total flux change to the time involved in that change, or

$$E_{av.} = \frac{\Phi}{t}\text{ e.m.u.} \qquad \text{Eq. 120}$$

Thus if the average flux change is at the rate of one line of force per second the average induced E.M.F. is 1 e.m.u. Obviously this is an extremely small E.M.F. The International Electrical Congress which met in Paris in 1881 decided to call the practical unit of P.D. and E.M.F. the *volt* and to fix its value at 10^8 e.m.u. This is a value of E.M.F. approximately equal to that developed by the Daniell cell, which was at that time considered to be the most dependable standard of E.M.F. On the basis of the above definition the E.M.F. in volts would be given by

$$E_{av.} = \frac{\Phi}{10^8 t}. \qquad \text{Eq. 121}$$

102. E.M.F. Developed in a Linear Conductor Due to Its Movement in a Magnetic Field.—We shall find it useful to have

available an expression giving the value of the E.M.F. induced in a linear conductor when subjected to a changing magnetic flux.

Referring to Fig. 130, suppose that a straight conductor is moved through a uniform field from the position *ab* to the position *a'b'*, perpendicularly to the direction of the flux, in the time t. Let the length of the conductor be designated by l, and the linear displacement by s. From eq. 110 we have

$$\Phi = A \cdot B,$$

where Φ is the total flux, A the area and B the flux density. The area swept out by the movement of the conductor will be $l \cdot s$. Hence

$$\Phi = lsB.$$

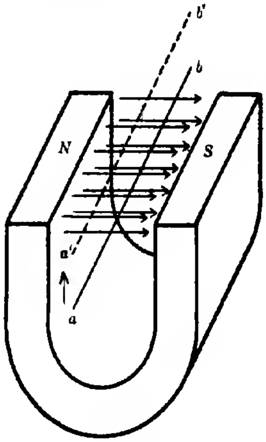


Fig. 130

Dividing by t we get

$$\frac{\Phi}{t} = lB\frac{s}{t}.$$

But $\frac{s}{t}$ = velocity = v. Hence we may write

$$E_{av.} = lBv \text{ (e.m.u.)}, \qquad \text{Eq. 122}$$

where l is the active length of the conductor in cm., B the flux density of the inducing field, and v the velocity of displacement in cm./sec. If E is to be expressed in volts we have

$$E_{av.} = \frac{Bvl}{10^8}. \qquad \text{Eq. 123}$$

It should be noted that the same result would have obtained had the conductor remained stationary and the magnet been moved.

103. E.M.F. Induced in a Rotating Coil.—The results of the last section may be extended to cover the case of a conductor having the form of a rectangular loop, as shown in Fig. 131*a*. Assume that the loop *abcd* is rotated in a clockwise direction about the axis *xx'*. It is evident that this case differs from the previous situation in two respects. First we have *two* active conductors, *ab* and *cd* connected in series, and secondly, both of these con-

ductors because of their angular displacement about the axis xx' *will "cut" flux* at a changing rate.

If we consider the side *ab* as it moves through some given position such as that shown in Fig. 131*a* the E.M.F. developed in

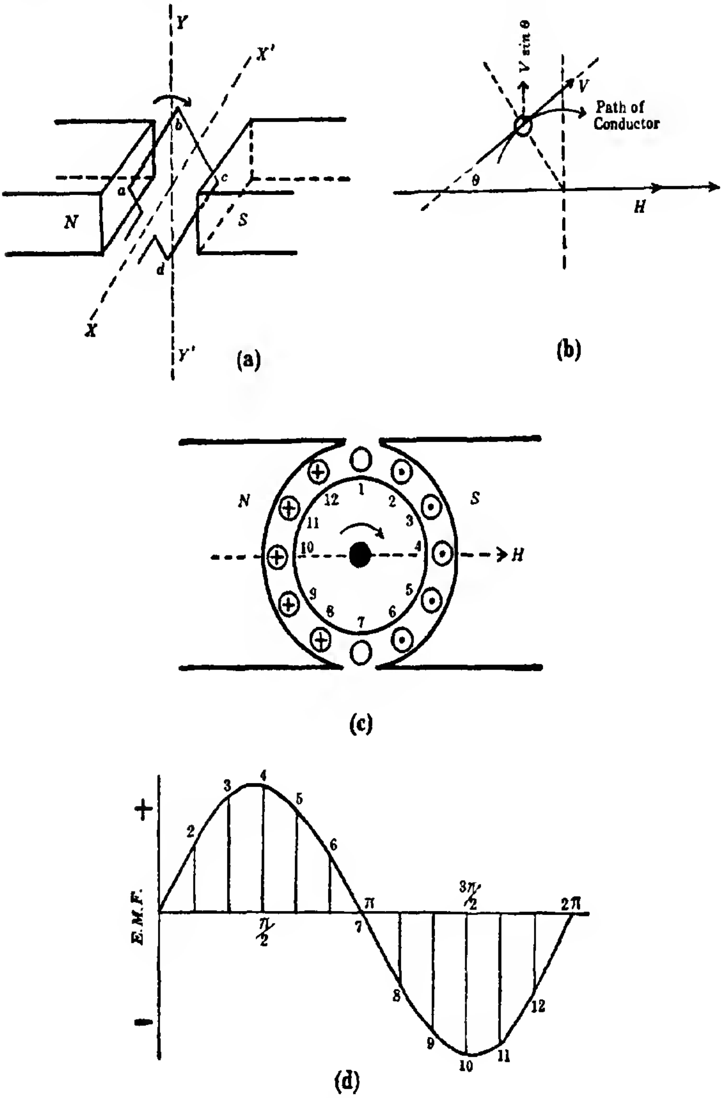


Fig. 131

this part of the conductor will be in a direction away from the reader. At the same time the E.M.F. produced in the side *dc* will be directed from *c* toward *d*. The two E.M.F's. are therefore

additive, and hence, by applying eq. 122, we would have as the total average E.M.F. through a very small interval of time

$$E_{av.} = \frac{2Bvl}{10^8}. \qquad \text{(i)}$$

Since the end sections of the loop, *bc* and *ad*, move in a direction parallel to the flux no E.M.F. will be produced in those parts, and hence they will contribute nothing to the E.M.F. generated by the loop as a whole.

While the linear velocity of the loop may be constant its *effective* velocity is variable. Referring to Fig. 131*b* it will be seen that at any instant the actual velocity vector makes an $\angle\theta$ with the direction of the flux, and that the component of the velocity perpendicular to the direction of the flux will be $v \sin \theta$. This component, $v \sin \theta$, is the effective velocity of the conductors with respect to the flux. From the geometry of the case θ is also the angle between a vertical plane through the axis of rotation and the instantaneous position of the plane of the coil, and it is this angle which will be meant whenever the angular displacement of the coil is referred to. Replacing v in (i) by its equivalent $v \sin \theta$ we have

$$e = \frac{2Blv \sin \theta}{10^8}, \qquad \text{(ii)}$$

where e is the value of the E.M.F. at any instant.

It will be more convenient to have the velocity of the conductors expressed in angular measure. To do this we make use of the relation

$$v = \omega r,$$

where ω is the angular velocity and r the radius of the rotating coil. But

$$\omega r = 2\pi Rr,$$

where R is the number of revolutions of the coil per second. Hence we may write

$$e = \frac{4\pi BRrl}{10^8} \sin \theta \cdot \qquad \text{(iii)}$$

It is evident that $2rl = A$, the area of the coil, and since $\Phi = BA$, eq. (iii) becomes

$$e = \frac{2\pi R\Phi}{10^8} \sin \theta.$$

If, as is commonly the case in practice, the coil consists of several turns connected in series, each loop would contribute its own E.M.F., and hence the total E.M.F. generated in the winding would be

$$e = \frac{2\pi NR\Phi}{10^8}\sin\theta, \qquad \text{Eq. 124}$$

where N is the number of turns in the coil.

From the last equation it may be seen that the maximum value of the E.M.F. will obtain when $\sin\theta = 1$, which will be when θ is 90°, i.e., when the plane of the coil is parallel to the direction of the flux. In that position a given conductor will be moving at right angles to the flux; therefore $\frac{d\Phi}{dt}$ will be a maximum. The minimum value of the E.M.F. will occur when $\theta = 0$, which is when the plane of the coil is normal to the flux. In this position the conductors will be moving parallel to the flux and hence $\frac{d\Phi}{dt} = 0$. At intermediate positions the E.M.F. will have values between zero and the maximum corresponding to the various values of $\sin\theta$.

There is one other significant aspect of the case which must be noted. Figure 131*c* is an attempt to represent a number of the positions which one of the conductors, say *ab* in Fig. 131*a*, will assume during one complete revolution of the coil. For convenience these positions are spaced 30° apart. When the conductor is in position 1 the E.M.F. will be zero, as noted above. As it moves through positions 2 and 3 to 4 the E.M.F. will increase in value and its direction will be toward the reader. As it passes on through 5 and 6 to 7 the E.M.F. will decrease, becoming zero again at 7. From position 7 to 10 the E.M.F. will again increase in value, but, since for this particular conductor the direction of the flux is in effect reversed, *the E.M.F. will be reversed also* and will accordingly be directed from *a* toward *b*. From 10 to 1 the E.M.F. will again decrease, becoming zero at 1. It will thus be seen that the E.M.F. is periodic or alternating in character. If these angular positions and the corresponding E.M.F. values be plotted the graph shown in Fig. 131*d* will result. This curve will represent one complete *cycle* of operations, and the time required for the completion of a cycle is known as the *period*. The

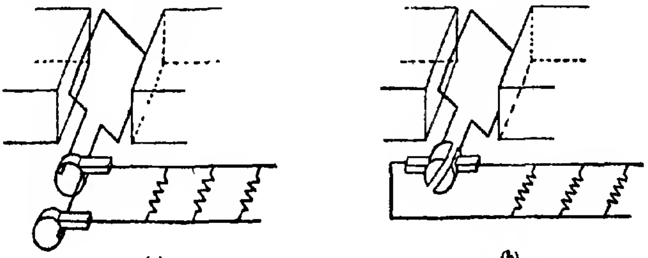
frequency would be the number of cycles which occur per second. The angle θ is known as the *phase angle.*

104. The Dynamo.—The dynamo is a practical application of the principles outlined in the last section. If the rotating coil is driven by a mechanical agency we have a means whereby mechanical energy may be converted into electrical energy. It should however be borne in mind that *a dynamo is a device for developing or generating an E.M.F.* If we connect the terminals of our rotating coil, by suitable means, to a lamp or other electrical load, thus completing the circuit, current will flow in response to the E.M.F. produced by the machine. The magnitude of the current will depend upon the value of the E.M.F. generated by the dynamo and also upon the resistance of the complete circuit. This may be expressed thus,

$$I = \frac{E_t}{R_a + R_e}, \qquad \text{Eq. 125}$$

where E_t is the terminal E.M.F. produced by the dynamo, R_a the resistance of the conductor making up the rotating coil, and R_e the resistance of the load or external circuit.

In the practical machine, in order to secure a maximum of magnetic flux in the region of the conductors, the coil is wound on an iron core (eq. 111). The pole pieces are shaped to conform to the shape of the rotating member, thus producing a field in the air gap between poles and the rotor which is nearly radial and hence at right angles to the moving conductors. The rotating winding and the supporting core is known as the armature.



(a) (b)

Fig. 132

The external or load circuit may be connected to the terminals of the rotating coil by two possible arrangements. In one method use is made of *slip rings* as diagrammed in Fig. 132*a*.

These consist of two metallic rings mounted on, but insulated from, the shaft which carries the armature winding. One terminal of the armature is connected to each ring; brushes to which the load circuit may be connected bear against the slip rings. By this arrangement the E.M.F. impressed on the load circuit is of the character shown in Fig. 131*d*, and the machine is called an *alternator.*

In some generators of this type two independent coils, each having their own pair of slip rings, are mounted on the same shaft, the planes of the two coils being placed at an angle of 90° to one another. Obviously in such a case the E.M.F. generated in one coil will be a maximum when that induced in the other coil is zero. In other words the two E.M.F's. will differ in phase by 90°.

In other alternators three independent coils are arranged on a common shaft with their planes positioned 120° with respect to each other. A generator having one winding is known as a *single phase* alternator; one with two coils, a *two phase* machine, and a three coil unit, a *three phase* alternator. Generators are made which will develop as many as six independent E.M.F's. Machines which produce two or more E.M.F's. are called *polyphase* alternators.

The second arrangement whereby the external or load circuit may be connected to the terminals of the rotating coil is by means of a *split ring commutator* and suitable contact brushes, as diagrammed in Fig. 132*b*. When the brushes are so adjusted that the segments of the commutator make contact while the coil is cutting flux at the maximum rate the machine will deliver an E.M.F. to the load circuit *which will always be in the same direction,* as set forth in Fig. 133*a*. Such a machine is known as a *direct current* generator.

It will be seen that the terminal E.M.F. of such a unit is uni-directional but pulsating in character. In order to make possible the production of a uni-directional E.M.F. in which the "ripples" will not be so pronounced it is the practice to provide the armature with a number of windings, the terminals of each coil being connected to a pair of commutator bars (Fig. 133*c*). As each coil in turn is electrically connected to the brushes through its own pair of commutator bars it will impress upon the load circuit the E.M.F. being produced in that individual coil during the time

contact obtains. This results in an E.M.F. of the nature shown in Fig. 133*b*. The greater the number of coils in the armature winding the more nearly will the E.M.F. approach a constant value.

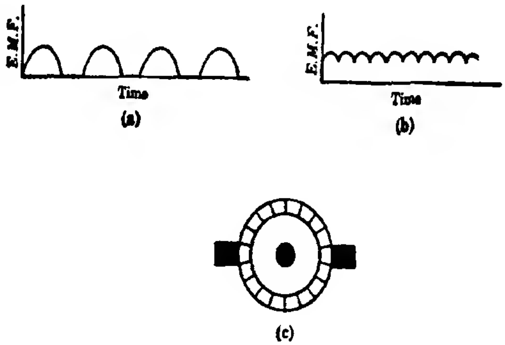


Fig. 133

In both alternating and direct current generators the magnetic field is supplied by powerful electromagnets. In the case of alternators the direct current for energizing or "exciting" the field must be supplied by some outside source such as a small direct current machine. In direct current generators the field current may also be drawn from an outside source, in which case the machine is said to be separately excited. However in most

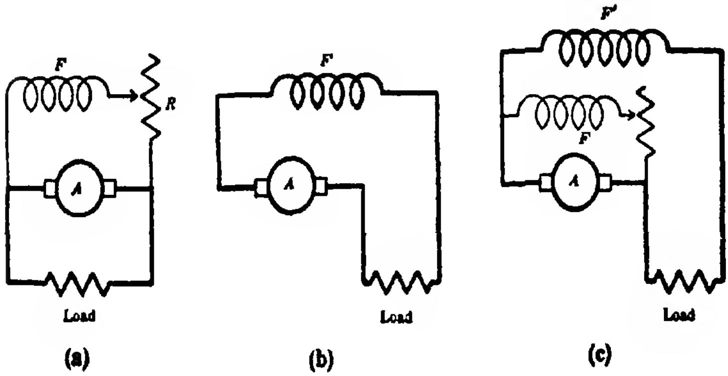


Fig. 134

cases the exciting current is supplied by the machine itself. This may be accomplished in one of three ways as sketched in Fig. 134. In the circuit arrangement shown in Fig. 134*a* the field winding F, consisting of many turns, is connected in parallel, or shunt,

across the brushes. Commonly there is enough residual magnetism in the field magnets to develop a small E.M.F. at the brushes when the machine is first started. This E.M.F. will produce a corresponding current in the field winding which in turn will build up a greater flux in the pole pieces. This building up process will continue until the normal E.M.F. of the machine is reached. Usually a variable resistance R is inserted in series with the field winding in order to be able to vary the value of the field current and thus control the terminal E.M.F. of the dynamo. A generator of this type is known as a *shunt* machine. Its load or *external characteristic* is shown as curve A in Fig. 135. The electrical load on a generator is increased by *decreasing* the load resistance. Since the external and field circuits are in parallel the current through the field winding will decrease as the load increases (Sec. 43), and hence the terminal E.M.F. will fall with increasing load.*

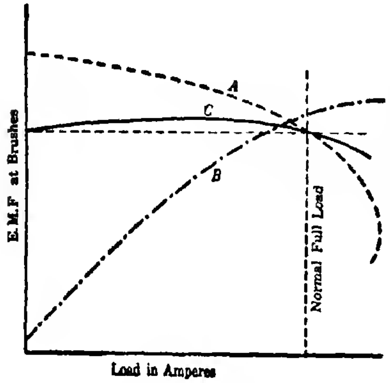


Fig. 135

A second circuit arrangement for exciting the field is indicated in Fig. 134*b*. In this case the field winding F is in series with the load circuit and the armature winding. All current delivered to the load will therefore pass through the field coils. The result is that the field flux will increase in value as the current drawn from the machine increases. The terminal E.M.F. will therefore tend to rise with increasing load, as shown in curve B, Fig. 135. Such a generator is referred to as a *series* machine, and is said to have a rising characteristic.

In some classes of service it is desirable to have a generator whose terminal E.M.F. does not change radically with load. By combining the characteristics of the two previously described machines in a single unit this end can be attained. The field circuits of such a dynamo are sketched in Fig. 134*c*. This

* There are other factors which operate to cause a decrease in terminal E.M.F. with increase in load. For a complete discussion of the performance of generators the reader should consult any standard text on the elements of electrical engineering.

machine is known as a *compound wound* generator. The field winding, as the name implies, consists of two parts, one composed of many turns in shunt with the brushes, and a second of a few turns in series with the armature and load. The external characteristic is shown as curve C in Fig. 135, and it will be noted that the E.M.F. does not fall appreciably unless the generator is overloaded.

105. Foucault or Eddy Currents.—Variations in magnetic flux give rise to induced currents not only in linear conductors, such as wires, but also in conductors having the form of plates or sheets. Before Faraday's discovery of magnetic induction, Gambey in 1824 noticed that the motion of a magnet suspended and set into oscillation was rapidly damped when a plate of copper was held just beneath the magnet. At about the same time Arago observed that when a pivoted magnetic needle is supported directly over a

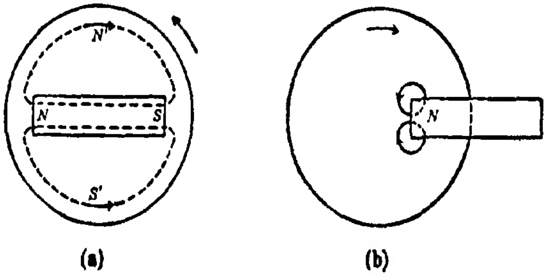


FIG. 136

rotating copper disk, the magnet is caused to rotate with the disk. Faraday explained these phenomena on the basis of electromagnetic induction. In the latter case the relative motion of the disk in the field of the magnet gives rise to E.M.F's. in the surface of the metal disk, which in turn set up currents which circulate in closed paths. The magnetic fields resulting from these induced currents are, by Lenz's law, of such direction as to oppose the motion which gave rise to them; hence the mechanical effect upon the suspended magnet. Referring to Fig. 136*a*, which represents Arago's experiment, it may be pointed out that the E.M.F's. induced directly beneath the north pole will cause a current to flow from the edge of the disk toward the axis; the current due to the south pole will be directed from axis to edge. The net result is a current which passes along a diameter of the disk from left to right in the figure, the path being completed

along the circumference of the disk, as shown by the dotted lines. The magnetic field resulting from these so-called *eddy currents* has a polarity as indicated by $N'S'$. The electromechanical reaction between these two sets of poles results in the magnet being rotated in the direction shown by the arrow.

If the magnet be fixed as in Fig. 136*b* the result will be an electromechanical force tending to oppose the force applied to rotate the disk. The direction of the eddy currents will be as indicated in the figure.

A number of important commercial applications of the magnetic drag effect have been made. For instance in one of the widely used types of automobile speedometers an annular magnet is caused to rotate, by means of a spiral driving shaft, in juxtaposition to an aluminum disk, the motion of the latter being controlled by means of a spiral spring. As the magnet turns, the eddy currents in the disk cause it to turn and thus indicate the speed. The calibration marks appear on the edge of the disk.

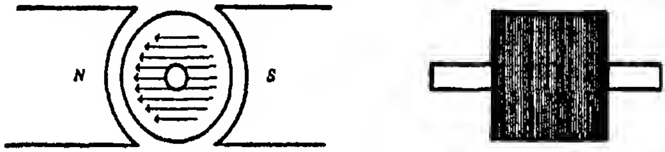
Another application of this principle is found in the control mechanism which forms a part of watt-hour meters (Sec. 52). Certain types of these meters consist of a set of fixed coils and a rotating coil, the latter acting, in effect, as the armature of a motor (Sec. 131). To control the speed of the rotating element in such meters a copper disk is fastened to the shaft of the rotor. One or more permanent magnets are fastened with the poles close to the surface of the disk, somewhat as in Fig. 136*b*. As the armature rotates the currents induced in the disk tend to oppose the motion of the rotor, thus acting as a stabilizing load. The speed of the rotor can be adjusted by altering the position of the magnets. This control disk is usually visible from the outside of such meters, and may be seen rotating whenever current is passing through the meter.

Still another application of magnetic damping is to be found in the aluminum coil form which supports the windings of D.C. ammeters and voltmeters. As the coil of the moving system rotates in response to the electromechanical torque eddy currents are set up in the metallic coil frame and the resulting magnetic field tends to dampen the motion of the coil and thus make the instrument "dead beat."

Perhaps the most important commercial application of eddy currents is to be found in the *induction motor.* By means of a

certain arrangement of polyphase alternating current circuits (Sec. 133) it is possible to produce a rotating magnetic field. If a rotor consisting of short circuited low resistance conductors is placed within the region of such a rotating field large eddy currents will be developed with the result that the conductors forming the rotor will experience a torque tending to produce rotation, thus converting electrical energy into mechanical energy.

The foregoing are illustrations of those conditions under which eddy currents serve a *useful* purpose. There are however circumstances under which such currents are very undesirable. It appears to have been first observed by Foucault that these eddy currents result in heat, and consequently give rise to loss of energy. When a metal disk is rapidly rotated between the poles of a strong electromagnet its temperature is decidedly raised.



(a) (b)

Fig. 137

If the armature core of a dynamo were made of solid iron as sketched in Fig. 137*a* and were rotating, E.M.F's. would be induced in the surface layers of the iron and the resulting currents would heat the core. Since, in this case, these eddy currents would serve no useful purpose their existence means a wasteful dissipation of energy. If however the armature core were to be built up of thin insulated laminations, as shown in Fig. 137*b*, the resistance offered by the metal as a whole to these eddy or Foucault currents would be materially reduced; indeed they may be practically suppressed by such a method of construction. The plane of the laminations must of course be at right angles to the direction of the Foucault currents. The thinner the laminations and the better they are insulated from one another the more effective are they in preventing these wasteful currents. In certain high frequency alternating current generators the laminations are extremely thin, being of the order of one mil in thickness.

In transformers (Sec. 123) the varying flux set up by the alternating current flowing in the windings tends to give rise to Foucault currents in the core, thus developing heat. By building up the core of thin insulated laminations placed as shown in the projection in Fig. 138, these undesirable currents are largely prevented, and the so-called core losses * materially reduced. The core laminations are usually insulated from one another by being varnished or enameled.

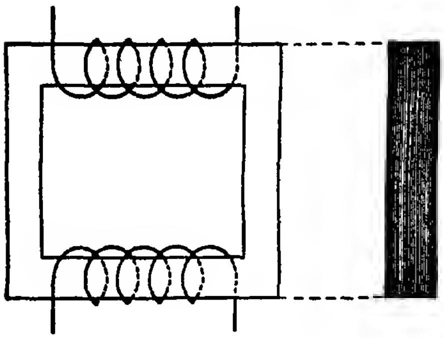


Fig. 138

In the construction of some transformers the error is made of fastening the laminations together by means of uninsulated bolts passing directly through the laminations. Such a plan of construction results in the short circuiting of the iron strip constituting the core and thus, in effect, causing the core to act as if it were of solid iron.

106. Self-Inductance.—It has already been shown (Sec. 101) that a changing magnetic flux will give rise to an induced E.M.F. in a neighboring circuit. It is equally true that if the conductor forming a solenoid is carrying a varying current, the resulting changing flux will *induce an E.M.F. in the circuit itself.* Each turn or loop of the solenoid reacts inductively upon its neighboring coils, just as if they were physically independent circuits. This phenomenon is known as *self-induction.* In conformity with Lenz's law, if the normal current in the solenoid be *increasing* in value, the *self-induced E.M.F.* will oppose the applied E.M.F. and hence *retard* the rise of the current in the circuit; similarly if the original current in the coil be *decreasing,* the self-induced E.M.F. will be *in the same direction* as the applied pressure and thus tend to *maintain* the current. Indeed we may define *self-induction or inductance as that property of a circuit by virtue of which it opposes a change in the existing current.* A circuit behaves as if the current in it possessed a form of inertia—a tendency to resist any change

* The core or "iron loss" in electrical equipment involving varying magnetic fields consists of two parts, viz., eddy current losses and losses due to hysteresis.

in value. In fact inductance is sometimes spoken of as "electromagnetic inertia." If, then, a circuit possess self-inductance the current will not rise immediately to its maximum value upon applying an E.M.F.; neither will the current instantly fall to zero upon opening the circuit. Figure 139 depicts the growth and decay of current in an inductive circuit. It will be noted that in the particular circuit examined 0.05 sec. was required for the current to reach a steady state and that there was a cor-

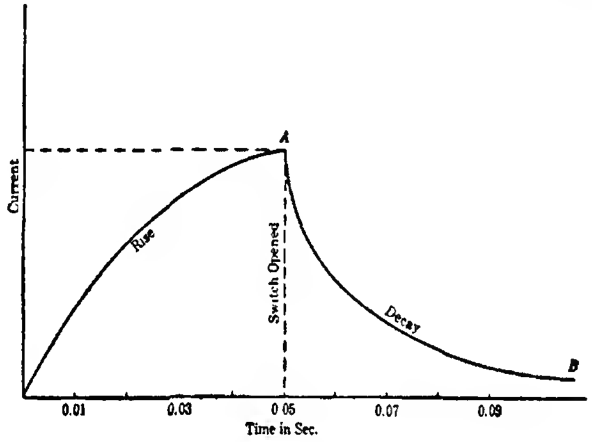


Fig. 139

responding lag in the decay of the current after the source of E.M.F. had been disconnected from the circuit. Since the time rate of change of flux is greater on "break" than on "make" the E.M.F. due to self-inductance is greater on opening the circuit than when the application of the E.M.F. is made. In fact the E.M.F. of self-inductance on "break" may have a value of several times the applied E.M.F., thus causing a flashover or arc at the switch. The energy stored in the magnetic field appears as the heat and noise of the flashover. We shall see that the property (self-inductance) of a circuit which gives rise to such an electrical behavior is of very great importance.

107. Coefficient of Self-Inductance.—The magnitude of the self-induced E.M.F. depends upon the time rate of change of the magnetic flux due to the variation of the original current in the coil. This in turn will depend upon the value of the flux, the total flux being governed by the current and the dimensions of the circuit. If the circuit has no iron the total flux will vary as

the current, or

$$\Phi \propto I.$$

We may write

$$\Phi = LI, \qquad \text{Eq. 126}$$

where L is a constant depending on the geometrical form of the circuit, that is, the shape, area and the number of turns. This proportionality constant becomes what is known as the *coefficient of self-inductance*, or the *inductance*, of a circuit. From the above relation it is evident that the inductance of a circuit is *numerically* equal to the total number of lines of magnetic flux included by the circuit and due to unit current in the circuit.

One may also express the inductance of a circuit in terms of the self-induced E.M.F. Referring to eq. 126, if we differentiate with respect to t we have

$$\frac{d\Phi}{dt} = L\frac{dI}{dt}.$$

Since, in general,

$$-\frac{d\Phi}{dt} = E,$$

it follows that

$$E = -L\frac{dI}{dt}, \qquad \text{Eq. 127}$$

where E is the E.M.F. due to self-inductance. In the last equation the negative sign indicates that when $\frac{dI}{dt}$ is positive the self-induced E.M.F. is opposed to the applied E.M.F.

From eq. 127 a second definition of the coefficient of self-inductance may be formulated. This would be to the effect that the self-inductance (inductance) of a circuit is *numerically* equal to the self-induced E.M.F. due to unit change of current per second in the circuit. Stated in another way, a circuit will have unit (c.g.s.) inductance when a current changing at the rate of one e.m.u. per second develops an opposing E.M.F. of one electromagnetic unit. The practical unit of inductance is the *henry*, which is equal to 10^9 e.m.c.g.s. units. In certain classes of work the henry proves to be an inconveniently large unit. In such cases the millihenry (one thousandth part of a henry) is used. The microhenry (millionth of a henry) is also sometimes employed. Our definition when put in terms of practical units would be stated as follows: *The inductance of a circuit has a value of one*

henry, if the E.M.F. developed is one volt, when the current in the circuit varies at the rate of one ampere per second.

It is highly important that the student secure a clear understanding of what is meant by the coefficient of self-inductance, and to always bear in mind that self-inductance *obtains only while the current is changing in value.* Further, it should be noted that the value of the self-inductance of a circuit containing iron (or other magnetic material) will be a function of the permeability.

108. Inductive and Non-Inductive Circuits.—Any circuit which forms a closed loop possesses appreciable inductance. If however a short conductor doubles back on itself without forming a closed loop it is practically non-inductive. Figure 140 illustrates this

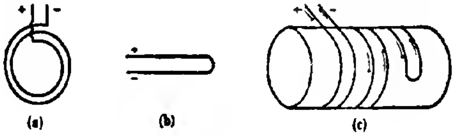


FIG. 140

point. In the arrangements represented by *b* and *c* the magnetic flux due to the current in one wire will be neutralized by the flux due to the return circuit and hence the circuit as a whole possesses practically no inductance. Standard resistance coils are wound in the manner shown by *c*. If however a single straight wire is of considerable length and of large size it may exhibit a certain amount of inductance.

109. Calculation of Inductance.—The calculation of the value of the coefficient of self-induction for any given circuit requires a knowledge of the magnetic flux distribution due to the current in the circuit. Except for a few cases the analytical treatment is more or less involved, and outside the scope of this text. We will however outline the treatment for two or three typical cases met with in practice.

Long Solenoid.—We have seen (Sec. 92, eq. 102) that the field strength within a long solenoid or helix is given by the expression

$$H = 4\pi nI,$$

where n is the number of turns per cm. length of coil. This might be written

$$H = \frac{4\pi NI}{l},$$

where N is the total number of turns, and l the length of the solenoid. Using the relations embodied in eqs. 110 and 111 the *total flux* within the helix would then be given by

$$\Phi = \frac{4\pi NIA}{l},$$

where A is the area of cross-section. Each turn in the solenoid incloses all of this flux; hence the *effective flux* which operates to produce self-induction effects will be given by

$$\Phi_{\text{eff.}} = \frac{4\pi\mu N^2IA}{l}.$$

By definition (Sec. 107) the coefficient of self-inductance is numerically equal to the flux due to unit current. Therefore, making I unity, we have the inductance of a solenoid given by

$$L = \frac{4\pi\mu N^2A}{l}. \qquad \text{Eq. 128}$$

In many practical cases the cross-section of the core is circular; hence

$$L = \frac{4\pi^2\mu r^2N^2}{l}, \qquad \text{Eq. 129}$$

where r is the radius, L being in c.g.s. units. To express the inductance in *henrys* our relation becomes

$$L = \frac{4\pi^2\mu r^2N^2}{10^9 l}. \qquad \text{Eq. 130}$$

In *millihenrys*, it would take the form

$$L = \frac{4\pi^2\mu r^2N^2}{10^6 l}. \qquad \text{Eq. 131}$$

In *microhenrys*, the equation becomes

$$L = \frac{4\pi^2\mu r^2N^2}{10^3 l}. \qquad \text{Eq. 132}$$

If the winding does not include magnetic material μ becomes unity and we have, for the case of an air core helix,

$$L = \frac{4\pi^2r^2N^2}{10^6 l} \qquad \text{Eq. 133}$$

in millihenrys.

An examination of the equations just deduced discloses the fact that the only unit entering into these relations is that of *length.* It therefore follows that *the e.m.c.g.s. unit of self-inductance is the centimeter.*

It should be borne in mind that the permeability μ is a function of the magnetizing force, and hence these equations cannot be applied to circuits containing a magnetic substance unless the mean value of μ is known or can be estimated.

Another consideration must also be noted, viz., that the foregoing relations were developed on the assumption that the magnetic field within the solenoid is uniform; hence eqs. 128 to 133 hold strictly true only for toroidal windings (Fig. 116). They are however applicable to those cases where the length of coil is great compared with the diameter, say not less than ten to one. Various correction factors have been worked out to take account of the end effects. Professor Nagaoka has developed a correction formula based on the ratio of the radius of the coil to its length and has prepared an accompanying table of constants to assist in making rapid and accurate calculations of the coefficient of self-inductance in those cases frequently met with in practice. Professor Nagaoka's table is to be found in a volume entitled *Calculation of Alternating Current Problems,* by Dr. Louis Cohen. These and other correction data are also incorporated in the *Circular of the Bureau of Standards No. 74.*

In dealing with any quantity such as self-inductance it is always well to know the magnitude in the case of certain familiar circuits. Such a knowledge serves to assist one in forming a rough estimate of the values which may be encountered in practice. For instance, the inductance of an ordinary door bell is something like 0.012 henry; a common telephone receiver (diaphragm in place) varies from 0.075 to 0.1 henry; the secondary of a 3/4-inch induction coil measures about 15 henrys; the coil of a sensitive galvanometer is from 1 to 2 henrys; a 50-turn coil (2-inch inside diameter) of the type frequently used in radio receiving sets will be about 0.15 millihenry.

EXAMPLE.—A coil having 300 turns is 30 cm. in length and 2.4 cm. in diameter. What is the coefficient of self-inductance in cm. and in millihenrys?

Substituting in eq. 133 we have

$$L = \frac{4\pi^2(1.2)^2(300)^2}{30} = 170{,}600 \text{ cm.}$$

$$= 0.17 \text{ mh.}$$

Two Long Parallel Wires.—In any given case the total magnetic flux at any point is the algebraic sum of the components due to the several contributing fields. In the case of two parallel wires ("lead" and "return") the total or resultant field will be the sum of the fields due to each wire. Referring to Fig. 141, let MM' and SS' represent two long parallel wires, as, for example,

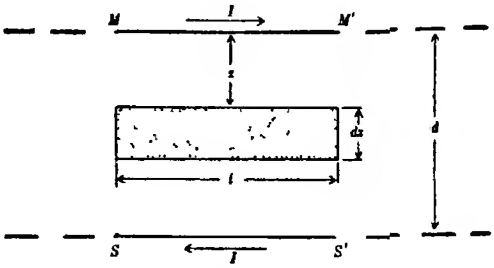


FIG. 141

the two leads constituting a telegraph or simple telephone pair. Let r be the radii of the wires, and d their distance apart between centers. Consider a length l of these wires and the magnetic field between them. In that part of the field indicated by the dotted area the total flux is equal to $Hldx$. The total field strength will be given (eq. 94, Sec. 89) by the sum of the fields due to each of the two current elements, or

$$\frac{2I}{x} + \frac{2I}{d - x} = H.$$

Recalling our definition of inductance (Sec. 107), we may make I unity, and thus the flux through the shaded area will be

$$\Phi = 2l\left(\frac{1}{x} + \frac{1}{d - x}\right)dx.$$

Considering the whole area between the wires

$$L = 2l\int_r^{d-r}\left(\frac{1}{x} + \frac{1}{d - x}\right)dx$$

$$= 2l\left(\log_e\frac{d - r}{r} - \log_e\frac{r}{d - r}\right)$$

$$= 4l\log_e\frac{d - r}{r}\cdot$$

Changing to the base 10, our equation becomes

$$L = 4l \log_{10} \frac{d - r}{r} 2.3026 \text{ e.m.u's. (cm.).} \qquad \text{Eq. 134}$$

In arriving at the above relation we have neglected the field in the wires themselves. If the wires are small compared with their distance apart the error thus introduced is negligible; in most practical cases this condition obtains.

EXAMPLE.—A two-wire circuit is 300 meters in length, the wires being spaced 30 cm. apart. If No. 12 B. & S. wire is used, what is the inductance of the line?

Wire of size No. 12 B. & S. gauge is approximately 0.2 cm. in diameter. Utilizing eq. 134 we have

$$L = 4 \times 30{,}000 \log_{10} \frac{29.9}{0.1} \times 2.3026$$

$$= 7 \times 10^5 \text{ cm. or } 0.0007 \text{ henry.}$$

Formulas giving the coefficient of self-inductance have been worked out for a number of special cases, particularly in connection with high frequency alternating current circuits. For a discussion of such cases the student is referred to the *Circular of the Bureau of Standards No. 74*, Revised Edition.

110. Coefficient of Mutual Inductance.—Two or more circuits are frequently electrically associated in such a manner that the magnetic flux produced by one winding threads through the winding of one or more of the associated circuits. There is therefore a mutual magnetic reaction giving rise to an induced E.M.F. in one of the coils whenever the current changes in value in the associated circuit. The winding into which energy is being fed is spoken of as the primary and the associated coil from which energy is being absorbed by some form of electrical load is referred to as the secondary. Such an arrangement of associated circuits has what is called a *coefficient of mutual inductance*, or, more simply, *mutual inductance*. Mutual inductance may be thought of as a property of associated circuits by virtue of which any change in the magnitude of a current in one of the circuits gives rise to an induced E.M.F. in the other circuit. The coefficient of mutual induction of two circuits is *numerically* equal to the total magnetic flux which obtains in one of the coils due to unit current in the other.

The flux in one of the coils due to a current in the other will be proportional to that current or

$$\Phi \propto I.$$

We may write $\Phi = MI$, where M is a constant known as the coefficient of mutual induction. We have seen (eq. 119) that

$$E = -\frac{d\Phi}{dt}.$$

Hence

$$E = -\frac{d(MI)}{dt}$$

$$= -M\frac{dI}{dt}. \qquad \text{Eq. 135}$$

In words this means that the coefficient of mutual inductance of two circuits is numerically equal to the E.M.F. developed in one circuit when the current in the other circuit changes at unit rate.

Because of the similarity of the factors involved in self and mutual inductance the same unit is used for both quantities, viz., the henry. On this basis the mutual inductance of two magnetically associated circuits will be one henry if an induced E.M.F. of one volt is developed in one of the circuits when the current changes at the rate of one ampere per second in the other circuit.

111. Coefficient of Mutual Inductance of Two Coaxial Solenoids.—A case frequently met with in practice is that of two coaxial windings one of which is receiving energy and the other of which is delivering energy to a load circuit. Suppose that we have two coaxial solenoids sketched in Fig. 142 and assume that all of the flux produced by the primary P is linked with the secondary winding S. Let N_1, A, l, and I represent the total number of turns, the cross sectional area, the length and the current in the primary windings, respectively. Then, by eqs. 110 and 111,

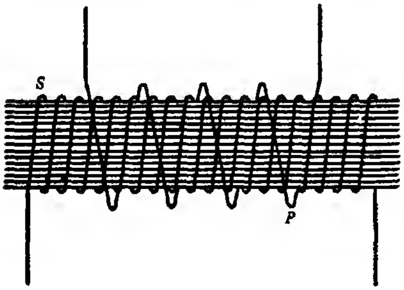


Fig. 142

$$\Phi = \frac{4\pi\mu N_1 A I}{l}.$$

Making the current unity, we have

$$\Phi = \frac{4\pi\mu N_1 A}{l}.$$

If N_2 represent the total number of turns in the secondary, the effective flux or "linkage" for unit current in the primary will be given by

$$\Phi_{\text{eff.}} = \frac{4\pi\mu N_1 N_2 A}{l}.$$

By definition this is numerically equal to the coefficient of mutual inductance, or

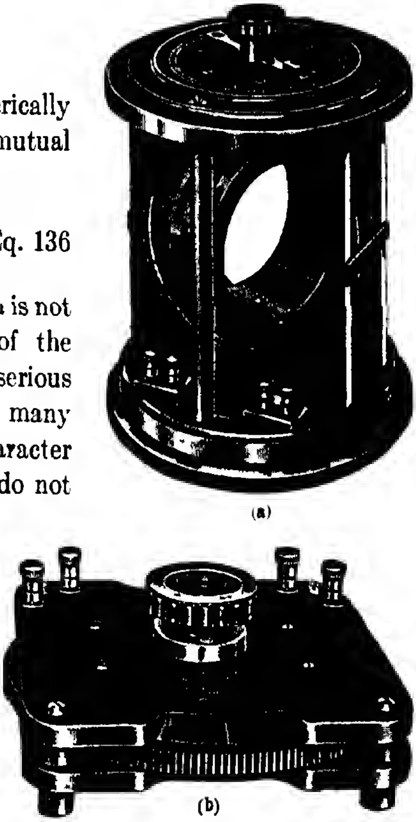
$$M = \frac{4\pi\mu N_1 N_2 A}{l}. \quad \text{Eq. 136}$$

If conditions are such that μ is not constant the application of the above relation presents serious difficulties. However in many cases circuits of the character assumed in the discussion do not contain a magnetic material as core; hence $\mu = 1$. In such cases the mutual inductance in centimeters would be given by

$$M = \frac{4\pi N_1 N_2 A}{l}. \quad \text{Eq. 137}$$

In henrys this becomes

$$M = \frac{4\pi N_1 N_2 A}{10^9 l}. \quad \text{Eq. 138}$$



(Courtesy Leeds and Northrup Co.)

FIG. 143.—(a) AYRTON AND PERRY STANDARD INDUCTOMETER; (b) BROOKS INDUCTOMETER

112. Measurement of Inductance.—Since inductance is an important quantity it becomes necessary to be able to accurately determine its value. This is most commonly accomplished by a method involving what is known as an inductance bridge. This consists of a network similar to the Wheatstone bridge whereby a comparison is carried out between the unknown inductance and a standard inductance.

Standards of inductance are made in various forms, some of which are of the variable type. A variable unit can be made from two coaxial coils joined in series, and arranged so that they may be rotated with respect to one another. The Ayrton and Perry standard shown in Fig. 143*a* is of this type.

Another form of standard inductance used in testing laboratories is known as the Brooks inductometer (Fig. 143*b*). This unit consists of two sets of flat coils, one of which is fixed, the other being movable in a fixed plane. There are also inductance standards having a fixed value.

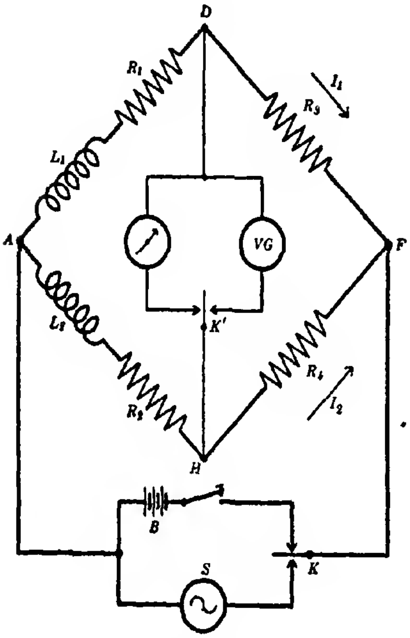


FIG. 144

Measurements by means of the inductance bridge involve, in addition to a standard inductance, a source of alternating current of constant frequency and also a means of detecting such a current. The two coils whose coefficients are to be compared form two arms of the bridge, as shown in Fig. 144. The method is based on the fact that, for alternating currents, the drop in potential due to the counter E.M.F. is proportional to the magnitude of the inductance over which the drop occurs.

In the figure let R_1, R_2, R_3, and R_4 be non-inductive resistances; L_1 a standard inductance, and L_2 the unknown inductance; S a source of alternating current; G a direct current galvanometer, and VG a vibration galvanometer or other form of A.C. detector. The bridge is first balanced for direct current in the usual manner (Sec. 45) by adjusting R_1 and R_2, utilizing the battery B and galvanometer G. Switch K is then thrown to connect the A.C. source, and K' closed to connect the A.C. detector. If the standard inductance (L_1) be of the variable type its value is varied until a balance (zero deflection or silence) is secured. If

the unknown inductance (L_2) is of a value which falls within the limits of L_1, R_3 and R_4 may be equal in value. If the unknown inductance is greater or less than the maximum or minimum values of the standard, R_3 and R_4 are adjusted to secure a balance. For this reason the resistances R_3 and R_4 are frequently referred to as the ratio coils. When a final balance obtains, the difference of potential at any instant between A and D will equal that between A and H; likewise the drop over DF will equal that over HF. Hence we may write

$$R_1I_1 + L_1\frac{dI_1}{dt} = R_2I_2 + L_2\frac{dI_2}{dt}.$$

The terms $L_1\frac{dI_1}{dt}$ and $L_2\frac{dI_2}{dt}$ represent the fall of potential due to the counter E.M.F. developed in the inductances. When a D.C. balance obtains,

$$R_3I_1 = R_4I_2.$$

Differentiating we get

$$R_3\frac{dI_1}{dt} = R_4\frac{dI_2}{dt}.$$

Combining to eliminate I_2 and $\frac{dI_2}{dt}$, there results

$$R_4R_1I_1 + R_4L_1\frac{dI_1}{dt} = R_2R_3I_1 + R_3L_2\frac{dI_1}{dt}.$$

Since the bridge was first balanced for direct current,

$$R_1R_4 = R_2R_3;$$

hence

$$L_1R_4 = L_2R_3,$$

or

$$\frac{L_1}{L_2} = \frac{R_3}{R_4}. \qquad \text{Eq. 139}$$

The vibration galvanometer referred to early in this discussion is an instrument which does not differ essentially from the galvanometer described in Sec. 94. In the vibration form of instrument the coil through which the current to be detected is passed is so designed and suspended that it has a natural frequency of mechanical vibration equal to the frequency of the alternating current being observed. Due to this fact it will respond to an

extremely minute alternating current and hence is well adapted for use as a detector in determining the nul point in bridge measurements involving alternating currents. A telephone receiver is sometimes employed to detect when the inductance bridge is in balance, but is not as satisfactory for this purpose as the vibration galvanometer.

113. Growth of Current in a Circuit Having Self-Inductance.—Reference has already been made (Sec. 106) to the fact that the counter E.M.F. of self-induction acts to retard the rise of current in an inductive circuit. It remains to derive an expression for the value of the current at any time t after an E.M.F. is applied to the circuit. Referring to Fig. 145, let E be the impressed E.M.F.; R the total resistance; i the value of the current at any time t after closing the circuit (instantaneous value). I_m represents the maximum (or steady) value of current, as given by E/R. In order to set up a current in the circuit LR an E.M.F. must be applied which shall be equal to the iR drop plus the E.M.F. of self-inductance. This may be expressed thus,

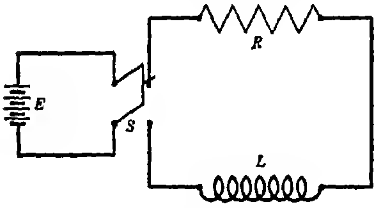


Fig. 145

$$E = iR + L\frac{di}{dt}\cdot$$

Integrating this equation we arrive at the solution

$$i = I_m(1 - \epsilon^{-(R/L)t}) = \frac{E_m}{R}(1 - \epsilon^{-(R/L)t}), \qquad \text{Eq. 140}$$

which indicates how the current increases in value with time. In the term $\epsilon^{-(R/L)t}$, ϵ is the base (2.7183) of the Naperian system. It is thus evident that the rise of current in a circuit containing inductance and resistance is logarithmic. It will be seen that when L is small the current reaches its steady value in a very short interval of time.

It has been pointed out (Sec. 109) that the c.g.s. unit of inductance is the unit of length. It will also be shown later (Sec. 138) that the units which enter into resistance are the same as the fundamental units in velocity, viz., length and time. The recip-

rocal of the ratio $\frac{R}{L}$ would therefore involve only the fundamental unit of time. Hence the ratio $\frac{1}{R/L}$ is spoken of as the *time constant* of a circuit and is commonly represented by the Greek letter τ. Equation 140 may then take the form

$$i = I_m(1 - \epsilon^{-(t/\tau)}). \qquad \text{Eq. 141}$$

When $t = \tau$, this reduces to

$$i = I_m - \frac{I_m}{2.7182} = 0.632 I_m. \qquad \text{Eq. 142}$$

This means that the current reaches 0.632 of its final value in an interval of time equal to the time constant τ. This factor, τ, is a measure of the growth of the current under the conditions specified. It thus becomes evident that *the rate of increase of current depends not alone upon the inductance L, but upon L and R conjointly.* The graph shown in Fig. 139 should be re-examined in the light of the foregoing discussion.

114. Decay of Current.—Employing the same designations as in the previous section, let us suppose that, after the current has reached its maximum or steady value, the switch is short circuited and battery removed. What will be the law of the decay of the current in such a case? We may represent the physical conditions by the equation

$$L\frac{di}{dt} + Ri = 0.$$

The solution * of this differential equation leads to the relation

$$i = I_m \epsilon^{-(R/L)t}. \qquad \text{Eq. 143}$$

Substituting the *time constant* for $\frac{1}{R/L}$ we have

$$i = I_m \epsilon^{-(t/\tau)}. \qquad \text{Eq. 144}$$

It is thus evident that the decay of the current in a circuit having self-inductance and resistance is also logarithmic, and the greater the value of τ, the more slowly does the current diminish in value.

* The student should carry out this operation, as well as the corresponding step in Sec. 113.

It is of interest to note in this connection that the growth and decay of the current are complementary. This is shown by the fact that if we add $I_m(1 - \epsilon^{-(t/\tau)})$ to $I_m\epsilon^{-(t/\tau)}$ there results I, the maximum or steady value of the current.

PROBLEMS

1. It is desired to wind a coil having an inductance of 0.2 mh. Circumstances are such that the length of the coil must not exceed 15 cm. and its diameter 2 cm. How many turns will be required?

2. A two-wire circuit is one mile in length. The wires are No. 10 B. & S. and are spaced one foot apart. What is the inductance of the circuit in henrys?

3. Two coaxial windings consist of 500 and 5000 turns respectively. They are wound on a square form whose dimensions are $5 \times 5 \times 15$ cm. What is the mutual inductance in millihenrys?

4. Plot the growth and decay of the current in a circuit having the following constants:

$$\begin{aligned} \text{Resistance} &= 5 \text{ ohms}, \\ \text{Inductance} &= 10 \text{ henrys}, \\ \text{Maximum current} &= 5 \text{ amperes}. \end{aligned}$$

What is the value of the current 0.5 sec. after the E.M.F. is applied; also 0.5 sec. after the E.M.F. is disconnected? What is the time constant of the circuit?

CHAPTER XVIII

ALTERNATING CURRENTS

115. Effective Value of Alternating Current and E.M.F.—We have examined (Sec. 104) the means by which an alternating E.M.F. may be produced and it was found that such an E.M.F. would, in general, be sinusoidal in character. The relation between the instantaneous and maximum values may be expressed thus:

$$e = E_m \sin(\omega t), \qquad \text{Eq. 145}$$

where e represents the instantaneous value of the E.M.F., E the maximum value, $\omega = (2\pi \times \text{frequency})$ and t the time. If such a sinusoidal E.M.F. is impressed upon a circuit containing only resistance the instantaneous current will be given by the relation

$$i = \frac{E_m}{R} \sin(\omega t) = I_m \sin(\omega t), \qquad \text{Eq. 146}$$

where i is the value of the current at any instant and I_m the maximum value.

The graph representing the relation between E.M.F. or current and time (Fig. 147) indicates what is known as the *wave form.* While the E.M.F. available from a commercial power circuit does not always have a strictly sine wave form yet it is so nearly sinusoidal in character that calculations based on that assumption will approximate very closely the actual conditions. In fact unless the sine wave is assumed the calculations become highly involved and beyond the scope of this volume. A photographic record of a commercial E.M.F. wave form is shown in Fig. 146.

Since an alternating E.M.F. or current is periodic in character, varying from a maximum in one direction to a corresponding maximum in the other direction, the question presents itself as to what a volt or an ampere means in this connection. Obviously the maximum value cannot be taken; hence it becomes necessary to find some expression for what might be called the *effective* value.

Since the heating effect of the current does not depend upon

the *direction* of the current we may take the thermal effect of the current as a basis for our calculation. With this in mind it may be said that the effective value of an alternating current is the value of the direct current which will develop the same thermal

FIG. 146

effect as the alternating current during one complete cycle of the latter.

It has been shown (Sec. 51, eq. 74) that the heating effect of a current passing through a pure resistance varies as the square of

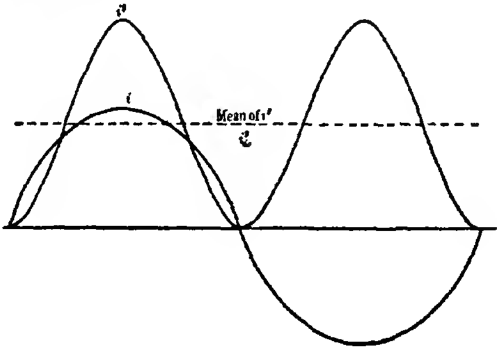


FIG. 147

the current. Therefore a curve constructed by using as ordinates the square of the instantaneous current values will represent the variations in the rate at which heat is developed by an alternating current. The mean ordinate of this heat curve also represents

the square of the direct current which would produce the same quantity of heat in the same period of time as would the original alternating current. This is graphically represented in Fig. 147. It follows therefore from our definition of effective value that the mean ordinate of the curve representing i^2 is equivalent to the square of the effective value of the alternating current. In other words,

$$I^2 = \text{mean value of } i^2, \tag{i}$$

where I is the effective value of the alternating current. Then

$$I = \sqrt{(\text{mean value of } i^2)} \tag{ii}$$

$$= \sqrt{[\text{mean value of } I_m^2 \sin^2(\omega t)]}. \tag{iii}$$

It now remains to evaluate the expression under the radical sign. This may be accomplished by the aid of a few simple trigonometric transformations.

$$I_m^2 \sin^2(\omega t) = \frac{I_m^2}{2} - \frac{I_m^2}{2}\cos 2(\omega t).^*$$

In general, if n is an integer the mean value of $\cos n\alpha$ over a complete cycle is zero. Therefore the above expression reduces to $\frac{I_m^2}{2}$, which from (i) is the square of the effective value of the current. Hence the effective value of an alternating current is given by the expression

$$I = \frac{I_m}{\sqrt{2}} = 0.707 I_m. \qquad \text{Eq. 147}$$

Since we have assumed that the current and the E.M.F. are both sinusoidal we may write a corresponding expression for the effective value of an alternating E.M.F., thus,

$$E = \frac{E_m}{\sqrt{2}} = 0.707 E_m. \qquad \text{Eq. 148}$$

Because of the method of determining the effective value of current and E.M.F. the term *square-root-of-mean-square*, or, more simply, *root-mean-square*, is sometimes used to designate what we have called effective value. Some writers use the term *virtual* as an equivalent of effective. In engineering practice root-mean-square (abbreviated r.m.s.) is the term most commonly employed.

All ordinary alternating current instruments are calibrated to

* The student should verify this statement.

read in effective (r.m.s.) values. When such instruments are first calibrated with direct current their readings when employed on alternating current circuits will give r.m.s. values, i.e., 0.707 of the maximum values. If for example an electrostatic voltmeter be connected to an alternating current circuit in which the E.M.F. is varying between + 100 and − 100 volts, the instrument will read 70.7 volts, and 70.7 volts constant E.M.F. would give the same reading.

It should be noted in passing that the formulas applying to alternating current and E.M.F. which we have already developed, or may deduce as we proceed, are valid without change for r.m.s. values.

116. Average Values and Form Factor.—The average value of an alternating E.M.F. or current for any number of complete cycles is zero, because there are as many negative values as positive ones. The mean value for a half cycle however is a quantity which is not zero, and which is useful in certain alternating current calculations. To express this in terms of the maximum value of the current we have but to find the area under the sine curve and divide it by the length of the base line expressed as an angle. In terms of the calculus this would be

$$I_{\text{av.}} = \frac{I_m}{\pi}\int_0^{\pi} \sin(\omega t)d(\omega t) = 0.636 I_m. \qquad \text{Eq. 149}$$

The ratio of the effective to average values is known as the *form factor*, because of the fact that it serves as an indication of the wave form. This ratio is

$$\frac{0.707}{0.636} = 1.11. \qquad \text{Eq. 150}$$

117. Relation between Current and E.M.F. (Alternating).—If a circuit has any appreciable self-inductance or capacitance the relation of the current to the applied E.M.F. cannot be expressed by the simple relation indicated in Sec. 115 for a non-inductive circuit. We shall proceed to deduce an expression for the current in terms of the E.M.F. and the several constants of the circuit. We shall first consider the following case.

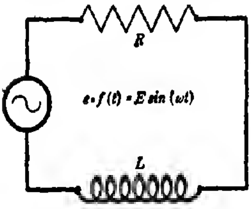


Fig. 148

Case I. *Inductance and Resistance.* Let the applied voltage be a sine function of the time as expressed by $e = E_m \sin \omega t$, where $\omega = 2\pi f$, f being the frequency (Fig. 148). In order to establish a current i in such a circuit the applied E.M.F. must be equal to the ohmic drop (Ri) + the counter E.M.F. of self-inductance $\left(L\frac{di}{dt}\right)$. This may be written in the form

$$E \sin \omega t = Ri + L\frac{di}{dt} \cdot \qquad \text{Eq. 151}$$

There is a linear differential equation of the first order, and the procedure to be followed in arriving at a solution is outlined in Murray's *Differential Equations.* The solution is

$$i = \frac{E}{\sqrt{(R^2 + \omega^2 L^2)}} \cdot \sin (\omega t - \phi) + A_1{}^{-\epsilon(Rt/L)} + \cdots.$$

After a brief interval of time the exponential terms (transients) * become negligible. The remaining part of the equation shows that the current is a sine function of the time, and that *it lags behind the impressed E.M.F. by an angle ϕ.*

The greatest value that $\sin (\omega t - \phi)$ can have is unity; hence the maximum value of the current will be given by

$$I = \frac{E}{\sqrt{(R^2 + \omega^2 L^2)}} \cdot \qquad \text{Eq. 152}$$

It has previously been shown that the virtual or r.m.s. values of both E.M.F. and current bear a definite ratio to the maximum values; hence this equation holds for virtual or r.m.s. values.

The denominator of the right-hand member of eq. 152 is known as the *impedance* of the circuit, the first term being ordinary resistance and the second the square of two factors, angular velocity and inductance. This term ωL is known as *reactance.* Reactance is commonly expressed by X, in this case X_L. To rewrite, we have

$$I = \frac{E}{\sqrt{(R^2 + X_L{}^2)}} \cdot \qquad \text{Eq. 153}$$

* Immediately after an alternating E.M.F. is applied to a circuit there are, in general, in addition to the principal current one or more other currents of small magnitude which exist for only a brief interval of time. These temporary currents are called "transients."

Both reactance and impedance are measured in ohms. It should also be noted that the reactance, and hence the impedance, is a function of the frequency. If the resistance is small compared with the reactance, as is frequently the case in practice, the above relation becomes

$$I = \frac{E}{X_L} = \frac{E}{\omega L} = \frac{E}{2\pi f L}. \qquad \text{Eq. 154}$$

In eqs. 153 and 154 the impedance is sometimes represented by Z, thus,

$$Z = \sqrt{(R^2 + X_L^2)}. \qquad \text{Eq. 155}$$

Both this simple form of the equation and the more complete relation serve as highly useful mathematical tools. For instance, if one measures the drop over a given inductive winding for known current and frequency values the magnitude of the inductance may be computed. Knowing the inductance, the current and the frequency, one may compute the "reactance drop" or fall in potential due to a given inductance. It will be evident that the magnitude of the current in an A.C. current may, if desired, be controlled by means of a variable inductance rather than by a variable resistance, thus saving the energy that would ordinarily be dissipated as heat if the latter method were employed. An inductance which is utilized for current control in this manner is known as a *choke coil.*

EXAMPLE.—Suppose we have a circuit whose resistance is 10 ohms and inductance 25 henrys. What will be the magnitude of the current if the applied voltage is 2000 at 60 cycles? If the frequency were 1000 cycles?

$$Z = \sqrt{(10^2 + 4\pi^2 \times 60^2 \times 25^2)}$$
$$= 9425 \text{ ohms.}$$
$$I = \frac{2000}{9425}$$
$$= 0.212 \text{ amp.}$$

The student should solve the second part of the problem.

It is important to note the phase relations which obtain between the current and the E.M.F. in a circuit of this character. If we assume that the current has the same wave form as the E.M.F., then

$$i = I \sin \omega t. \qquad \text{(i)}$$

We know that the E.M.F. of self-induction is given by

$$E_L = -L\frac{di}{dt}. \qquad \text{(ii)}$$

Differentiating eq. (i) we have

$$\frac{di}{dt} = I\omega \cos \omega t. \qquad \text{(iii)}$$

Combining eqs. (ii) and (iii) we get

$$E_L = -IL\omega \cos \omega t. \qquad \text{(iv)}$$

From (i), when $t = 0$, $i = 0$. Under the same conditions from (iv)

$$E_L = L\omega I \text{ (a maximum).}$$

Further, from the same two relations, when

$$t = \frac{T}{4}, \quad i = I \text{ (a maximum),}$$

and

$$E_L = 0.$$

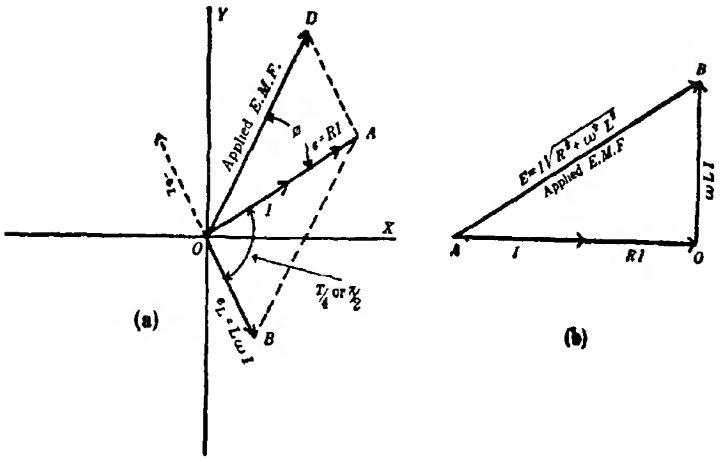


FIG. 149

It is therefore evident that the current and E.M.F. of self-inductance differ by one quarter period, E_L passing through its maximum value one quarter period later than i; E_L is therefore said to lag i by 90°.

It will frequently be found useful to represent the above relation graphically by means of a "clock" or vector diagram, as

shown in Fig. 149*a*. The student should study this diagram very carefully. Remembering that $AB = DO$, and redrawing the vector triangle as in Fig. 149*b*, we may write

$$\overline{AB} = \sqrt{(\overline{AO}^2 + \overline{OB}^2)},$$
$$E = \sqrt{(R^2I^2 + L^2\omega^2I^2)},$$
$$I = \frac{E}{\sqrt{(R^2 + \omega^2L^2)}},$$

a relation previously deduced as eq. 152.

The phase difference between the applied E.M.F. and the resulting current, which is an important factor in A.C. practice, may be expressed in terms of the constants of the circuit. From the foregoing vector relation it will be evident that the magnitude of this phase difference, expressed as an angle ϕ, will be given by the relation

$$\tan\phi = \frac{\omega LI}{RI} = \frac{\omega L}{R} = \frac{2\pi fL}{R}. \qquad \text{Eq. 156}$$

Physically, of course, the phase difference is *an interval of time* and not an angle, the relation $\phi = \omega t$ serving to connect the time and angle concept. It is to be noted that the angle of lag (ϕ) varies directly as the self-inductance in the circuit and also directly as the frequency. This may be well illustrated by one or two practical problems.

EXAMPLE.—Suppose we have a circuit in which there is an inductive winding whose inductance is 0.002 henry and whose resistance is 10 ohms. If the frequency of the current flowing in the circuit be 60 cycles, the angle of lag will be given by

$$\tan\phi = \frac{2\pi \times 60 \times .002}{10}$$
$$= .0758,$$
$$\phi = 4^\circ\ 20'.$$

To express this as a time interval we have

$$t = \frac{\phi \text{ (in radians)}}{\omega}$$
$$= \frac{.0756}{2\pi \times 60}$$
$$= 0.0002 \text{ sec.}$$

What would be the angular phase difference if the inductance were doubled? If the frequency were 25?

CASE II. *Capacitance and Resistance.* In this case the applied E.M.F. must equal the ohmic drop plus the P.D. developed across the condenser as it is charged. Referring to Fig. 150 and following the same general plan of analysis as in Case I, we may write

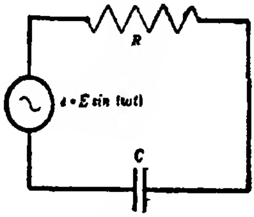


FIG. 150

$$e = f(t) = E \sin \omega t = Ri + \frac{Q}{C}.$$

But for a charging condenser

$$Q = \int i dt;$$

hence we may write

$$e = f(t) = E \sin \omega t = Ri + \frac{1}{C}\int i dt.$$

This equation is also of the first order, and gives as a solution *

$$i = \frac{E}{\sqrt{\left(R^2 + \frac{1}{\omega^2 C^2}\right)}} \sin (\omega t + \phi) + B_1 \epsilon^{-(t/RC)} + \cdots .$$

As in the previous case, the exponential term may be dropped, and we see that the current is again a sine function of the time. It is however not in phase with the applied E.M.F., but *leads* by the angle ϕ. The maximum value will be given by

$$I = \frac{E}{\sqrt{\left(R^2 + \frac{1}{\omega^2 C^2}\right)}}. \qquad \text{Eq. 157}$$

As in the case of the inductive circuit, the denominator of the above relation is known as impedance, and the term $\frac{1}{\omega C}$ is designated as *capacitive* (or negative) reactance, written

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}. \qquad \text{Eq. 158}$$

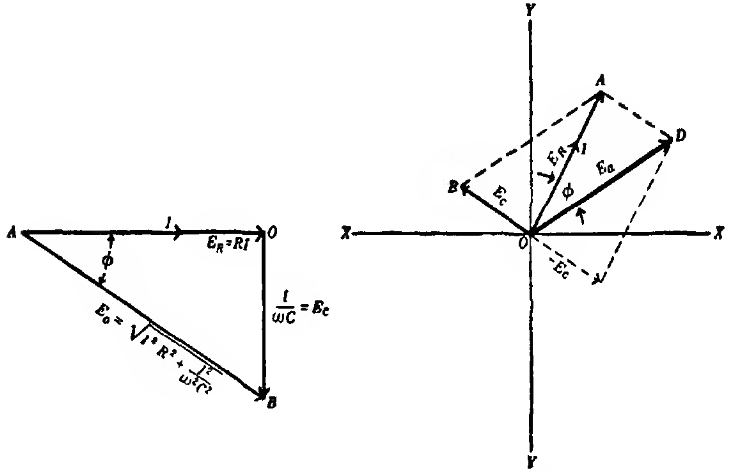
The graphic representation of the phase relations in this case is shown in Figs. 151*a* and *b*.

* Murray's *Differential Equations.*

The phase angle between the E.M.F. and current will be given by

$$\tan \phi = \frac{I}{\omega C} \Big/ RI = \frac{1}{\omega CR} \cdot \qquad \text{Eq. 159}$$

It should be noted that capacitive reactance varies *inversely* as the frequency and also inversely as the capacitance of the condenser. Therefore, when the resistance is negligible the current increases directly as the frequency. To illustrate this aspect of the case let us consider the following instance.



(a) (b)

FIG. 151

EXAMPLE.—Suppose that we have a condenser of 0.01 mf. capacitance. Assume $E = 100$ and $f = 800$ (mean voice frequency).* Neglecting the resistance, eq. 157 becomes

$$I = E\omega C = E2\pi fC.$$

Then

$$I = 100 \times 2\pi \times 800 \times 10^{-8}$$
$$= 0.005 \text{ ampere.}$$

If the frequency were 10^6 (common in radio practice) our solution becomes

$$I = 100 \times 2\pi \times 10^6 \times 10^{-8}$$
$$= 6.28 \text{ amperes.}$$

It is thus evident that a condenser whose capacity is of the order indicated above would pass practically no voice current but

* The frequency of the alternating current generated by the voice in a telephone circuit has a mean value of about 800 cycles.

would readily admit currents of the frequency employed in high frequency communication processes.

In charging a long transmission line the current may reach a high value as shown by the following case.

Example.—Assume a line having a capacitance of 2 mf., which is operated at 100,000 volts and 60 cycles. The charging current would be

$$
\begin{aligned}
I &= 2\pi \times 60 \times 2 \times 10^{-6} \times 10^{5} \\
&= 7.5 \text{ amperes.}
\end{aligned}
$$

At the voltage mentioned, this represents a substantial amount of energy.

Another aspect of capacitive reactance should also be noted. From the relation

$$I = \frac{E}{\dfrac{1}{\omega C}},$$

we may derive

$$E = \frac{I}{\omega C} = \frac{I}{2\pi f C},$$

which shows that for a given current the E.M.F. across a condenser varies inversely as the frequency and also inversely as the capacitance. Thus for a given current the E.M.F. across a condenser will be less the higher the frequency.

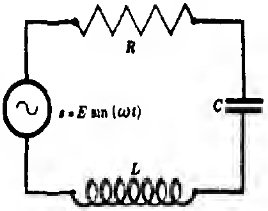


Fig. 152

Case III. *Resistance, Inductance, and Capacitance.* In this case (Fig. 152) we may express the conditions which obtain by the relation

$$E = f(t) = E_R + E_L + E_C.$$

This results in the equation

$$E \sin \omega t = Ri + L\frac{di}{dt} + \frac{q}{C}.$$

But since

$$i = \frac{dq}{dt},$$

$$\frac{di}{dt} = \frac{d^2q}{dt^2}.$$

Hence

$$E \sin \omega t = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{c}.$$

This is a differential equation of standard form * and yields as a solution (neglecting transient terms)

$$i = \frac{E}{\sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}} \sin(\omega t - \phi).$$

When $\sin(\omega t - \phi) = 1$, the current will have maximum value, and be given by the expression

$$I = \frac{E}{\sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}}. \qquad \text{Eq. 160}$$

This is the most important relation in the theory of alternating currents. It is sometimes written in either of the two following forms:

$$I = \frac{E}{\sqrt{[R^2 + (X_L - X_C)^2]}} = \frac{E}{Z}. \qquad \text{Eq. 161}$$

As in Cases I and II the denominator of this expression gives the impedance, Z, the resultant or net reactance being $\left(\omega L - \frac{1}{\omega C}\right)$.

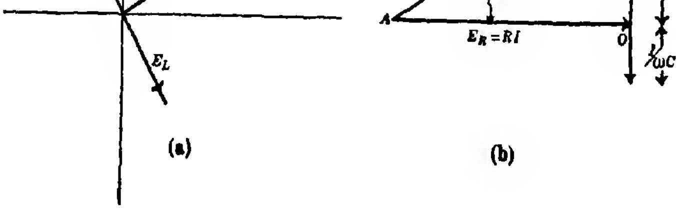


Fig. 153

Both X_L and X_C are expressed in ohms. The angle ϕ will be either positive or negative depending upon whether inductance or capacitance predominates. It will thus be seen that a circuit containing *both* inductance and capacitance may under some

* See Murray's *Differential Equations*.

circumstances act, in effect, as an inductance or as a capacitance, depending upon circumstances.

The vector diagrams corresponding to this case are shown in Fig. 153*a* and *b*. As Fig. 153*b* is drawn the inductive reactance exceeds the capacitive reactance by BO and hence the current is shown lagging the applied E.M.F. If

$$\omega LI = \frac{I}{\omega C} \quad \text{or} \quad \left(\omega L = \frac{1}{\omega C}\right),$$

the current and applied E.M.F. would be in phase and AB would coincide with AO, and under these conditions the circuit would be operating at its natural frequency (Sec. 118). In the event that the capacitive reactance were to exceed the inductive reactance, AB would fall below AO, the current leading the E.M.F.

EXAMPLE.—Find the current in a circuit having the following constants:

$$\begin{aligned} R &= 10 \text{ ohms}, \\ C &= 4 \text{ mf.}, \\ L &= 5 \text{ henrys}, \\ E &= 100 \text{ volts}, \\ f &= 60 \text{ cycles}. \end{aligned}$$

Substitution in eq. 160 gives

$$\begin{aligned} I &= \frac{100}{\sqrt{\left[10^2 + \left(2\pi 60 \times 5 - \dfrac{1}{2\pi 60 \times 4 \times 10^{-6}}\right)^2\right]}} \\ &= \frac{100}{\sqrt{[100 + (1885 - 648)^2]}} \\ &= .08 \text{ amp.} \end{aligned}$$

If the condenser in this case were shorted and a constant E.M.F. applied to the circuit the current would be 10 amperes.

118. Resonance.—An examination of the reactance term $\left(\omega L - \dfrac{1}{\omega C}\right)$ discloses the fact that for a given frequency it is possible for the inductive reactance to equal the capacitive reactance. When this condition obtains we have

$$\omega L = \frac{1}{\omega C}, \qquad \text{Eq. 162}$$

and the value of the current in the circuit for a given E.M.F. will depend only upon the ohmic resistance. In such a case the

inductance and capacitance nullify one another, so far as their effect upon the current is concerned, and the current is in phase with the E.M.F.

It should however be noted in this connection that the two factors, inductive and capacitive reactance, are affected differently by frequency. If we examine the last equation when put into the form

$$2\pi fL = \frac{1}{2\pi fC},$$

it will be noted that low frequencies make the first term small compared with the second, while a high frequency causes the first term to be very large compared to the last. There is then an intermediate and definite value of frequency at which the two terms will be equal, and this quite regardless of the particular values of L and C. The particular frequency at which ωL equals $\frac{1}{\omega C}$ may be determined by solving this equality for f, thus,

$$2\pi fL = \frac{1}{2\pi fC},$$

$$f^2 = \frac{1}{4\pi^2 LC},$$

$$f = \frac{1}{2\pi\sqrt{(LC)}}. \qquad \text{Eq. 163}$$

When the frequency, then, has the value given by this equation ωL will equal $\frac{1}{\omega C}$ and we have what is known as a state or condition of electrical *resonance*. In the above equation L is in henrys, C in farads and f in cycles per second. In practice it is more common to express C in microfarads, and hence our relation becomes

$$f = \frac{1000}{2\pi\sqrt{(LC)}}. \qquad \text{Eq. 164}$$

In certain high frequency measurements it is convenient to express L in millihenrys. The equation then becomes

$$f = \frac{5034}{\sqrt{(LC)}}, \qquad \text{Eq. 165}$$

C being in microfarads and L in millihenrys.

If a periodic E.M.F. having the frequency given by the foregoing equation be applied to any circuit the magnitude of the current will be determined by R alone. In such a case, if we plot current against frequency there results a graph similar to that shown in Fig. 154 where f' is the resonant frequency and I' the current at resonance. In securing the data for the above curve the capacity and inductance were held constant, the frequency of the applied E.M.F., only, being varied. The effect of resistance is to reduce the value of the current at resonance, and also to make the resonance curve less peaked. This is graphically shown by the curves in Fig. 155.

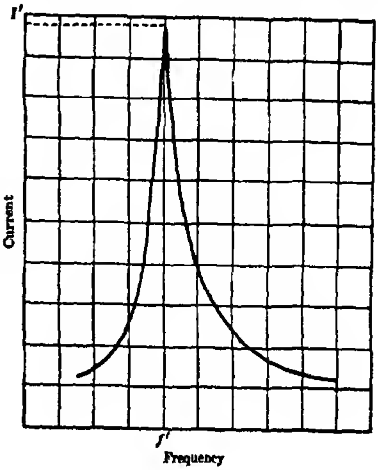


FIG. 154

In the study and application of alternating currents of high frequency the sharpness of the peak of the resonance curve for a given circuit is an important factor. This leads to the use of an expression known as "*sharpness of resonance.*" It may be shown that sharpness of resonance is given by the ratio $\frac{\omega L}{R}$. It is thus evident that not only the resistance but the ratio of the inductance to the resistance is a factor in determining the slope of the resonance curve. Thus, in a circuit containing capacitance, inductance and resistance, the resonance curve will be comparatively "flat" if the inductance is relatively small and the capacitance high. This is apparent from the curves set forth in Fig. 156. In setting up the two curves shown in the figure the resistance of the circuit was held constant, the frequency being varied.

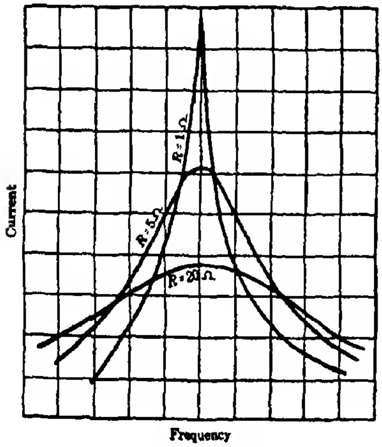


FIG. 155

In dealing with the question of resonance it is important to note that the E.M.F. developed across both the condenser and the inductance coil may be many times the value of the impressed E.M.F. This may be well illustrated by a practical case.

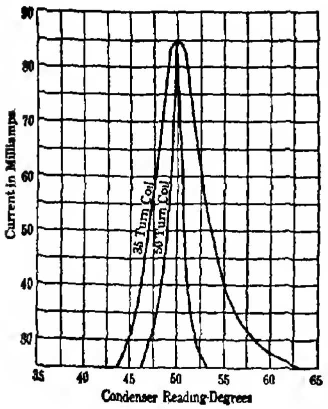


FIG. 156

EXAMPLE.—In a certain circuit operated at a frequency of 890,000 cycles the resistance is 3 ohms, the capacitance 0.0003 mf., and the inductance 0.017 mh. The current at resonance is 6 amperes. The value of the impressed E.M.F. may be found from the relation $E = RI_r$, where I_r is the current when resonance obtains. Substituting known values we have

$$E = 3 \times 6 = 18 \text{ volts.}$$

The P.D. across the inductance is given by ωLI_r, or

$$E_L = 2\pi \times 890{,}000 \times 0.000107 \times 6 = 3577 \text{ volts.}$$

The P.D. developed across the condenser $= \dfrac{I}{2\pi fC}$ and is, of course, equal numerically to the P.D. across the inductance. It is thus seen that the pressure developed across both the condenser and the inductance is nearly 200 times the value of the impressed E.M.F. Obviously the condenser, if operated under resonance conditions, must be so constructed as to withstand this electrical pressure without being punctured.

119. Reactance Curves.—In the practical application of the principle of electrical resonance many problems arise in the solution of which certain graphs known as *reactance curves* are found useful.

In the case of a circuit such as that shown in Fig. 152 the reactance is given by the expression $\left(\omega L - \dfrac{I}{\omega C}\right)$. At high frequencies, the inductive reactance, ωL, predominates, while at comparatively low frequencies the capacitive reactance, $\dfrac{I}{\omega C}$, predominates. In Fig. 157, these two parts of the total reactance are plotted for a given case, the inductive reactance being positive and the capacitive negative. The algebraic sum of these two

factors is indicated by the line marked *total* reactance. An examination of the curve shows that at the frequency ω' the inductive reactance equals the capacitive reactance with the result that the *total reactance* is zero. In other words a condition of resonance obtains, and the current in the circuit will be a maximum.

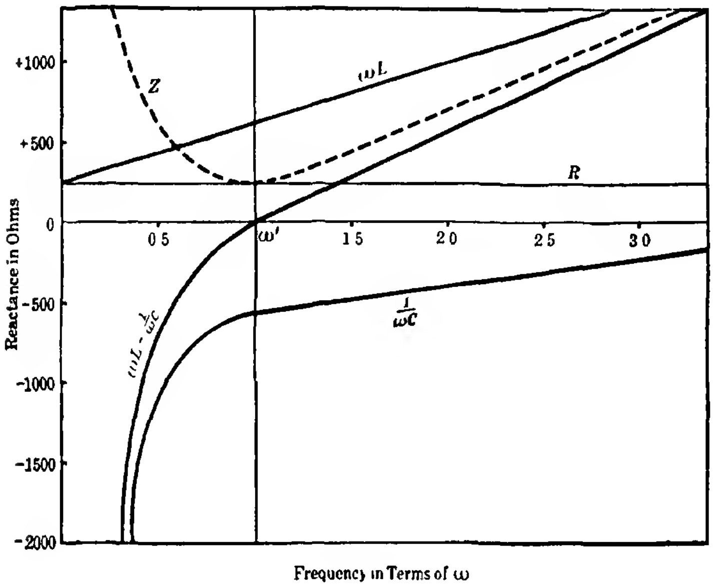


FIG. 157

While the total reactance is zero at the resonant frequency, *the impedance is not zero,* and hence the value of the current will be determined by the remaining part of Z which is R. The values for the impedance Z are shown by the dotted curve.

120. Parallel Resonance.—Thus far in our discussion of resonance we have considered only those cases in which the inductance, capacitance and resistance were in series with the applied E.M.F. Such an arrangement is known as a *series circuit* and resonance in such cases is sometimes referred to as series resonance. There is however a case of equal importance in which

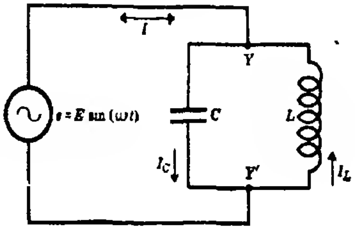


FIG. 158

the inductance and capacitance are in parallel with the applied E.M.F. Such an arrangement is shown in Fig. 158. An organization of this character behaves very differently from the series arrangement previously discussed. In the parallel circuit the alternating E.M.F. is applied at YY' and any current I supplied by the source will divide between the capacitive and inductive branches in inverse proportion to their respective impedances, and the total current will be the vector sum of the currents through L and C. This may be expressed by

$$I = I_C + (-I_L).$$

From this it follows that

$$I = \frac{E}{\sqrt{(R^2 + \omega^2 L^2)}} - \frac{E}{\sqrt{\left(R^2 + \frac{1}{\omega^2 C^2}\right)}}.$$

For our present purposes we may neglect the resistance R and our equation becomes

$$I = \frac{E}{\omega L} - \omega CE$$

$$= \frac{E}{2\pi f L} - 2\pi f CE.$$

If the frequency of the applied E.M.F. be varied (increased for example) the capacitive reactance, $2\pi fC$, will tend to increase while the inductive reactance, $\frac{1}{2\pi fL}$, will decrease. For a certain critical (resonant) frequency these two reactances will be equal. Hence

$$2\pi fC = \frac{1}{2\pi fL},$$

which makes

$$I = 0.$$

It is therefore evident that at resonant frequency the net current between Y and Y' approaches zero as a limit. Thus we see that in the parallel circuit case we have *minimum* current when resonance obtains while in the series case we have *maximum* current. It is however important to bear in mind that, though the sum of the currents between Y and Y' is zero (or very small),

large currents may and do exist in both the inductance and the condenser branches.

It is frequently useful to represent the foregoing facts regarding parallel resonance graphically. To do this we may conveniently make use of two quantities, one of which is known as *admittance* and the other as *susceptance.* Admittance (Y) is defined as the reciprocal of impedance $\left(\frac{1}{Z}\right)$, and corresponds in alternating current work to conductance $\left(\frac{1}{R}\right)$ in direct current practice. As has been shown, impedance is made up of resistance and reactance. The reactive part of admittance is what is designated as susceptance.

Referring to Fig. 158, the total current I is the sum of the currents in the inductance L and the condenser C. Neglecting resistance, this may be expressed by the equation

$$I = \frac{E}{\omega L} - \omega CE = E\left(\frac{1}{\omega L} - \omega C\right).$$

Now $Z = \frac{E}{I}$, and hence

$$\frac{1}{Z} = \frac{I}{E} = \text{Admittance} = Y. \qquad \text{Eq. 166}$$

The reactive part of the admittance (susceptance) is made up of two factors, viz., the inductive susceptance, $\frac{1}{\omega L}$, and the capacitive susceptance, ωC, the former predominating at low frequencies and the latter at high frequencies. The student should observe that each of the susceptances is the reciprocal of the corresponding reactance, the resistance factor of the admittance being negligible.

If we plot both the inductive and capacitive susceptance for the parallel circuit being studied we have the curves shown in Fig. 159. By combining the ordinate values for $\frac{1}{\omega L}$ and ωC we may set up a curve which will represent the total susceptance, and it is this quantity which is significant in our present consideration. Bearing in mind the physical significance of susceptance, it is seen that the curve representing total susceptance passes through zero at a frequency ω_0 which is the resonance frequency for the

circuit being studied. This means that the current will be at a minimum when resonance obtains, and would be zero if the circuit had zero resistance.

Further, if we set up a reactance curve by taking reciprocals of values on the curve of total susceptance, we find that the curve will have two branches, as shown, each of which goes to infinity. It is thus evident that the reactance at resonance frequency is infinite, and that the impedance would also be infinite if the resistance in both branches of the circuit were zero. Thus we arrive at the same conclusions as previously deduced.

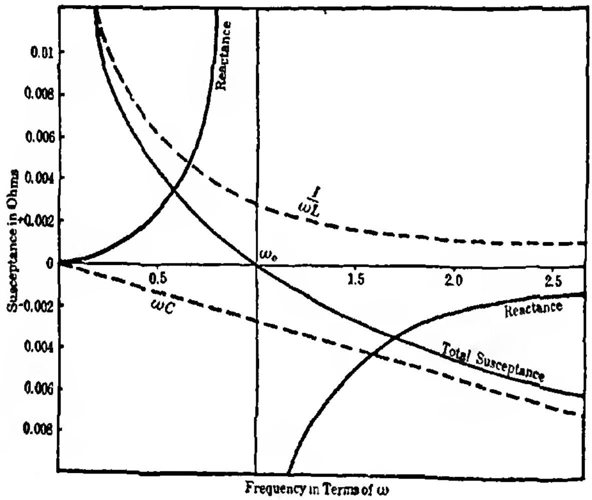


FIG. 159

121. Applications of Series and Parallel Resonant Circuits.— The principle of electrical resonance finds wide application in practice, particularly in the field of communication engineering. By utilizing the principles of resonance energy in the form of high frequency alternating currents is conveniently and efficiently transferred from one circuit to another, or if occasion demands, such currents may be completely suppressed. The suppression of a current of a certain frequency involves the design and use of so-called *filter* or *rejector* circuits. A specific case in point is an organization which will make it possible for a radio receiving set to completely "tune out" the signals from some particular transmitting station while listening to transmission from another given plant.

An antenna system incorporating a rejector circuit is an excellent illustration of both series and parallel resonance effects in one organization. While the component circuits which go to make up a complete modern radio receiving set are made quite selective by means of a group of associated resonant circuits they may not completely shut out all of the energy in those frequencies which are close to the desired frequency, particularly if the source of the interfering frequency is emitting a relatively large amount of energy. The interference may however be eliminated by the use of a rejector circuit made up of a parallel arrangement as sketched in Fig. 160*a*. In the diagram L and C represent the inductance and capacitance, respectively, of the rejector circuit. C' and L' are the corresponding elements of the receiving system proper, the capacitance in both cases being variable. The equivalent circuit is shown in Fig. 160*b*, C_A representing the capacitance of the antenna system with respect to the earth, and E the source of alternating E.M.F. The rejector circuit (parallel type) is first brought into resonance ("tuned") with the interfering frequency by adjusting C, thus making its reactance for that frequency infinite, and the current zero.* When this adjustment has been effected the antenna circuit (series type) as a whole is tuned to the desired frequency by means of C', thus making the current at the desired frequency a maximum. Many different circuits of this general character are possible, but the one cited is typical.

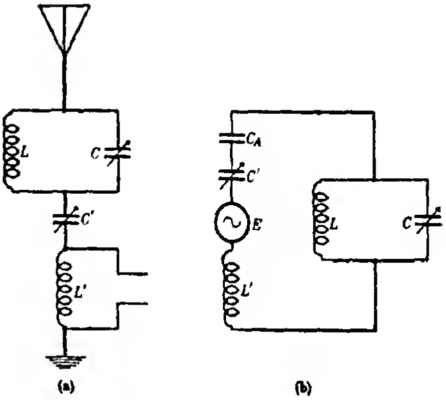


FIG. 160

In the designing of such circuits advantage is taken of the facts disclosed by reactance diagrams, similar to those discussed in the last section. A reactance diagram corresponding to the case just cited is shown in Fig. 161.

* The current will not actually be zero, but will approach zero as a limit, its minimum value depending upon the resistance in the two branches of the rejector circuit.

The reactance of C and L in parallel is shown as X''. Since this combination is in series with C' (including C_a), we must add the capacitive reactance of C', and this gives the reactance curve X. Thus we see that the reactance to the desired frequency ω' is zero while it is infinite to the interfering frequency ω_0. Hence there will be a maximum current at the desired frequency and a minimum at the interfering frequency, which is the end sought.

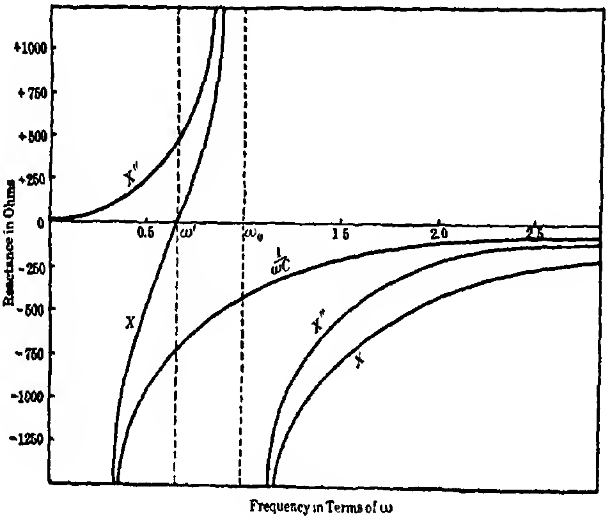


FIG. 161

Another field in which networks consisting of capacitance and inductance units find wide application is that of telephone and telegraph engineering. Both in the transmission of speech and telegraphic signals it becomes important to be able to suppress or enhance currents of certain frequencies. Filters are also widely used for the purpose of "smoothing out" the current obtained from rectifying devices (Sec. 143). Electrical filters utilized for such purposes fall in general into three groups, being known as high pass, low pass, and band filters. A high pass filter will allow currents *above* a certain frequency to pass without attenuation, while a low pass filter permits the existence of currents whose frequency is *below* a certain value. The band filter passes currents whose frequencies lie *between* certain limits. Figure 162 shows the general arrangement of the elements in the three typical forms of filters above mentioned. The actual values of the capacitance and inductance employed depend in a given case upon

the frequency limits for which the filter is designed. Under certain conditions the constants of the elements which go to make up the successive sections of a filter may not be the same for all sections. The effect of such circuits on current values, and

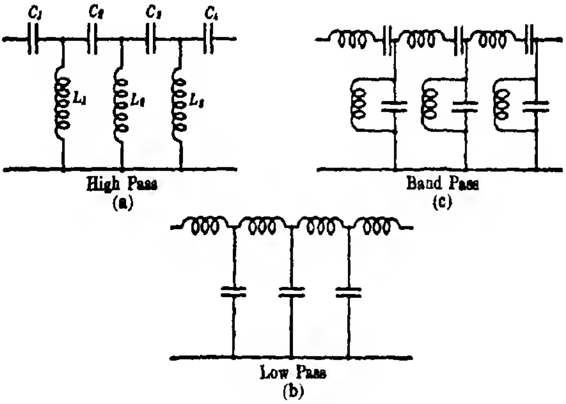


FIG. 162

hence on the energy transfer, is strikingly shown by the curves sketched in Fig. 163. Such graphs are known as attenuation curves. Attenuation is expressed in various ways, but for our purposes it may be defined as the ratio of the power input to the power output, and the above curves should be examined with this relation in mind.

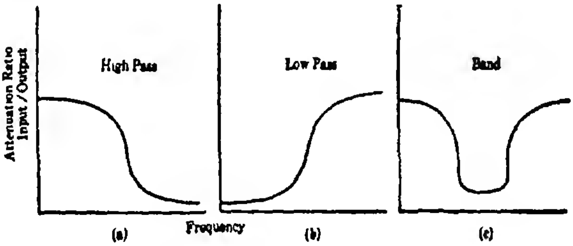


FIG. 163

For a comprehensive treatment of the subject of electrical filters the student is referred to *Electrical Oscillations and Electric Waves*, Chapter XVI, by G. W. Pierce; U. S. Patent No. 1,227,113, 1917, G. A. Campbell; "Transmission Characteristics of Electric Wave Filters," O. J. Zobel, *Bell System Technical Journal*, October 1924. A paper on Wave Filters in the same *Journal* for January 1925 gives a good bibliography of this subject.

122. Power and the Power Factor.—In the case of constant current circuits the value of the power (time rate of energy dissipation) is given by the product of the current and the E.M.F. In the case of alternating currents, however, that simple relation does not account for all of the factors involved in the situation. In the latter case both the E.M.F. and the current vary harmonically and they may also differ in phase. Thus if the E.M.F. be represented by

$$e = E_m \sin(\omega t),$$

the current would, in general, be given by

$$i = I_m \sin(\omega t - \phi),$$

ϕ being the phase difference between the E.M.F. and the current. The instantaneous rate of doing work will be given by

$$P = ei = E_m \sin(\omega t) \times I_m \sin(\omega t - \phi),$$

which reduces to

$$P = \tfrac{1}{2} E_m I_m [\cos\phi - \cos(2\omega t - \phi)].$$

During a complete cycle $\cos(2\omega t - \phi)$ will have all values between $+1$ and -1, and hence its mean value will be zero. The above relation will accordingly reduce to

$$P = \tfrac{1}{2} E_m I_m \cos\phi.$$

Numerically, however,

$$\frac{E_m I_m}{2} = \frac{E_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}.$$

But we have seen (eqs. 147 and 148, Sec. 115) that $\frac{E_m}{\sqrt{2}}$ and $\frac{I_m}{\sqrt{2}}$ represent effective or r.m.s. values of E.M.F. and current respectively; hence we have

$$P = EI\cos\phi \qquad \text{Eq. 167}$$

as an expression giving the power (P) or time rate of energy dissipation in terms of the r.m.s. values of the alternating E.M.F., current, and the angle of phase difference. The term $\cos\phi$ is known as the *power factor*. If it were found, for instance, that the current in a given circuit lagged the E.M.F. by an angle of 37° the power factor would be approximately 0.8, the cosine of

37° being 0.799. In passing it may be said that the angle of lag or lead, and hence the power factor, is largely governed in any given case by the nature of the electrical equipment which serves to convert the electrical energy into mechanical energy. The value of the power given by eq. 167 is referred to as the *actual* or *true* power, in contra-distinction to the *apparent* power, which is given by the simple product of the E.M.F. and current. The apparent power may be defined as the rate of doing work when the power factor is unity or, in other words, when the current and E.M.F. are in phase. The power factor is sometimes defined as the ratio of the actual or true power to the apparent power, thus,

$$\text{Power factor} = \frac{\text{true power}}{\text{apparent power}}.$$

True power is expressed in watts or kilowatts, while apparent power is expressed in terms of the units which go to make up the quantity, viz., "volt-amperes" or, more commonly, "kilovolt-amperes." In practice this is frequently abbreviated kv-a.

The significance of the power factor can be well illustrated by a consideration of the following typical case. Suppose we have a load requiring 10 hp. and assume that this load is supplied from electrical mains at 220 volts and that conditions are such that the power factor is 0.9. Under these circumstances the current supplied by the service mains would be given by

$$I = \frac{7460}{220 \times 0.9} = 37 \text{ amperes.}$$

Suppose that operating conditions so changed that the power factor dropped to 0.6. Computation will show that the current would then be 56.5 amperes. It is thus evident that the service mains and other associated circuits would carry a much larger current for the same power transfer when the power factor is low. Since the energy lost in the wiring due to heating varies as the square of the current it will be apparent that the losses will be decidedly greater at the lower power factor. To prevent this and also to obviate other undesirable effects due to a low power factor power companies take steps to keep the power factor of their service as near unity as possible.

The study of a diagram showing the relation of current and

E.M.F. to power will assist the student in arriving at an understanding of the important relations discussed above.

In Fig. 164 the power curve has been plotted by determining points such as Y_3 by taking the product of the ordinates YY_1 and YY_2. The areas inclosed by the power curve and the time ordinate represent the energy delivered to and received from the circuit during a complete cycle. The areas above the zero line (marked positive) represent the energy delivered to the line and those below the line (negative) the energy return by the line to the generator. The total power developed during any cycle is equal to the algebraic sum of the positive and negative areas.

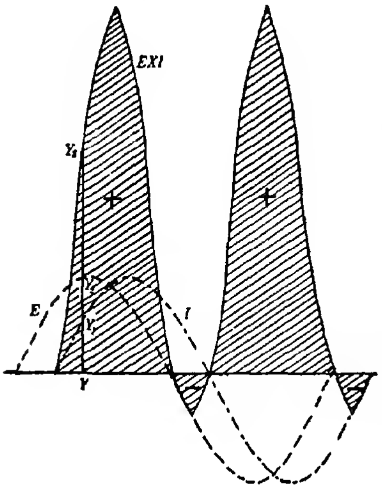


Fig. 164

Theoretically it is possible to have a condition under which the phase angle would be 90°, as for instance in the case of a circuit containing inductance but no resistance. In such a case the power factor would be zero and the current would be in quadrature with the E.M.F., as shown in Fig. 165.

The diagram discloses the fact that the net power is zero, since the sum of the positive areas, in any given interval of time, will

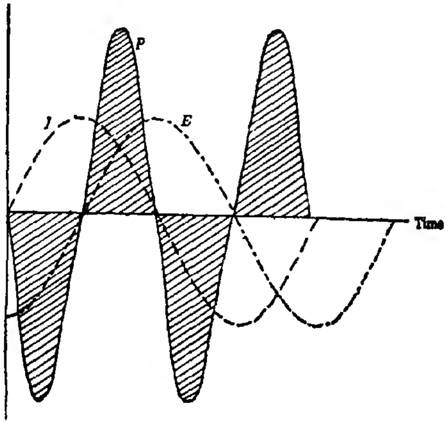


Fig. 165

equal the negative areas. There is however current flowing in the circuit under such a condition, but this current does not accomplish any work. It is therefore known as the "wattless current."

Owing to the fact that all circuits have some resistance the extreme condition referred to above is never met with in practice. However, there is a "wattless component" which obtains in all cases where the phase angle is not zero or unity.

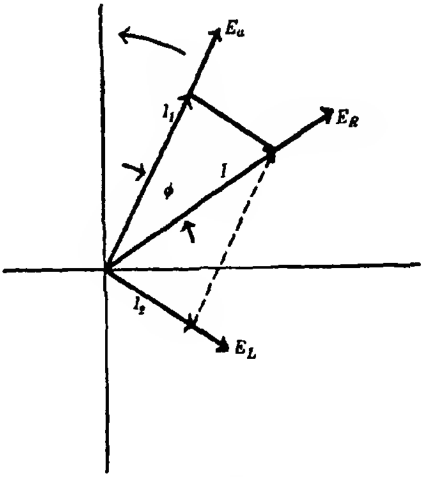


FIG. 166

If one considers the case of a circuit in which a lagging current obtains, as shown in Fig. 166, the current I may be resolved into two components, one of which, I_1, will be in phase with the applied E.M.F., and a second, I_2, in quadrature with I_1 and the applied E.M.F. Now $I_2 = I \sin \phi$ and $I_1 = I \cos \phi$. The component $I \cos \phi$ is the useful or *watt-component* while the component $I \sin \phi$ is the so-called *wattless* or *idle component.* This latter component represents that part of the current which, in an inductive circuit, gives rise to the magnetic field.

CHAPTER XIX

THE TRANSFORMER

123. Fundamental Theory of the Static Transformer.—Perhaps the most widely used device in the field of applied electricity is the static transformer. The term static signifies that there are no moving parts, potential transformations being effected by changes in magnetic flux within fixed windings grouped about a common iron core. The possibility of the commercial distribution of electric power over extended areas is largely due to this agency. In communication engineering, particularly in physical telephone and radio practice, the transformer also finds wide application.

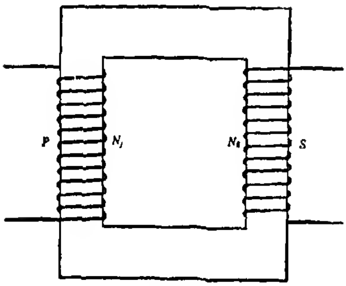


FIG. 167

In its simplest form a transformer consists of a closed magnetic core upon which are wound two coils (Fig. 167), one of which usually having a greater number of turns than the other. That winding *to which* energy is supplied is known as the *primary* and the coil *from which* energy is taken as the *secondary*. It is important that this distinction be clearly kept in mind, as the transformer may be utilized to increase ("step up") or to decrease ("step down") the E.M.F. It may be, and often is, used simply as a coupling agent between two parts of an electrical organization, in which case no pressure transformation occurs, the two windings having an equal number of turns.

We will first examine the relation which obtains between the E.M.F's. in the two windings. We will assume that sufficient current is flowing in the primary, as the result of an applied E.M.F., to completely magnetize the iron core, and that the secondary winding is open. As in dealing with a simple inductive circuit the applied E.M.F. must equal the sum of the ohmic drop (in this case due to the magnetizing current) and the counter

E.M.F. of self-induction. This may be expressed thus,

$$e_1 = Ri_1 + L_1\frac{di_1}{dt}, \tag{i}$$

where the subscripts indicate the instantaneous E.M.F., current and self-inductance in the primary respectively.

It has been previously shown (Sec. 107) that the E.M.F. of self-inductance is equal to the rate of change of the flux. Hence we may write

$$L_1\frac{di_1}{dt} = \frac{d\Phi_1}{dt}, \tag{ii}$$

where Φ_1 = the total flux existing in the iron due to the current in the primary. We are interested in the relation of the number of turns to the existing flux. Hence the relation

$$\Phi_1 = N_1\Phi \tag{iii}$$

will prove useful, where Φ indicates the flux due to each turn and N_1 the turns in the primary winding, the term $N_1\Phi$ being what is called the "flux turns" of the primary. Substituting eqs. (ii) and (iii) in eq. (i), we have

$$e_1 = R_1i_1 + N_1\frac{d\Phi}{dt}. \tag{iv}$$

Since in a transformer that part of the total applied E.M.F. which is required to supply the Ri drop is very small compared with that necessary to balance the reactance drop (in practice about one per cent), we may neglect the Ri term, and, for our purposes, consider that

$$e_1 = N_1\frac{d\Phi}{dt}. \tag{v}$$

In general,

$$\text{E.M.F.} = \frac{d\Phi'}{dt} = e',$$

or

$$d\Phi' = e'dt;$$

hence

$$\Phi' = \int e'dt, \tag{vi}$$

the primes signifying values in general. From eq. (iii)

$$\Phi = \frac{\Phi_1}{N_1}. \tag{vii}$$

Combining eqs. (vi) and (vii), we have

$$\Phi = \frac{1}{N_1}\int e_1 dt. \qquad \text{(viii)}$$

To evaluate the expression $\int e_1 dt$ we will assume that the applied E.M.F. is a sine function, thus,

$$e_1 = E_1 \sin \omega t.$$

Substituting this value for e_1 in eq. (viii),

$$\Phi = \frac{E_1}{N_1}\int \sin \omega t \cdot dt,$$

which yields

$$\Phi = \frac{-E_1}{N_1\omega}\cos \omega t. \qquad \text{(ix)}$$

We thus have an expression for the flux per turn in terms of the maximum applied E.M.F. and the number of turns in the primary.

Passing now to a consideration of what takes place in the secondary, it may be noted that the flux turns of the secondary may be written

$$N_2\Phi = \Phi_2,$$

and hence

$$\frac{d\Phi_2}{dt} = N_2\frac{d\Phi}{dt}. \qquad \text{(x)}$$

But

$$e_2 = \frac{d\Phi_2}{dt},$$

which from eq. (x) becomes

$$e_2 = N_2\frac{d\Phi}{dt}. \qquad \text{(xi)}$$

We may eliminate $\frac{d\Phi}{dt}$ by differentiating eq. (ix) and substituting in eq. (xi). This yields

$$e_2 = \frac{N_2}{N_1}E_1 \sin \omega t. \qquad \text{(xii)}$$

But

$$E_1 \sin \omega t = e_1.$$

Hence eq. (xii) becomes

$$e_2 = \frac{N_2}{N_1} e_1,$$

or

$$\frac{e_1}{e_2} = \frac{N_1}{N_2}. \qquad \text{Eq. 168}$$

It is thus evident that *the ratio of the applied and developed E.M.F's. is equal to the ratio between the turns in the primary and secondary.*

Thus far we have not considered the effect of the current which may exist in the secondary if the latter be shorted through a load impedance. When current flows in the secondary the magnetic effect of this current, by Lenz's law, will oppose the flux in the core due to the magnetizing current in the primary. This means that the back E.M.F. due to self-inductance of the primary is lessened, with the result that a greater current will tend to flow in the primary winding. A transformer thus becomes largely self-regulating, the flux not decreasing more than one per cent between no-load and full-load. This is but another aspect of the law of the conservation of energy. It will thus be evident that if the load impedance of the secondary is made zero (short circuit) not only the secondary but the primary may burn out.

The magnitude of the E.M.F. developed in the secondary of a transformer, in terms of the flux, frequency and number of turns may be easily found. The periodic flux in the core due to the magnetizing current in the primary threads through the secondary winding and induces therein an E.M.F. whose value is given by

$$e_2 = -N_2 \frac{d\Phi}{dt}.$$

Assuming that the flux follows a sine wave, having a maximum value Φ, we may write

$$e_2 = -N_2 \frac{d}{dt}(\Phi \sin \omega t)$$

$$= -\omega N_2 \Phi \cos \omega t,$$

which may be written

$$e_2 = 2\pi f N_2 \Phi \sin (\omega t - 90).$$

Thus it is seen that the induced E.M.F. is also a sine wave and lags the flux by 90°. The maximum value will be

$$E_2 = 2\pi f N_2 \Phi,$$

and the effective value

$$E_2 = \frac{2}{\sqrt{2}} \pi f N_2 \Phi = 4.44 f N_2 \Phi,$$

where E is in e.m.u's. In practical units

$$E = 4.44 f N_2 \Phi 10^{-8} \text{ volts,} \qquad \text{Eq. 169}$$

which reduces to

$$E = 4.44 f N_2 A B 10^{-8} \text{ volts,} \qquad \text{Eq. 170}$$

where A is the cross-sectional area of the core and B the flux density.

Modern power transformers have an efficiency of the order of 95 per cent, so that, approximately,

$$P_1 = P_2,$$

and, except for the slight copper and iron losses, one might write

$$e_1 i_1 = e_2 i_2,$$

or

$$\frac{e_1}{e_2} = \frac{i_2}{i_1}. \qquad \text{Eq. 171}$$

Upon the relations given as eqs. 168 and 171 the wide utility of the transformer in electrical power distribution rests. Electrical energy may thus be transmitted at high pressure and low current values and "stepped down" for the consumer's use. In power work this procedure results in an enormous saving in the cost of line conductors, though the cost of insulation is considerably increased. The net gain is, however, large. This economy results from the fact that thermal line losses vary as the square of the current.

124. Types of Transformers.—Transformers are of three general designs (Fig. 168) which are commonly designated as the *core type*, the *shell type*, and the *open core type*. In the former (a) the core consists of a single continuous magnetic path (rectangular or circular), the primary and secondary being disposed about different sections of the core. In certain forms of this type both

the primary and secondary are divided into two sections, a primary and a secondary component being placed on separate sides of a rectangular core. In other forms both the primary and secondary are concentrically wound on the same "leg," or all of the primary may be about one part of the core and the entire secondary about another section of the iron.

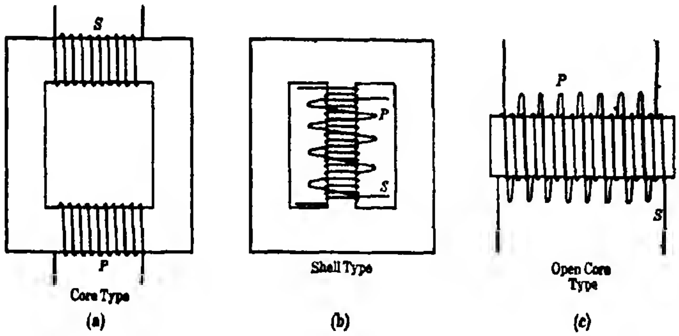


Fig. 168

Transformers of the shell type (*b*) differ from the core type in that the windings are disposed about a central section of the iron and the magnetic circuit is completed through two or four paths. In some cases the primary and secondary are concentric while in others both the primary and secondary are arranged in alternate sections, or "pied," as it is sometimes called.

The open core transformer (*c*) is the most simple of all the types, consisting of a straight core upon which the primary and secondary are concentrically disposed. This type is usually made only in small sizes, but is extensively used in telephone equipment.

The particular type of transformer which is employed in a given case depends largely upon the special use to which it is to be put.

125. Constant Current Transformer.—Thus far in our discussion we have dealt only with transformers designed to give a constant potential. There is however a class of service, particularly in illuminating engineering practice, in which it is desirable to have available a transformer unit which will produce a constant current and a variable potential. Such a transformer is shown diagrammatically in Fig. 169. It will be noted from the drawing that the secondary is movable and so suspended that it is nearly balanced by a counter-weight. When the transformer is in operation the currents in the primary and secondary at any

given instant are in opposite direction. Hence the magnetic field will cause repulsion between the coils, and the secondary will be forced upward and away from the primary. This gives rise to a greater magnetic leakage between the two coils with a

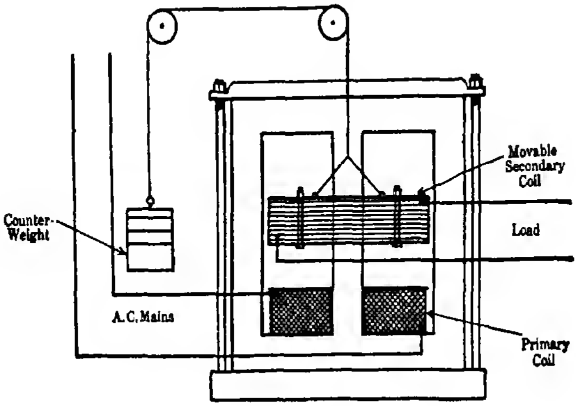


FIG. 169

resulting decrease in the voltage developed in the secondary. This tends to decrease the current in the secondary and the connected load circuit, thus lessening the magnetic repulsion. The falling of the secondary will then immediately readjust the voltage conditions so that a constant current is automatically maintained. The constant current transformer is used in supplying energy to series arc and series tungsten street lamps (Sec. 55 and Sec. 56).

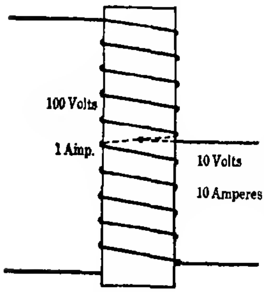


FIG. 170

126. Auto-Transformer.—Another type of transformer which has certain special but important uses is known as an auto-transformer, because of the fact that the primary and secondary constitute a single continuous winding. A schematic drawing of such an organization is shown in Fig. 170. Such a transformer may be used either for increasing or decreasing the voltage. The currents in the primary and secondary circuits being of opposite phase, that part of the winding which is common to both circuits carries a current whose value is the *difference* between primary and secondary currents. It is thus evident that in this type of transformer the whole of the

energy does not undergo transformation. For instance, in the case of a step-down unit of this type the transformer serves to supply the required increase in current by subtracting from the applied pressure. As a result of this relation the auto-transformer is more efficient than a unit of the standard type, and the gain in efficiency becomes more evident as the ratio of transformation approaches unity. Such transformers are employed in connection with small rectifying outfits (Sec. 143), in balancing units in certain power distributing systems, and to a certain extent in high frequency circuits.

127. Instrument Transformers.—When measuring potential differences in connection with high potential alternating current circuits it is the practice to connect the voltmeter to the line through a *potential* or instrument transformer as shown in Fig. 171*a*. It is customary to step down the potential to 110 volts. The output of such a transformer is only sufficient to operate one or more voltmeters.

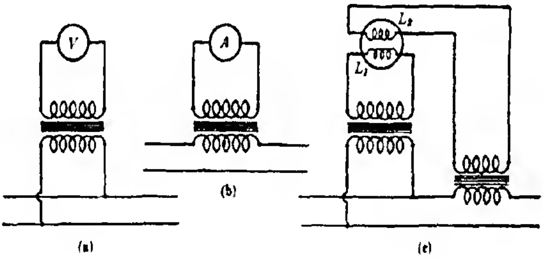


Fig. 171

In measuring current in high tension alternating current circuits, it is impracticable to employ shunts; hence recourse is had to what is known as a *current* (or series) *transformer*. The manner of connecting such a device is shown in Fig. 171*b*. In dealing with current values of the order of 1000 amperes the primary consists simply of the main conductor itself. The winding is so arranged that the secondary will deliver a few watts, and is figured on the basis of five amperes in the secondary.

In the case of wattmeters and power factor meters for use on high voltage circuits a combination of two such instrument transformers is employed. A sketch of the connections for a wattmeter layout is shown in Fig. 171*c*. The potential coil of the wattmeter is designated L_1, and the current winding as L_2.

128. Welding and Furnace Transformers.—In the operation of electric furnaces and in electric welding processes current values as high as 25,000 amperes obtain. In order to secure currents of this magnitude use is made of special step-down transformers having a few turns on the primary side and a single heavy copper conductor for the secondary or load circuit.

129. Telephone and Radio Transformers.—It is an interesting fact that there are probably a larger number of transformers used in the field of communication engineering than in power practice. It is also worthy of note that the theoretical problems involved in the design of transformers used for communication purposes are far more complex than those encountered in power generation and distribution. The reason for the fact last mentioned is that in power work a given transformer is designed to operate at a definite and fixed frequency and wave form, while in the case of telephone transformers the unit must function efficiently over a relatively wide range of frequencies when supplied by a current having a complicated and varying wave form. Since the fidelity of speech reproduction depends upon the preservation of the voice current wave form it is evident that the problems involved are of an entirely different order than those encountered in power practice. While it is outside the scope of this text to enter into a detailed discussion of the many special types of transformers employed in the communication field, mention may be made of one or two of the forms most commonly encountered in practice.

The simple open core transformer used in practically all desk and wall telephone sets has already been referred to. This unit serves to transform the variable direct current produced by the action of the sound waves upon the microphone button into an alternating current of higher voltage. The device is commonly spoken of as an induction coil rather than as a transformer because the primary, as just noted, is supplied by a direct current (variable) rather than alternating current. Fundamentally however it is a transformer, and the principles of transformer design are applicable.

The toroidal type of transformer also finds extensive use in telephone practice. A diagrammatic sketch of a common form is shown in Fig. 172. In this unit the core is made either of soft iron wire or thin laminations, and, as the name implies, the wind-

ing, consisting usually of four sections, is toroidally arranged, thus giving high efficiency and comparative freedom from stray fields. All eight terminals are brought out to separate connections, thus making a very flexible unit. This type of transformer is used in both talking and ringing circuits. These transformers are frequently spoken of as "repeating coils."

During the past few years small transformers have also come to be widely used in radio receiving sets. Most commonly these transformers are of the shell type. They must be designed to give a uniform "step-up" ratio over a wide range of frequencies, the limits in music being approximately 50 to 5000 cycles. At the higher frequencies the electrostatic capacitance which exists between the layers of the winding will bypass (Case II, Sec. 117) an appreciable part of the current. This capacitance, together with the inductance, forms a resonant circuit for a narrow band of frequencies, thus giving non-uniform voltage transformation. The effect of either process is to give rise to distortion. By arranging the windings in alternate sections this capacitance effect may be minimized. In these and all transformers used in communication circuits the unit must be so designed as to reduce "stray" magnetic flux to a minimum; otherwise "cross-talk" will occur.

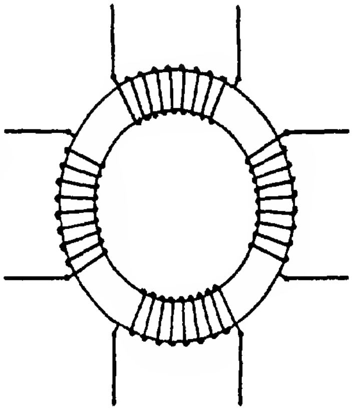


Fig. 172

In speaking of transformers it is pertinent to point out a fundamental principle which finds application wherever transformers are utilized in communication circuits.

It may be shown that, in order to secure maximum transfer of energy, the impedance of the primary of a transformer used in telephonic or radio circuits *must be equal to* (or match) the impedance of the line or other device from which it receives energy. Further, the impedance of the secondary should also match the impedance of the circuit to which it is to transfer energy. If, for instance, a transformer is to be used for the purpose of coupling

two circuits one of which has an impedance of 500 ohms to another whose impedance is 2000 ohms, a transformer should be employed whose windings have impedances of 500 and 2000 *respectively,* the low impedance side of the transformer being connected to the line of low impedance and the high impedance side to the high impedance circuit.

130. Resonance Transformers.—There is a type of transformer which is of special interest and utility in connection with the study and application of alternating currents having frequencies expressed in kilocycles. It is possible to arrange two associated circuits in such a manner that, without the presence of a magnetic material, comparatively high potentials may be developed. Figure 173 represents two circuits made up of capacitance and inductance and a minimum of resistance. Circuit A differs from circuit B in that the magnitude of the inductance L_2 is much greater than that of L_1. The two circuits can be brought into resonance with a common frequency by varying some one or all of the constants involved. If an alternating E.M.F. having a frequency determined by the relation $\frac{1}{2\pi\sqrt{LC}}$ (eq. 163) be impressed on circuit A a maximum current will obtain in that circuit. If now the constants of circuit B are so adjusted that it is resonant to the same frequency as obtains in A, an E.M.F. will be established by induction at the terminals of the inductance L_2. The magnitude of this induced E.M.F. will depend upon the ratio of the magnitude of the two inductances L_1 and L_2.* By making L_1 consist of a few turns while L_2 is composed of a large number of turns extremely high potential differences may be established between the terminals p and p'. This is the principle utilized in the design of the so-called "Tesla coil" and in many other high frequency transformers.

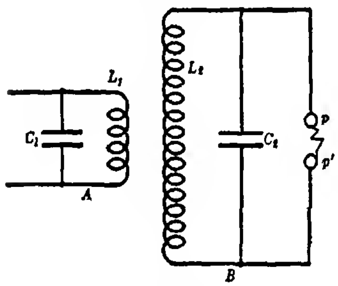


FIG. 173

* The complete theory of the resonant transformer is more complex than here stated. However the approximate explanation given will serve the purposes at hand.

CHAPTER XX

MOTORS

131. Principles of the Direct Current Motor.—In Section 93 we examined the laws governing the force action experienced by a conductor carrying a current and located in a magnetic field. We found that a rectangular coil through which a current was flowing was acted upon by an electromechanical couple whose value is $IHAN \sin \theta$ (eq. 108), where I is the current through the coil, H the field intensity in the region of the conductors, A the average effective area of the individual turns, N the number of turns in the coil winding, and θ the angle which the plane of the coil makes with the direction of the field. In applying this relation to the moving coil galvanometer (Sec. 94) the insertion of an iron core within the region of the coil winding produced a radial field, thus making the direction of the flux at all points at right angles to the direction of movement of the coil. Under these circumstances the equation for the torque developed becomes

$$L = IHAN,$$

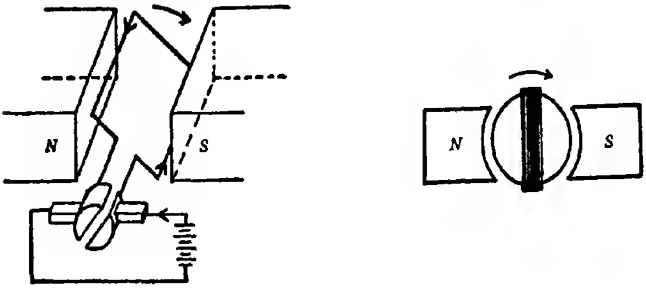
which may also be written

$$L = I\Phi N, \qquad \text{Eq. 172}$$

where Φ is the total flux threading the coil.

Essentially, a constant (direct) current motor does not differ, so far as torque action due to the current is concerned, from the D'Arsonval type of galvanometer. In its main electrical and mechanical details it is practically identical with a constant current generator (Sec. 104). The electromechanical reactions taking place in a motor of this type may be readily followed by reference to Fig. 174. Let us assume that current is sent from an outside source through the armature winding as indicated in (a). In response to the torque developed (eq. 172) the coil will rotate in the direction indicated. When the coil reaches the position shown in (b) the torque will be practically zero. Due however to the rotational inertia of the armature it will, if not too heavily loaded mechanically, continue to turn past the vertical position.

As it does so the commutator bars shift to opposite brushes, *thus reversing the current through the armature winding.* As a result the torque tending to rotate the armature will still be clockwise, and a continuation of this process will accordingly cause the rotor to continue in motion. Thus we have a means whereby a constant current may be converted into mechanical energy, this being the converse of what takes place in the case of the dynamo.



(a) (b)

Fig. 174

Several aspects of the process just outlined require consideration. As the motor armature coils rotate they move through a magnetic field and hence, as in the case of a dynamo, *an E.M.F. will be developed in the windings of the rotor.* By Lenz's law the direction of this induced E.M.F. will be opposite to the applied E.M.F. Before the armature begins to rotate the counter E.M.F. is zero; hence the current taken by the armature will depend wholly on the magnitude of the applied E.M.F. and the value of the resistance of the armature winding. As this resistance is usually only a fraction of an ohm excessive current would result when the E.M.F. is first applied. To avoid this a "starting resistance" is cut into the armature circuit until the rotor has attained nearly full speed. As the angular speed increases the counter E.M.F. increases, in accordance with the fundamental laws of the dynamo. The counter E.M.F. however never equals the applied E.M.F. If the motor is loaded sufficiently to cause the speed to decrease the counter E.M.F. will correspondingly decrease, thus resulting in an increase in armature current. It will thus be seen that a constant current motor is, within limits, self-regulating as to speed. The current taken by the motor will

be given by the relation

$$I = \frac{E - E'}{R}, \qquad \text{Eq. 173}$$

where E is the applied pressure, E' the counter E.M.F. and R the resistance of the armature winding.

In Section 104 reference was made to series, shunt and compound wound generators. The fields of constant current motors are designed in a similar manner, and each type of motor has correspondingly distinct operating characteristics. Series motors develop a large starting torque and are therefore used in such service as the operation of electric railways and cranes. Shunt motors operate at nearly constant speed at all normal loads and are therefore useful in driving lathes, wood-working machinery, etc. A compound motor gives even more constant speed than the shunt motor. It is used in comparatively small units where constant speed is an important consideration.

132. Synchronous Motors.—The necessary reversal of the direction of the current in the armature coil of the motor can be brought about without the use of a commutator. Reference to Fig. 132*a*, which shows the essentials of an alternating current generator, will indicate how this may be accomplished. It will be evident that if an alternating current is supplied to the armature through the slip rings and the rotor, by some outside mechanical means, brought up to the speed it would have if generating an E.M.F. of the same frequency as the applied pressure the torque will always be in the same direction and motor action will obtain. The angular speed which will bring about the reversal of the current as a given conductor passes from one of the field poles to another of opposite sign is known as the synchronous speed and a motor operating in the manner just described is called a synchronous motor. A motor of this character operates at constant speed when not overloaded. If however it is greatly overloaded so that the speed is materially decreased it will "pull out" of synchronism and stop. Synchronous motors are extensively used, particularly in large units. They are usually of the polyphase revolving field type corresponding to polyphase generators.

133. Rotating Magnetic Field.—It will be recalled from one's study of the elements of S.H.M. that it is possible to produce circular motion by combining two simultaneous linear S.H.M's.

The conditions which must be fulfilled in order to attain such an end are that the two component S.H.M's. shall be of equal amplitude; that they shall be at right angles to one another; and that they differ in phase by 90°.

Since alternating currents, and the resulting magnetic fields, obey the laws of S.H.M. it is possible by combining two alternating fields to produce a rotating magnetic field. An analysis of the case is not difficult.

Suppose two coils A and B carrying alternating current of the same frequency and amplitude are positioned at right angles to one another as shown in Fig. 175*a*. Assume that the current in

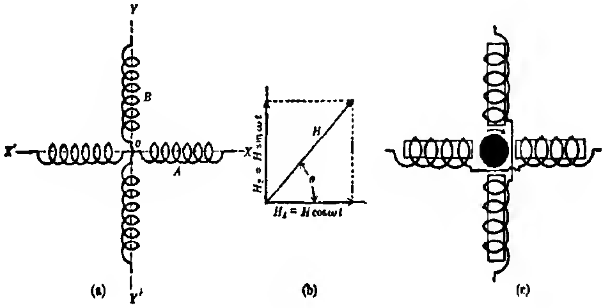


FIG. 175

coil B differs in phase by 90° from the current in coil A. The alternating current in A will give rise to a harmonic field along XX' and the current in B a corresponding sinusoidal field along YY'. Since the currents differ in phase by 90° the two resultant fields will also be in quadrature. The character and magnitude of the field which results from two such component fields may be determined by finding the resultant according to the usual vector methods. Referring to Fig. 175*b*, the instantaneous value of the field (H) in the direction OX due to the current in coil A will be given by

$$H_1 = H_m \cos(\omega t),$$

where H_m is the maximum value of the field in that direction. The corresponding value of the field (H_2) along OY will be expressed by

$$H_2 = H_m \sin(\omega t).$$

The resultant field at the center O will then be

$$H = \sqrt{(H_1^2 + H_2^2)}$$
$$= H_m\sqrt{[\cos^2(\omega t) + \sin^2(\omega t)]} = H_m. \qquad \text{Eq. 174}$$

It is therefore evident that *the resultant field is constant in magnitude.* Further, the instantaneous position of the vector representing the resultant field will be given by

$$\tan\theta = \frac{H_2}{H_1} = \frac{\sin(\omega t)}{\cos(\omega t)} = \tan(\omega t),$$

from which it follows that

$$\theta = \omega t. \qquad \text{Eq. 175}$$

It is therefore evident that *the resultant field H rotates with an angular velocity of ω radians per second*, and since $\omega = 2\pi f$ the resultant field will rotate a number of times per second corresponding to the frequency of the current in the coils A and B.

In our discussion of Foucault currents (Sec. 105) it was noted that, in the case of the automobile speedometer, a body of conducting material such as copper or aluminum when suspended in the presence of a rotating magnetic field was caused to rotate due to the development of eddy currents in the conductor. We have just seen that a rotating field of fixed magnitude and constant angular velocity may be established by combining the fields due to two alternating currents. If then a rotor made up of conductors of low resistance be mounted on a shaft in such a manner that it will be free to rotate within the region of a rotating field produced as outlined above the field due to the eddy currents generated in the rotor by the rotating field will, in conformity with Lenz's law, cause the rotor to revolve and thus develop motor action (Fig. 175c). This is the basis of the *induction motor.* Nicola Tesla was the first to apply these principles in the production of a practical motor of this type. In practice the rotor of the induction motor commonly consists of a series of heavy copper bars supported on the periphery of an iron frame-work and parallel to the axis of rotation. The conducting bars are short-circuited by being welded to a copper or brass ring at each end, thus forming what is frequently referred to as a "squirrel cage" rotor. The field coils are usually supplied with either two- or three-phase alternating current. In the three-phase units three

sets of field coils are so arranged that they produce three component fields separated by 120°, and also differing in phase by 120°.

It is possible to "split" a single-phase supply and thus produce what amounts to a two-phase current. This may be accomplished by introducing some inductance or capacitance into the circuit forming one of the phase windings, thus causing a lagging or leading current in that particular field coil.

By means of such an arrangement a motor can be constructed which will operate from a single phase supply. Such a unit is known as a *single phase motor.*

Two- and three-phase induction motors are widely used, the sizes ranging from a fraction of a H.P. to 200 H.P. Single-phase motors are usually made only in the smaller units. Induction motors are extremely rugged in construction and require little operating attention.

CHAPTER XXI

ELECTRICAL UNITS

134. Fundamental and Derived Units.—We have now reached a point in our study where we may properly consider the question of electrical units as a whole. In order to arrive at a comprehensive understanding of the inter-relations which exist in any system of units it is important to note that in making a statement concerning the magnitude of any physical quantity two factors are involved, one of which is a mere number and the other the unit in which the quantity is expressed. It is this second factor which we now proceed to examine.

Obviously there are many types of quantities and hence many possible forms of units. For instance there are mechanical, thermal, electrical and other types of quantities. It is conceivable that we might have entirely independent systems of units in which to express the various quantities occurring in these several fields. But since experience shows these various fields to be intimately related and in fact interdependent it is obviously desirable to adopt some basic system of units to which all special units may be referred. Experience has shown that all quantities with which science is now familiar can ultimately be expressed in terms of three fundamental units, viz., length, mass and time, length being expressed in centimeters, mass in grams, and time in seconds. In the electrical field there are two other physical concepts which are sometimes involved in the expression of quantities, viz., the dielectric constant (k) and magnetic permeability (μ). However these two latter concepts are not commonly referred to as fundamental units.

Units which are derived from the fundamental units just mentioned are referred to as *absolute or c.g.s. units.* For instance, linear velocity is a ratio of linear displacement to time, or $v = \frac{d}{t}\cdot$ Since linear displacement is naturally expressed in terms of length, velocity is expressed in terms of length and time. Therefore the absolute unit of velocity is the centimeter per second.

Again, it has been shown (Sec. 27) that capacitance is expressed

in terms of a unit of length; hence the absolute or c.g.s. unit of capacitance is in reality the centimeter. Other examples might be cited but the foregoing will suffice to illustrate what is meant by absolute or c.g.s. units.

135. Kinds of Electrical Units.—At various points in the preceding chapters we have referred to three kinds of electrical units, the electrostatic, the electromagnetic and the so-called "practical" units. In all electrical measurements the magnitudes with which we deal can only be determined by means of accurate observations of mechanical effects. As pointed out in our discussions it has been found convenient to adopt the mechanical effects due to unit charge or to unit pole as a basis of electrical calculations. The magnitudes of these effects can be and are expressed in absolute units, and all of the basic equations which we have developed have involved only fundamental units, or units derived directly from such units. It therefore follows that both *the electrostatic and electromagnetic quantities resulting from the application of any of our basic equations are expressed in absolute or c.g.s. units.*

It will be evident that the establishment of an absolute system of units in the fields of electricity and magnetism tends to simplify and unify both the theoretical and practical aspects of these intimately related sciences. The fact that we have two systems of absolute units in electrical and magnetic computations does not present a serious difficulty for the reason that the units in one system may be readily converted into those of the other, or into the "practical" units.

While the development of a system of units resulting from theoretical analysis based on the three fundamental units has done much to further the progress of electrical science, these c.g.s. units are in some instances inconveniently large and in other cases too small to be applicable in practice. For instance the absolute unit of resistance in the e.m. system is equal to the resistance of a piece of copper wire one millimeter in diameter and about one twenty-thousandth of a millimeter in length. By international agreement (1908), as already mentioned (Sec. 41, *et seq.*), units of a more convenient size have been adopted for use in the electrical industry, these practical units being defined *in terms of c.g.s. electromagnetic units.* These subsidiary units are derived from the absolute units by multiplying or dividing by powers of ten.

136. Practical Units.—For purposes of reference we list below the practical units in which the more common electrical quantities are expressed.

Resistance.—The *ohm* is the practical unit of resistance and is defined as 10^9 c.g.s. electromagnetic units. For purposes of reference in electrical measurements the ohm is specified as the resistance at 0° C. to a constant current of a column of mercury having a uniform cross-section of one square millimeter and a length of 106.300 cm.

E.M.F.—The unit of E.M.F. and P.D. is the *volt*, and has been made equal to 10^8 absolute e.m.u's. This is within about 2 per cent of the E.M.F. produced by a Weston standard cell.

Current.—The practical unit of current strength is designated as the *ampere.* An ampere is the current which exists in a conductor having a resistance of one ohm when a potential difference of one volt is maintained at its terminals. From the defining equation of current (eq. 54) we may write

$$I \text{ (in absolute e.m.u's.)} = \frac{E \text{ (in ab. e.m.u's.)}}{R \text{ (in ab. e.m.u's.)}}.$$

By definition,

$$I \text{ (in amperes)} = \frac{E \text{ (in ab. e.m.u's.)}/10^8}{R \text{ (in ab. e.m.u's.)}/10^9}.$$

Therefore the ampere is equivalent to one-tenth of an e.m.u. of current. The ampere has also been defined as that current which will deposit 0.0011183 gm. of silver per sec. from an aqueous solution of silver nitrate.

Quantity.—The unit of quantity employed in the electrical industry is the *coulomb* or *ampere-second.* The coulomb is that quantity of electricity which traverses in one second the cross-section of a conductor in which a constant current of one ampere is flowing. It is equal to one-tenth of an absolute e.m.u.

Capacitance.—The practical unit of capacitance is designated the *farad.* A conductor or electrical system has a capacitance of one farad when a charge of one coulomb will give rise to a potential of one volt. The farad is defined as the equivalent of one thousand-millionth (10^{-9}) of an absolute e.m.u. Even this small part of the c.g.s. unit is much too large for most practical purposes; hence the smaller unit known as the microfarad (mf. or μf.)

is commonly employed. The microfarad is equal to 10^{-15} e.m.u. or 10^{-6} farad.

Inductance.—The *henry* is the practical unit of both self and mutual inductance. In the case of self-inductance a circuit is said to have an inductance of one henry if a counter E.M.F. is developed in the circuit when the current in the circuit is changing at the rate of one ampere per second. Two electromagnetically coupled circuits have a mutual inductance of one henry when a change in the current through one of the circuits at the rate of one ampere per sec. gives rise to an E.M.F. of one volt in the related circuit. The henry would therefore be defined as 10^9 absolute e.m.u.

Power.—When a current of one ampere flows between two points in a conductor between which the P.D. is one volt, the rate of energy expenditure is said to be a watt. This is equivalent to 10^7 ergs per second.

137. Comparison of Practical and Absolute Units.—In the preceding section we have dealt only with the relation of the practical to the c.g.s. electromagnetic units. It will be found useful to have available in tabular form not only these equivalents but also the relations of the practical unit to the c.g.s. electrostatic units. Such a list follows.

Quantity	Name of Practical Unit	Equivalent in Absolute Units	
		e.m.u.	e.s.u.
Resistance	ohm	10^9	$\frac{1}{9 \times 10^{11}}$
Electromotive force and potential difference	volt	10^8	$\frac{1}{300}$
Current	ampere	10^{-1}	3×10^9
Quantity	coulomb	10^{-1}	3×10^9
Capacitance	farad	10^{-9}	9×10^{11}
	microfarad	10^{-15}	9×10^5
Inductance	henry	10^9	$\frac{1}{9 \times 10^{11}}$

138. Dimensions of Electrical Units.—In dealing with physical quantities, particularly in the electrical field, it is useful to know the extent to which the fundamental units of length, mass and time enter into the unit in which a given quantity is measured.

As pointed out in Sec. 134, linear velocity is the ratio of linear displacement to time; in other words length enters once into a computation of velocity and time enters once also but *inversely.* These facts may be expressed by what is known as a dimensional equation, thus,

$$[v] = \frac{L}{T} = LT^{-1},$$

which is to be read, the dimensions of the unit of velocity are LT^{-1}, the term *dimension* meaning the extent to which the fundamental units are involved in the quantity being examined. The *exponents* of the symbols for the fundamental units indicate the *dimensions* of derived units in terms of the fundamental units.

To extend our example somewhat, we might find the dimensions of acceleration, the defining equation of which is

$$a = \frac{v}{t}.$$

Since we already know the dimensions of velocity we may at once write

$$[a] = LT^{-2}.$$

Again, force is defined thus,

$$F = m \times a;$$

hence the dimensional equation for force becomes

$$[F] = MLT^{-2}.$$

It is thus seen that the fundamental unit of mass enters once in the unit of force, the unit of length also once and the unit of time inversely twice.

The dimensions of the electrical units are built up in a similar manner. Beginning with the mechanical force action between charges the dimensions of the absolute electrostatic units are readily determined. The force between two charges is given by eq. 2 in the form

$$F = \frac{q_1 q_2}{Kd^2}.$$

Making $q_1 = q_2$ we have

$$F = \frac{q^2}{Kd^2},$$

or

$$q = d\sqrt{(FK)} = dF^{1/2}K^{1/2}.$$

The dimensional equation for electrical quantity then becomes

$$[q] = L(MLT^{-2})^{1/2}K^{1/2}$$
$$= M^{1/2}L^{3/2}T^{-1}K^{1/2}.$$

By continuing this process the dimensions of all of the electrostatic units may be determined.

The dimensions of the c.g.s. electromagnetic units can be set up in a similar manner. Starting with the force in air between two magnetic poles as given by eq. 85 we have

$$F = \frac{m_1m_2}{d^2}.$$

If however we take into account the character of the medium separating the poles the above relation becomes

$$F = \frac{m_1m_2}{\mu d^2}, \qquad \text{Eq. 176}$$

where μ is the magnetic permeability (Sec. 95). If we make the two poles of equal value eq. 176 becomes

$$F = \frac{m^2}{\mu d^2}.$$

Solving for m as we did in the corresponding electrostatic equation, we get

$$m = d\sqrt{(\mu F)} = dF^{1/2}\mu^{1/2},$$

from which the dimensional equation for the unit of pole strength may be written thus,

$$[m] = M^{1/2}L^{3/2}T^{-1}\mu^{1/2}.$$

This line of reasoning may be extended to other quantities in the electromagnetic system. Take, for instance, the case of magnetic field intensity as expressed in eq. 87, which is

$$F = mH,$$

or

$$H = \frac{F}{m}.$$

On the basis of this defining equation the dimensional equation would be

$$[H] = M^{1/2}L^{-1/2}T^{-1}\mu^{-1/2}.$$

Thus we have a statement of the extent to which the fundamental units of length, mass and time enter into the c.g.s. unit of magnetic field intensity, which is the gauss.

One further example of the method by which the dimensions of electrical units are arrived at will suffice. The relation between the field strength due to the current in a long straight conductor and the magnitude of that current is given by eq. 94 as

$$H = \frac{2I}{x},$$

or

$$I = \frac{xH}{2}.$$

The numeric 2 may be neglected in setting up the resulting dimensional equation,

$$[I] = LM^{1/2}L^{-1/2}T^{-1}\mu^{-1/2} = M^{1/2}L^{1/2}T^{-1}\mu^{-1/2},$$

thus giving the dimensions of the absolute e.m.u. of current. By a continuation of this general process the dimensions of the remaining e.m.u's. may be deduced. Following is a tabular list of the dimensions of a number of units in both the electrostatic and electromagnetic system.

In addition to the utility of dimensional formulas in checking the corrections of equations by which calculations are to be made, a knowledge of the dimensions of the units entering into a given relation will often assist one in arriving at a conclusion as to the nature of the units in which the resulting quantity must be expressed, and also as to how the fundamental units enter into the final result. A case in point is that of inductance. The dimension of inductance in the e.m.u's. is length to the first power. The c.g.s. unit is therefore the centimeter.

Furthermore, dimensional formulas sometimes serve to reveal intimate relations between magnitudes which might by simple inspection appear to be quite independent. This utility of dimensional formulas is strikingly shown by an examination of the ratio of the dimensions of the various electrical units as given in

Unit of	Dimensions in the		Ratio of Electrostatic to Electromagnetic Units
	Electrostatic System in Terms of L, M, T, K	Electromagnetic System in Terms of L, M, T, μ	
Quantity	$L^{3/2}M^{1/2}T^{-1}K^{1/2}$	$L^{1/2}M^{1/2}\mu^{-1/2}$	v
E.S. Field Intensity	$L^{-1/2}M^{1/2}T^{-1}K^{-1/2}$	$L^{1/2}M^{1/2}T^{-2}\mu^{1/2}$	v
Potential	$L^{1/2}M^{1/2}T^{-1}K^{-1/2}$	$L^{3/2}M^{1/2}T^{-2}\mu^{1/2}$	v^{-1}
Capacitance	LK	$L^{-1}T^{2}\mu^{-1}$	v^{2}
Resistance	$L^{-1}TK^{-1}$	$LT^{-1}\mu$	v^{-2}
Current	$L^{3/2}M^{1/2}T^{-2}K^{1/2}$	$L^{1/2}M^{1/2}T^{-1}\mu^{-1/2}$	v
Inductance	$L^{-1}T^{2}K^{-1}$	$L\mu$	v^{-2}
Pole Strength	$L^{1/2}M^{1/2}K^{1/2}$	$L^{3/2}M^{1/2}T^{-1}\mu^{1/2}$	v^{-1}
Magnetic Field Intensity	$L^{1/2}M^{1/2}T^{-2}K^{1/2}$	$L^{-1/2}M^{1/2}T^{-1}\mu^{-1/2}$	v
Magnetic Flux	$L^{1/2}M^{1/2}K^{-1/2}$	$L^{3/2}M^{1/2}T^{-1}\mu^{1/2}$	v^{-1}

the two systems of c.g.s. units. Take for instance the case of the unit of current and assume the medium surrounding the conductor to be air, thus making both K and μ unity. The ratio of the electrostatic to the electromagnetic dimensions is

$$\frac{L^{3/2}M^{1/2}T^{-2}}{L^{1/2}M^{1/2}T^{-1}} = LT^{-1} = [v].$$

It is thus evident that the ratio of the c.g.s. electrostatic and electromagnetic units of current is of the nature of velocity.

Again if we examine the ratio of the dimensions in the case of the units of potential we find that

$$\frac{L^{1/2}M^{1/2}T^{-1}}{L^{3/2}M^{1/2}T^{-2}} = \frac{1}{LT^{-1}} = \left[\frac{1}{v}\right],$$

and for capacity the ratio is

$$\frac{L}{L^{-1}T^{-2}} = L^2T^{-2} = (LT^{-1})^2 = [v^2].$$

If we were to continue this process for the remaining units we should have the ratio values shown in the last column of the last table. An examination of the ratios there listed discloses a most remarkable fact, namely, that *in all cases the ratio is some power of a velocity or a reciprocal of a power of a velocity.* The question then naturally presents itself as to the numerical value of the velocity. Fortunately it is not difficult to make at least an approximate determination of this quantity. It is possible to accurately construct a condenser and to compute its capacitance from its geometrical dimensions. The result will be in c.g.s. electrostatic units. By charging the same condenser to a known potential and discharging it through a suitable galvanometer we may measure its capacitance in c.g.s. electromagnetic units. If then we take the ratio of the two results thus obtained we find it to be approximately 3×10^{10}. We are thus confronted with the astonishing fact that the ratio of the two absolute systems of electrical units involves a velocity, and that this velocity, as closely as can be determined, is the same as the velocity of light in free space. This significant relation was one of the factors which led Maxwell to advance the theory that light is essentially an electromagnetic phenomenon.

In passing, one other aspect of the interpretation of dimensional formulas should be noted. Many of the dimensions appear as fractions. It is difficult to attach any meaning to such an expression as $L^{3/2}$ or $M^{1/2}$, in and by themselves. Permeability (μ) and the dielectric constant (K) are factors which serve to specify, in certain limited respects, the properties of the medium. We do not know the absolute dimensions of K and μ. If we had this information and it were incorporated in our dimensional equations it is possible that those terms which now appear with fractional exponents would be rationalized, and it might be that the dimensions of the units in the two systems would prove to be the same. In other words, we do not at present know what the mechanism is by which one charge attracts another or by which one pole exerts a force upon another. It may however prove to be significant in this connection that the product of K and μ may be expressed in terms of velocity. This relation between K, μ and v may be readily reduced by equating the e.s. and e.m. dimensional expressions for any one of the units, say capacitance. The writing of such an equality is justified because of the fact that it

is unreasonable to assume that any given quantity can have essentially two different sets of dimensions of the fundamental units. Proceeding on this basis we have

$$LK = L^{-1}T^2\mu^{-1},$$

which yields

$$\frac{L^2}{T^2} = \frac{1}{K\mu} = [v^2],$$

or

$$[v] = \frac{1}{\sqrt{(K\mu)}}\,.$$

This relation indicates that $\frac{1}{\sqrt{(K\mu)}}$ has the dimensions of a velocity. When we have more complete information concerning the nature of the ether it is possible that we may be able to assign definite dimensions to both the dielectric constant and permeability.

CHAPTER XXII

ELECTRONICS

139. Ionization in Gases.—In our discussion thus far we have thought of the electric current as the orderly migration of electrons due to an E.M.F. In metals, as we have seen, the movement of the so-called free electrons constitutes the current. In considering electrolytic processes (Sec. 57) we noted that the moving charges are associated with certain atoms or groups of atoms which go to form the electrolyte. We are now to consider a group of phenomena in which electrons which are *wholly detached from solids and liquids* and, in some instances, even from gases are given definite motion. In other instances we shall examine cases in which one or more electrons having been detached from a gas atom the remainder of the atomic structure is also given definite motion. In the first instance we have what may be called an *electronic current* and in the latter case an *ionic current.* Several means are available whereby electrons may be separated from an atomic structure; by whatever method this is accomplished the process is known as *ionization.* If a single electron is removed from a neutral atom (one having an equal number of positive and negative electrons) the remainder of the atom will manifest a positive charge and is known as a positive ion. If a free electron becomes attached to a neutral atom or a group of atoms or molecules the group thus formed will manifest a negative charge and is spoken of as a negative ion. We shall now proceed to examine the means whereby ionization may be effected and the results which may follow from such a process.

140. Ionization by Collision.—One of the most common and important methods of bringing about a liberation of electrons from neutral atoms is known as ionization by collision. If a substance is composed *entirely* of neutral atoms its conductance is nil, but even in the case of dry and dust-free gases there are probably a few free electrons and hence a corresponding number of ions present. If therefore two plates, separated for instance by air, are maintained at a difference of potential there will be a slight transfer of charge from one plate to the other due to the

enforced movement of the few ions present. If the potential gradient be increased the free electrons and the positive ions will acquire correspondingly greater velocities, and if this electronic and ionic velocity becomes great enough electrons will be detached from some of the neutral atoms as a result of collisions. Additional ions will thus be formed and a greater current between the plates will result. This process may be cumulative up to that point where all the neutral atoms existing between the plates have been ionized by collision. After the ionizing process has started the potential gradient necessary to maintain a given current is much less than that required to initiate the discharge. This is a common phenomenon in connection with the formation of the electric arc. In general the gas pressure will have a bearing on the difference of potential required to initiate the discharge. The higher the pressure the greater will be the potential gradient necessary to bring about ionization. This is due to the fact that in a more dense gas the mean free path of the ions is comparatively short and hence for a given potential the electronic velocity is relatively small. For a given set of conditions an electron must have a certain minimum velocity, and hence a definite amount of kinetic energy, before ionization by collision will occur. It is the custom to express this energy in terms of the potential through which the electron has fallen. (See eq. 18.) The potential difference through which an electron must fall in order to produce ionization by collision in a given gas or vapor is referred to as the *ionization potential* of that particular substance. An electron, for example, must fall through a potential difference of 2.09 volts in order to bring about ionization in attenuated sodium vapor; in the case of hydrogen the drop must be 13.6 volts. Mercury vapor, which is used in certain electric lamps, has an ionization potential of 10.3 volts. Since the ions have greater mass than the electrons a higher potential difference is required before the positive ions can produce ionization. When however the potential fall is sufficient the positive ions will also possess enough energy to knock electrons from neutral atoms. When this condition obtains ionization multiplies rapidly and there is a marked increase in the space current.

It is, however, not alone in an electrostatic field that ionization by collision obtains. The disintegration of the atoms of radioactive bodies (Sec. 159) gives rise to three types of radiation,

namely, the α-rays, the β-rays, and the γ-rays. It has been established that the so-called α-rays are helium nuclei and hence, for present purposes, may be thought of as positive ions. These alpha particles or ions leave the atom of the radio-active substance with a velocity of several thousand miles per second, which is sufficient to cause marked ionizing effects by collision.

The β-rays are in reality electrons, that is, discrete negative charges. They leave the radio-active atom with a velocity varying from one third to one tenth of the velocity of light, and accordingly also give rise to ionization by collision. We shall see presently that ionization by collision plays an important part in connection with the phenomena occurring in connection with the transfer of charges in gases at greatly reduced pressures.

141. Ionization by Radiation.—It is possible to bring about ionization by means other than that of collision. Electrons may be detached from atoms by the action of certain forms of radiation, notably x-rays and ultra-violet light. In fact ether waves which lie within the visible spectrum, under certain conditions, may also effect a liberation of electrons.

Ionization by X-rays and Gamma Rays.—If, for instance, an x-ray tube (Sec. 150) be put into operation several meters from a charged electroscope the latter will be immediately discharged. Some of the electrons in the gas atoms acquire enough additional energy from the high frequency ether vibrations constituting the x-rays to be able to escape from the atom and thus become, for a time at least, free electrical entities. The electrons and ions thus produced cause the air to be conducting. Indeed it is by such ionizing means that the process of ionization in general, as well as the behavior of ions themselves, may be conveniently investigated. For instance an arrangement of apparatus as shown in Fig. 176 affords a convenient means of studying ionization phenomena. If the electroscope be charged and air drawn through the system as shown, the x-ray tube being in operation, the

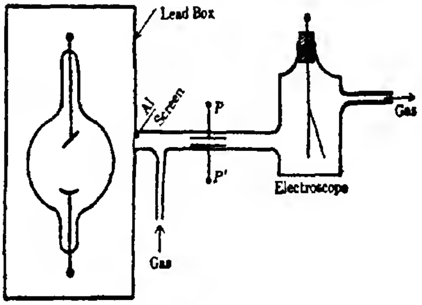


Fig. 176

electroscope will be immediately discharged. If, however, a plug of glass wool be placed in the gas line between the box and the electroscope the current of air will not discharge the electroscope. If the air be drawn through a liquid it will also lose its ionizing power. If the plates P and P' are maintained at a relatively high potential it will be found that the effect of the moving air on the electroscope is nil. The results of these and other experiments show that there must be present in the air, as a result of the action of the x-rays, either free charges (electrons) or atoms less or plus one or more electrons (ions) which when combining with the charge on the electroscope cause its charge to disappear. It is also evident that these ions can be removed from the gas by mechanical or electrical means, and that in ordinary air (any gas in fact) the atoms are electrically neutral. By means of equipment similar to that described above, the mobility and the rate of recombination of ions may be investigated.

As we shall see when we come to study radioactivity, one of the results of the automatic breaking down of the atomic structure in such elements as radium is the production or generation of what are known as gamma rays, which are waves having a length of about one tenth that of x-rays. These so-called rays are capable of ionizing a gas and will therefore cause the discharge of insulated conductors. Indeed one method (Sec. 157) of studying such rays is by means of their ionizing effects.

Photoelectric Effect.—As before indicated ultra-violet light, and even light within the visible spectrum, may be caused to liberate electrons from the atoms of certain elements. For instance if a piece of clean zinc be charged negatively and then be subjected to the action of ultra-violet light it will be found that the charge will disappear. If the metal be positively charged no action results. This phenomenon is known as the *photoelectric effect.* A much more pronounced electronic emission may be secured by employing one of several of the alkali metals, particularly when the surface is treated with hydrogen and the electrodes are inclosed in a vacuum. Experiment has shown that the *velocity* of the liberated electrons from such a photoactive body is independent of the intensity of the incident light, but that the *number of electrons liberated per unit of time is proportional to the intensity of the exciting radiation.* The latter property makes possible a wide field of practical utility. Photoelectric "cells" are now made in a

number of different types. The commercial unit as usually made consists of an evacuated glass or quartz bulb on the inside of a part of the wall of which is deposited the photoactive material, electrical contact with which is effected through a lead-in wire sealed in the cell wall. A second electrode, usually in the form of a loop or grid of wire, is placed a short distance in front of the sensitive surface. A difference of potential of 100 to 200 volts is commonly maintained between the two electrodes, the sensitive surface being negative. The circuit usually employed is shown in Fig. 177*a*. Light falling upon the photoactive surface liberates electrons which, under the action of the potential difference,

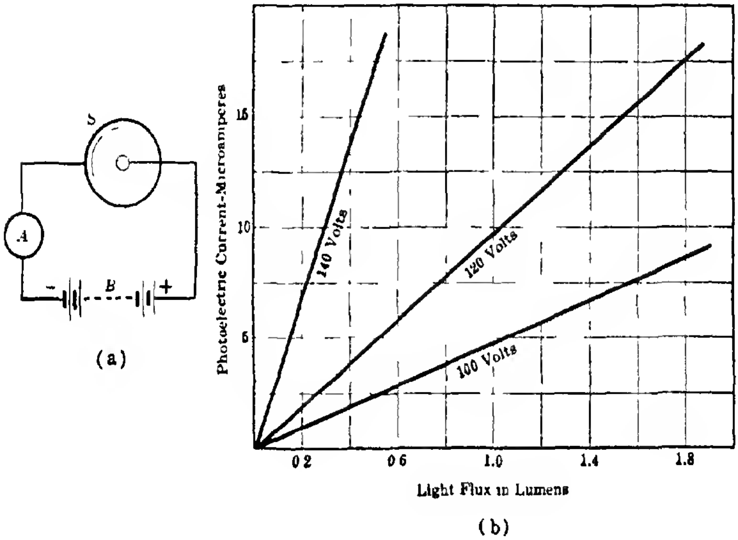


Fig. 177

move from S to E, thus establishing a current. The magnitude of the current is of the order of a few microamperes. If a small amount of gas exists in the tube ionization by collision will obtain and the current through the cell will be relatively large. Figure 177*b* shows the approximately linear relation between light intensity and cell current which may be expected in the case of a well-made potassium hydride vacuum type cell. Photoelectric cells are in general most sensitive to some particular part of the spectrum, depending upon the nature of the photoelectric surface employed. They will give rise to a measurable space current when excited by exceedingly small quantities of incident energy,

and, what is still more important, the response is instantaneous. In practice these minute photoelectric currents are commonly amplified (Sec. 145) by means of thermionic tubes. Photoelectric cells find extensive application in photometry, the transmission of photographs by wire and radiotelegraphy, and in talking moving pictures.

142. Ionization by Means of Heat.—Because of the extent of its use heat is at present the most important agency whereby electrons are liberated from metals. The electrons are held within the spheres of influence of the atomic groups by reason of the electrostatic force exerted by the nuclei. In order that an electron may escape from the surface of a metal a certain amount of work must be done, and this work is sometimes referred to as the "*electron evaporation constant.*" By the application of heat the mean electronic kinetic energy is increased and certain of the electrons may acquire enough energy to overcome the attraction of the metallic atom at the surface (sometimes called "surface constraint") and escape as a free electric entity. The process is quite analogous to evaporation in the case of liquids. The ratio between the electron evaporation constant and the electronic charge is a constant, sometimes referred to as the *electron affinity.* Expressed algebraically this relation becomes

$$V = \frac{W}{e},$$

where V is the electron affinity, usually expressed in terms of potential. This is a very important constant, for its value determines the number of electrons which will be liberated from a particular metal at a given temperature; it is characteristic of the material which is being used as a source of electronic emission. Since, in practice, the desideratum is to secure a maximum number of electrons for a minimum expenditure of energy in heating the metal, it is the aim of investigators to secure a substance which will yield a copious supply of electrons when operated at comparatively low temperature; in other words to have the above ratio as small as possible. Since devices which utilize thermoelectronic emission usually consist of a filament directly or indirectly electrically heated it is also desirable to operate the filament at a relatively low working temperature in order that the life of the filament may be as great as possible. A relation

due to Richardson * which connects temperature and electronic emission has the form

$$i = \alpha\sqrt{T}\epsilon^{-(b/2T)},$$

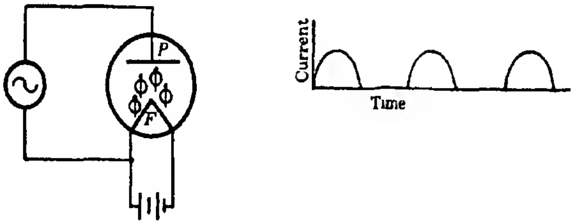
in which i is the emission current per sq. cm. of hot surface. This equation shows that by using metals which have a high melting point, such as platinum or tungsten, relatively high electronic emission can be secured.

However unless means are provided for removing the liberated electrons a state of electronic equilibrium will soon come to obtain corresponding to what we find in a saturated vapor; that is, as many electrons return to the filament in unit time as are detached. In other words, we have what is called a *space charge* surrounding the heated filament. If however we arrange a positively charged plate near the hot filament (cathode) the liberated electrons will be attracted to it, and if the potential of the plate be sufficiently high all of the emitted electrons will move to the plate, and we have the maximum electronic current which can obtain under a given set of operating conditions. Such a current is known as the *saturation current.* Various factors, as for instance the presence of a slight trace of oxygen, greatly influence the emission from a metal filament. Langmuir has shown that in order to realize the current values predicted by Richardson's equation cognizance must be taken of these modifying factors and also that the filament must be operated in the highest possible vacuum. It has been found that by introducing into the tungsten, which is to be utilized as filament material, certain impurities, as, for example, thorium, or by coating the filament with certain alkaline earths, a much more copious yield of electrons may be had. Thermionic devices utilizing such treated filaments may be operated at a dull red heat, and hence have a life of several thousand hours. However if the filament in such tubes be operated at a temperature even slightly in excess of normal the life is decidedly lessened. In certain tubes the electron-emitting body consists of a refractory rod or plate composed of alkaline oxids, this *emitter* being heated by an incandescent filament located within or near the source of electrons. Evacuated tubes designed to yield electrons by the application of heat are called thermionic tubes. There are various important practical applica-

* *The Emission of Electricity from Hot Bodies,* by O. W. Richardson.

tions of the process of ionization by heating, and these will now be discussed.

143. Two-Electrode Vacuum Tube.—The production of the modern two- and three-electrode tubes dates back to the extended researches of Elster and Geitel which were carried on from 1882 to 1889. These investigators disclosed important facts regarding electronic emission from heated wires or filaments. In 1883 Edison noted that a small current flowed from the filament to a plate located within an electric light bulb, if the plate were connected to the positive side of the filament. This has come to be known as the *Edison effect;* however the observation was not followed up by the discoverer and it was not until the phenomenon had been studied by Fleming and J. J. Thomson that its significance began to be appreciated. Eventually Fleming applied the principle in the production of his so-called *valve* for use in rectifying high frequency currents in radiotelegraphy and in 1905 obtained his historic *valve patent.* The Fleming device consisted of a hot cathode and a plate in an evacuated inclosure, and was made to serve as a detector for electric waves. It is probable that in the original Fleming valve there were ions present in considerable numbers due to residual gas. However, Professor Fleming's valve served as the beginning of a series of developments which have resulted in a number of important electronic devices.



(a) (b)

FIG. 178

From what has already been said it is evident that a two-electrode tube, such as the Fleming valve, Fig. 178*a*, will exhibit unilateral conductivity, the current passing between the plate P (anode) and the filament F (cathode) having the form shown in Fig. 178*b*. If there be an appreciable amount of residual gas present the current through the tube will be almost wholly due to gaseous ionization (Sec. 140); if a high vacuum obtains the

transfer of energy is effected by means of electrons, the ionization being negligible.

There are several rectifying devices based on the above principle in extensive industrial use. Two of these commercial units are known as "Rectigon" and "Tungar" rectifiers. A unit of this type is shown diagrammatically in Fig. 179. The "tungar" tube consists of a bulb filled with some inert gas such as argon in which are located a short heavy filament F and a cold electrode A which is a graphite plate. The filament is heated from part of the secondary winding of the step-down or auto transformer T.

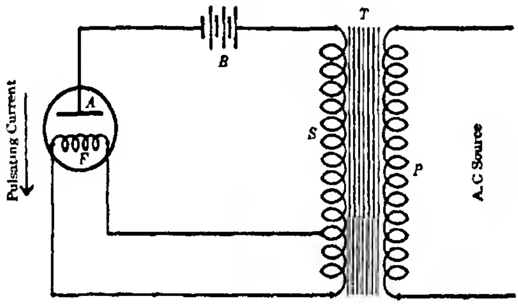


FIG. 179

In the diagram B represents a storage battery to be charged. If the filament F be heated a copious supply of electrons will be liberated and during the positive half cycle of the alternating E.M.F. these free electrons will be attracted to the anode A. During the negative half cycle the electrons will be repelled by A and hence the space current will be zero. In moving toward the plate A the electrons usually acquire a velocity of sufficient magnitude to ionize the small amount of gas present in the bulb. The energy transfer through the tube will be in the nature of a pulsating unidirectional current which may be utilized to charge the secondary cells B. Rectigon and Tungar rectifiers are made to deliver currents up to 6 amperes and pressures as high as 75 volts. The organization just described gives only half-wave rectifications. It is possible however to arrange a circuit that will yield full-wave rectification. Such a lay-out is shown diagrammatically in Fig. 180.

Another widely used two-electrode rectifying device is known as the *mercury arc rectifier.* A diagrammatic sketch of such a

unit is shown in Fig. 181*a*. While this device actually has more than two electrodes, essentially it is a two-electrode unit. It is so designed that it will handle relatively large currents and give full wave rectification. The glass pear-shaped bulb has four connections sealed into the wall and terminating on the inside in graphite or iron electrodes. Mercury covers two of these electrodes, C and D. The two electrodes A and A' are commonly called anodes, though during a part of the cycle they are actually negative. The bulb is thoroughly exhausted of air, only mercury vapor remaining. When cold the internal resistance of the tube is so high that current will not pass between either anode and the cathode. If however the mercury vapor is ionized current will pass from either A or A' to C, depending on which of the anodes is positive. In order to start the rectifier arrangements are provided for tilting the bulb so that a bridge of mercury is formed between the auxiliary electrode, D, and C. Upon being returned to the normal vertical position an arc between D and C is formed and some of the mercury is thus vaporized. The heated mercury at C liberates electrons which by collision ionize the mercury vapor in the region between the cathode and the two anodes A and A'. During one half of each cycle one of the electrodes, A for instance, is positive and current will pass from it to C. It will then be seen that the current in the load circuit is always in one direction, and this is in effect a direct current. Unless some provision is made for maintaining the current while the E.M.F. at the terminals of the secondary of the transformer T passes through zero the tube would cease functioning. In order to obviate this a reactance coil, L (Case I, Sec. 117), sometimes called

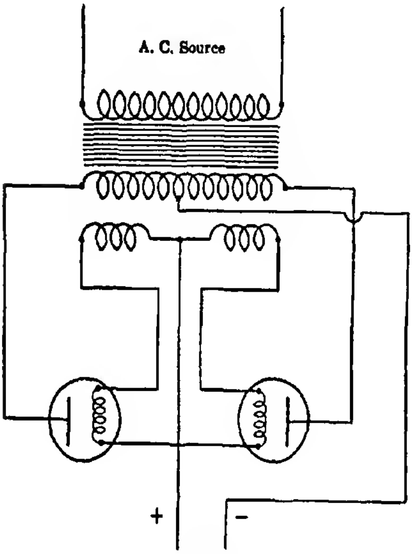


Fig. 180

a sustaining coil, having a large inductive reactance but low ohmic resistance, is included in the load circuit. This reactance causes the current to lag somewhat behind the E.M.F. with the result that there is still some current flowing from A to C, for instance, while it is beginning to flow from A' to C; thus the current in the load circuit at no time reaches zero; hence, once started, the ionizing process is continuous.

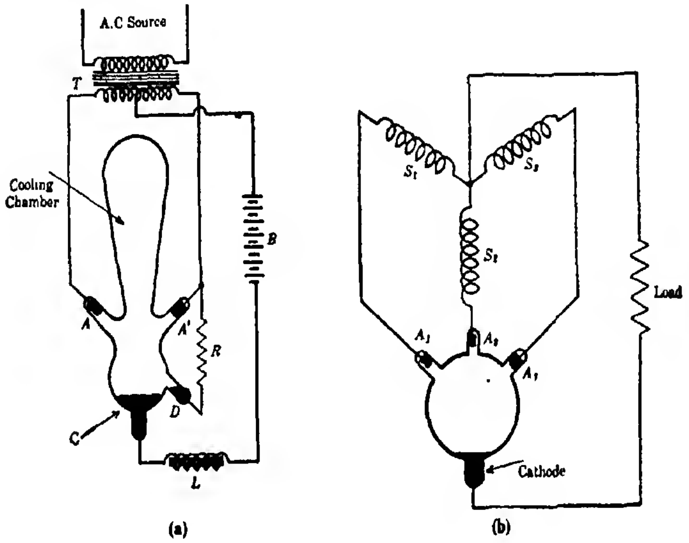


FIG. 181

The unit just described would be called a single-phase rectifier. Rectification of the current in a three-phase circuit may also be accomplished. The essentials of a three-phase organization are shown schematically in Fig. 181b. In the sketch S_1, S_2 and S_3 are the secondaries of a three-phase transformer; A_1, A_2 and A_3 are anode terminals; and R the load resistance. In the three-phase rectifier no sustaining reactance is necessary.

During operation the voltage drop across a single-phase rectifier of the mercury type is about 14 volts. The overall efficiency runs from 80 to 90 per cent, with a power factor approaching 90 per cent.

Mercury arc rectifiers are made in a variety of sizes and are used for charging storage batteries, for arc lighting purposes, and for supplying current to moving picture equipment. For arc lighting use they are made in capacities which will supply current to as

many as one hundred lamps; units which are designed for charging storage batteries in telephone and telegraph plants will deliver D.C. pressures up to 320 volts. It is predicted by some engineers that the mercury arc rectifier will in the near future play an important part in heavy power service such as the operation of railway systems.

"Kenotrons" and "Rectons."—When a high potential direct current is required for any purpose it is now customary to utilize a two-electrode tube rectifying organization which depends for its operation on a pure electronic discharge. Units of this character have already been referred to in Sec. 142. It only remains to point out that tubes used for this purpose must be entirely free from gas. This is accomplished by long continued pumping, during the last stages of which the metallic elements within the tube are thoroughly heated and also subject to an electric discharge for the purpose of detaching any occluded or adherent gas molecules. The current passed by such tubes is comparatively small, seldom reaching one ampere, but the pressure may be as high as 100,000 volts. In the larger tubes the heat developed at the anode by the impact of the electrons is, in certain types, dissipated by means of circulating water. Such tubes are known under the trade names of kenotrons and rectons.

144. Three-Electrode Tubes.—Certain discoveries or achievements in the realm of science stand out preeminently as marking the beginning of new epochs in the world's progress. Such was the invention of the magnetic telegraph by Morse, the telephone by Bell, the discovery of radioactivity by Becquerel, electromagnetic waves by Hertz, and x-rays by Röntgen. The production of the three-electrode thermionic tube by Dr. Lee de Forest, and called by him the *audion*, ranks in importance with the achievements just mentioned. Indeed it is probably not overstating the case to say that this device ranks with the telescope, the microscope, and the spectroscope as a scientific tool, and this quite apart from its extremely wide practical engineering applications.

The three-electrode tube, or triode as it is sometimes called, differs from the corresponding two-electrode unit in that a "control" electrode has been introduced between the cathode and the anode, the control member usually taking the form of a wire grid surrounding the cathode. Let us now consider the function

of this so-called grid or control element. The part played by the grid will be understood by reference to Fig. 182. Let us assume that we have a three-electrode tube (audion) so evacuated that practically no residual gas is present. Suppose the anode battery B gives an E.M.F. of 100 volts. With the filament F heated and the grid G at zero potential the current which flows will depend upon the internal resistance of the tube (plate-to-filament) and any resistance existing in the external circuit. If now a negative potential be applied to the grid by connecting a source of E.M.F. between the grid and filament at D the movement of the electrons to the anode will tend to be inhibited because of the repelling force existing between the negatively charged grid and the electron themselves. As a result the so-called "plate current" as indicated by the ammeter A will be diminished. If the negative potential of the grid is made higher the space current will be still further decreased. Indeed it is usually possible to give the control member a negative potential such that the plate current will be zero.

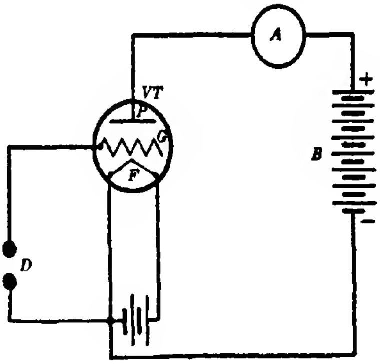


FIG. 182

It is also to be noted that if a positive potential be applied to the grid the movement of the electrons toward the anode will be increased with the result that the anode current will be increased.

If we plot grid potential against anode current for a given plate potential a graph is secured which is known as the static characteristic curve of the tube. A family of such curves is shown in Fig. 183. By referring to these curves it will be noted that a change of 5 volts in the grid voltage on either side of zero (ten volts total) will, at an anode voltage of 50, produce a change of 8 milliamperes in the plate current, or 0.8 ma. per volt. Now if the grid potential be held constant, say at zero, and the anode potential changed from 50 volts to 100 volts the plate current will change approximately 6 ma., thus giving 0.12 ma. per volt. Thus we see that a change in potential of one volt in the grid produces a change in the plate 6.6 times as great as does a change

of one volt in anode potential. In other words one volt applied to the grid is equivalent to 6.6 volts applied to the plate. Algebraically this may be expressed thus,

$$V_g = kV_p,$$

where k is the "amplification" factor. The ratio V_g/V_p, representing as it does the relative effects of grid and plate potential on the anode current, is known as the *amplification constant* of the tube. The value of this constant for any given tube depends upon the spatial relations of the mechanical parts of the tube.

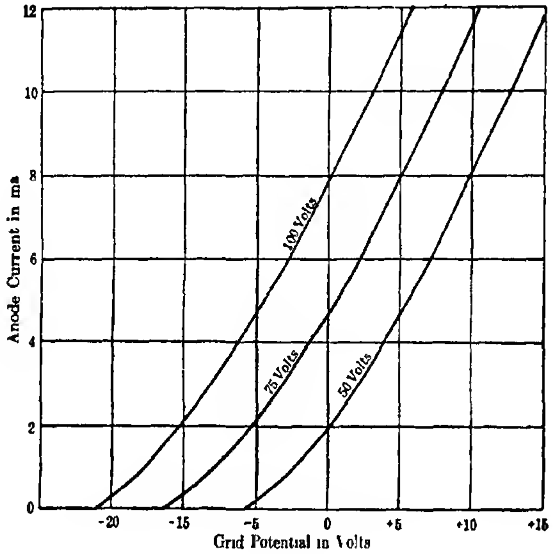


FIG. 183

The spacing of the grid wires and the distance of the grid from the filament are the principal controlling factors; the finer the grid mesh and the nearer to the filament the higher the amplification constant. The amplifying factor is to some extent a function of the anode voltage. Three-electrode tubes are designed having constants ranging from 2 to 40.

145. The Three-Electrode Tube as an Amplifier.—From the discussion of the preceding section it is evident that we have in the audion a device whereby a relatively small change in potential may be caused to control a comparatively large amount of energy. It is this property of the triode which is responsible for many of

its important uses, particularly in connection with the reproduction and amplification of variable potentials. When the three-electrode tube is used for such a purpose it is frequently designated as a *thermionic or audion amplifier*. The manner in which such amplification may be effected in practice is shown diagrammatically in Fig. 184.

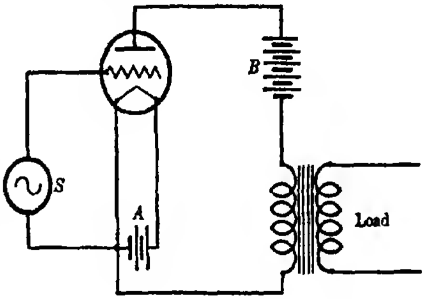


Fig. 184

If as shown in the diagram we apply to the grid of a tube arranged to be used as an amplifier an alternating potential there will, in effect, be superimposed on the constant anode potential an alternating potential whose value is given by the relation

$$V_g = kV_p,$$

where the factors have the significance previously indicated (Sec. 144).

The relation of the several factors involved is shown by the three curves of Fig. 185. Graph I is the grid potential-plate current curve showing the variation in anode current as a function of the grid potential. Curve II represents the variations in grid potential when a sinusoidal E.M.F. is applied between the filament and grid, while curve III depicts the corresponding changes in the anode current. If the variations in grid potential do not carry the operating point beyond the comparatively straight part of the characteristic curve (a to b) the plate current curve will be a replica of the variations in grid potential. In other words the anode current will be in

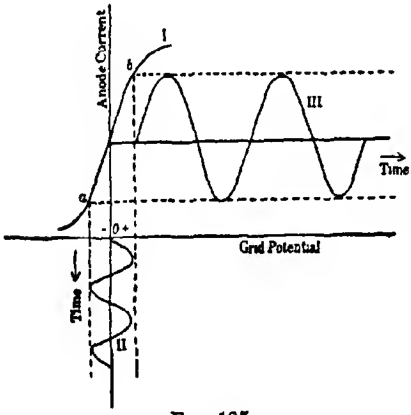


Fig. 185

synchronism with the grid potential as shown in Fig. 186. The amplitude of the alternating component in the anode circuit will be a function of the slope of the characteristic curve of the tube; the more steep the curve the greater the amplitude. The grid current supplied by the source S (Fig. 184) is extremely small and hence the power absorbed in the grid circuit is correspondingly small. The alternating current in the anode circuit is however relatively large; hence it is possible to secure relatively large power amplification.

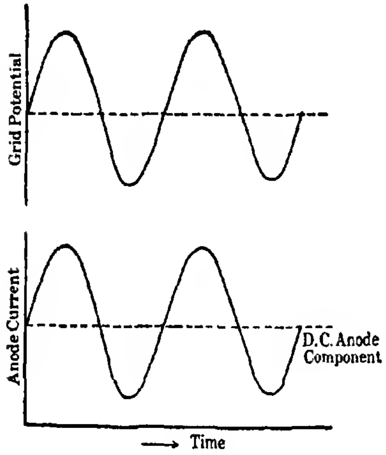


FIG. 186

The operating conditions may be so arranged that one may secure either potential amplification or power amplification. It may be shown that to obtain *maximum potential amplification* the external impedance (load circuit) should be as high as possible, theoretically infinite. In order to secure *maximum power amplification* the load impedance should be equal to the internal impedance (plate-to-filament) of the tube.

By means of a suitable transformer T (Fig. 184) the output of a tube may be made to supply the grid circuit of a second unit and thus additional amplification secured. Frequently several such "stages" or "steps" are employed.

We thus have in the thermionic tube a relay or "repeater," the moving element of which is an electron and therefore of negligible inertia; such a device is capable of releasing energy from a local source. When used under proper operating conditions the thermionic tube will give faithful reproduction of the wave form of the applied E.M.F. at any frequency, and is in practical use for frequencies ranging from a few cycles per second to 8×10^8.

It has been shown by De Forest, Armstrong and others that the degree of amplification obtained by a single tube may be increased several fold by feeding back to the grid a part of the out-

put from the plate circuit, thus giving rise to what is called *regenerative amplification.* If the alternating component of the anode current passes through a coil L_p (Fig. 187) which is so related to L_g that its magnetic field will tend to establish an E.M.F. in L_g in phase with the original E.M.F. in that circuit, the result will be an increased difference of potential between the grid and filament and hence an increase in the amplitude of the alternating current in the plate circuit.

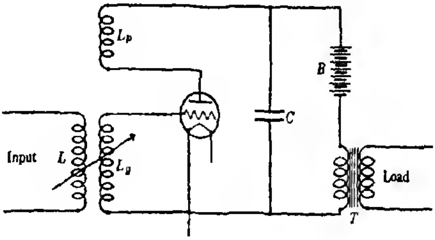


Fig. 187

A very slight energy transfer from the anode to the grid circuit is sufficient to decidedly increase the effect of the original applied E.M.F. and thus secure greatly increased amplification.

146. The Three-Electrode Tube as a Generator of Alternating Currents.—If the regenerative amplification, as outlined in the last section, be increased by bringing about a closer inductive relation between the anode and grid inductances the tube may become a generator of alternating current. The discussion which follows will serve to give the student a general understanding of the theory involved in the production of high frequency alternating currents by means of an electron tube. The explanation is only approximate, but is correct as to the main facts and will serve present purposes.

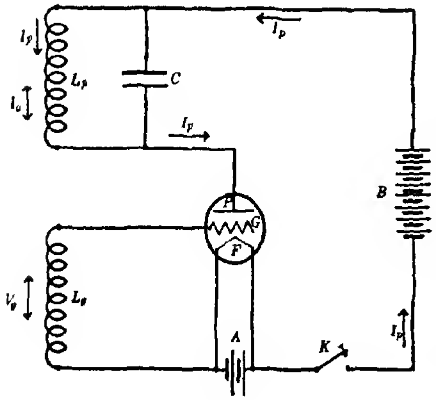


Fig. 188

Referring to Fig. 188, the inductance units L_p and L_g are so placed with respect to one another that any inductive effect due to a change in a current in one (say L_p) will give rise to an E.M.F. in the other. In addition the two coils are wound in the same

direction. These two coils constitute in fact a transformer, L_p being the primary.

For convenience in analysis let us suppose that the grid is at zero potential with respect to the filament, and that the filament is burning. Upon closing the anode (plate) circuit by means of k the current through L_p will change from zero to some value, I_p, which will be determined by the total resistance of the anode circuit. This pulse or change in anode current from zero to a fixed value I_p gives rise to a changing self-induced E.M.F. at the terminals of L_p. This varying E.M.F. applied to the terminals of the condenser C initiates a minute oscillating current in the circuit L_pC. The frequency of these oscillations will be largely fixed by the constants L_p and C.

Now since L_p and L_g are inductively related or "coupled" the small alternating current in the circuit L_pC passing, as it does, through the inductance L_p gives rise by induction to an alternating E.M.F. in L_g. There is developed, therefore, on the grid a varying potential whose magnitude will depend, other factors being held constant, upon the value of the coupling (mutual inductance) between L_p and L_g. From what has already been said about the characteristics of the electron tube, it is evident that this varying grid potential will give rise to a corresponding variation in plate current, I_p. These variations in the plate current will, under suitable conditions, on passing through L_p, tend to increase the amplitude of the h.f. alternating current already initiated in the circuit L_pC. Energy is thus transferred from the plate circuit to the grid circuit, which in turn acts to release additional energy in the anode circuit. Thus an alternating current is established in the anode circuit and the tube will continue to act as a generator.

In order to make the conditions of operation more clearly understood it is well to examine the phase relation which obtains in the case. The E.M.F. developed in L_p because of the initial pulse, and subsequent cyclic variation, in the anode current will be 90° in advance of the varying plate current.* The circuit L_pC being non-reactive at the resonant frequency, the high frequency alternating current established in that circuit will be in phase with the E.M.F. Through the transformer action of this oscil-

* See previous discussion of self and mutual induction (Secs. 106–110); also phase relations in connection with alternating currents (Chap. XVIII).

lating current on the grid inductance the induced E.M.F. in the coil will be 90° behind the oscillating current in the plate coil, and hence in phase with the fluctuating anode current. It should be remembered in this connection that the alternating potential V_g developed between the grid and the filament is equivalent to an alternating E.M.F., whose value is KV_g, impressed between the anode and filament, K being the amplification constant of the tube. It is a property of the electron tube that, when the external plate circuit contains a resistance, and the tube is functioning as an oscillator (or as an amplifier), the tube has an apparent "negative resistance." That is, as the anode potential

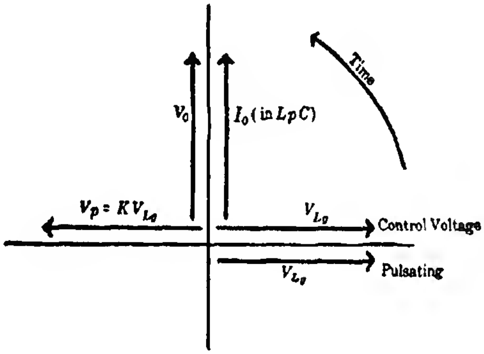


Fig. 189

increases the anode current will decrease, and vice versa. Thus the anode potential V_p is 180° ahead of the alternating component of the anode current I_p. These phase relations are delineated in Figs. 189 * and 190; I_0 and V_0 represent the high frequency or "oscillating" current and potential respectively. These diagrams should be very carefully examined as they depict graphically the theory of the audion as a generator of alternating currents.

It is important to note the conditions which determine the *amplitude* of high frequency oscillations which are set up in L_pC, as outlined above. If the alternating anode potential KV_g is smaller than V_0 the oscillations will decrease in amplitude, the oscillating current eventually ceasing. If KV_g is equal to V_0 the oscillations will just be sustained. If KV_g is greater than V_0 the oscillations will increase in amplitude and in turn produce a

* In Fig. 189 the length of the vectors has no significance.

greater grid potential, V_g, hence giving rise to a cumulative effect. There will be a limit, however, to the amplitude of the oscillations for the reason that the fluctuating plate current can neither be less than zero nor greater than the saturation values.

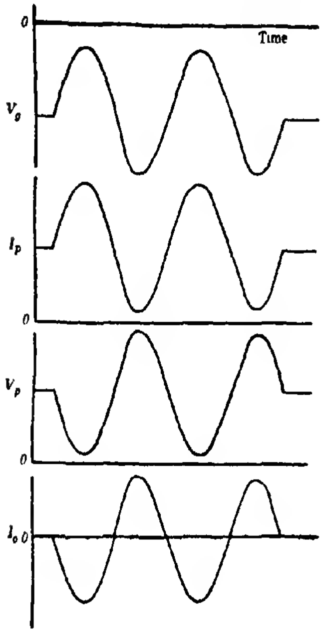


FIG. 190

Another way of dealing with this aspect of the case is to consider the *energy* involved in the process. If the energy supplied by the pulsating component of the anode current is *less than* that necessary to overcome the ohmic resistance of the circuit L_pC the oscillations will be dampened and die out. If the energy supplied by I_p is equal to the energy required to overcome the resistance of L_pC the oscillations will increase in amplitude until limited by the factors mentioned above. If for the purpose of utilizing the oscillations established in L_pC this circuit is coupled to an absorbing or "load" circuit the pulsating anode current must, in addition to the energy necessary to compensate for the resistance of L_pC as explained above, also supply an amount of energy equal to that absorbed by the "load" or "work" circuit. It will be evident that the energy thus required is supplied by the anode battery B.

It may be noted in passing that the oscillating current I_0 in the circuit L_pC, or in the load circuit, may be many times larger than the alternating current component of the anode or plate circuit. The reason for this will be apparent when it is observed that the internal anode resistance (plate-to-filament within the tube) is of the order of several thousand ohms, while the resistance of the circuit L_pC, and most load circuits, will not exceed a few ohms. The I^2R relation holds in this case as elsewhere.

In the electron tube we thus have available a device which is capable of producing an *alternating current of a definite frequency*, and this by means which are entirely static mechanically. The

frequency limits of the audion generator are very wide, ranging from a few cycles per second to above 6×10^7.

Owing to the fact that the grid potential-plate current curve of an electron tube is, in general, not a straight line, the wave form of the high frequency alternating current developed in the oscillating circuit will not be a pure sine curve, but will contain various harmonics. These harmonics can, nevertheless, be eliminated by the use of suitable filter circuits (Sec. 121). Their existence, however, tends to decrease the over-all power efficiency of the electron tube organization as a generator of alternating current. As the tube is commonly used the efficiency of transformations (D.C. to A.C. power) is of the order of 50 per cent, though a higher efficiency may, under certain circumstances, be secured.

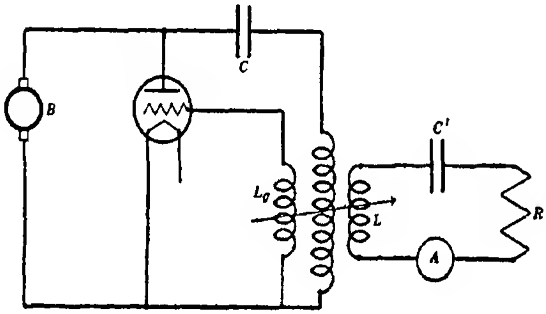


Fig. 191

In practice the load or absorbing circuit is usually inductively coupled to the anode circuit of the electron tube, though direct coupling is sometimes utilized. The load circuit may also be electrostatically coupled to the high frequency power generating circuit. Figure 191 shows one of the many circuits which may be arranged for the production of high frequency alternating current power, and the method of coupling to a load circuit. In the particular organization illustrated the D.C. anode supply is in parallel with the tube and its associated grid and plate circuit. As a generator the circuit, however, functions in the same general manner as the organization shown in Fig. 188. The load circuit consists of the coupling inductance L, the capacitance C' and a resistance. The condenser, C', represents the capacitance of an antenna system when the generator is employed for radio purposes (Chapter XXV), or it may represent the capacitance of a phys-

ical telephone line when the generator is utilized for guided wave (carrier current) telephony or telegraphy. The resistance *R* represents the effective resistance of any load circuit.

Electron power tubes are now made in sizes ranging from a few watts to several kilowatts output. The smaller tubes operate at an anode potential of a few hundred volts, while some of the high power units require a pressure as high as 12,000 volts.

In order to secure adequate electronic emission in tubes in which a pure tungsten filament is employed the filament current must necessarily be relatively high. In a certain foreign tube of this type rated to deliver 10 kw. the filament current is 47 amperes at 28 volts, and the electronic plate current is of the order of 3 amperes. Figure 192 shows a group of power tubes ranging in size from a few watts to one kilowatt.

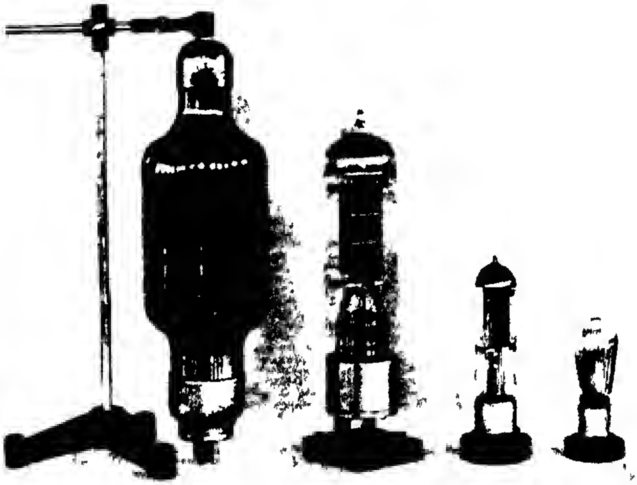


FIG. 192.—AIR-COOLED THERMIONIC POWER TUBES, RANGING FROM 5 TO 1,000 WATTS OUTPUT

As previously pointed out (Sec. 142), by using a material for the filament which contains a substance such as thorium, or by coating the filament with certain oxids, such as calcium or barium, the emission may be greatly increased, and the same electronic emissions may be secured with a current value of less than half that necessary with a pure tungsten filament.

In the operation of power tubes having an output of several kilowatts the anode itself becomes very hot and hence provision must be made for dissipating considerable thermal energy. For a given input to a tube the anode thermal dissipation is of course less when the tube is delivering high frequency energy to a load circuit than when oscillations do not obtain. Tubes are frequently rated on the basis of anode dissipation. By this is meant the power, in watts or kilowatts, that the tube is capable of dissipating at the anode in the form of heat when the t ube is functioning in an oscillating circuit.

(Courtesy Mullard Tube Co.)

FIG. 193.—QUARTZ THERMONIC TUBE

(Courtesy Western Electric Co.)

FIG. 194.—WATER-COOLED THERMIONIC TUBE

Various means are utilized for the purpose of dissipating the heat thus developed at the anode. In certain units the tube itself is made of silica, the anode being placed close to the quartz wall, silica being capable of withstanding very high temperatures without cracking or fusing. Such a unit is shown in Fig. 193. In other types of high power tubes the anode consists of a hollow metallic cylinder about which water is caused to circulate. This latter form of tube is common in this country. Its construction has been made possible as the result of the recent development of a method of sealing copper to glass. A water-cooled electron tube is shown in Fig. 194.

147. The Audion as a Detector.—In radio communication (Chapter XXV) it becomes necessary to, in some way, change the *frequency* of the carrier * wave to a value which may be sensed by the ear. This may be brought about in various ways but the method almost universally employed at the present time utilizes a three-electrode tube. The functioning of the audion in this capacity may be understood by reference to Fig. 195*a* and *b*.

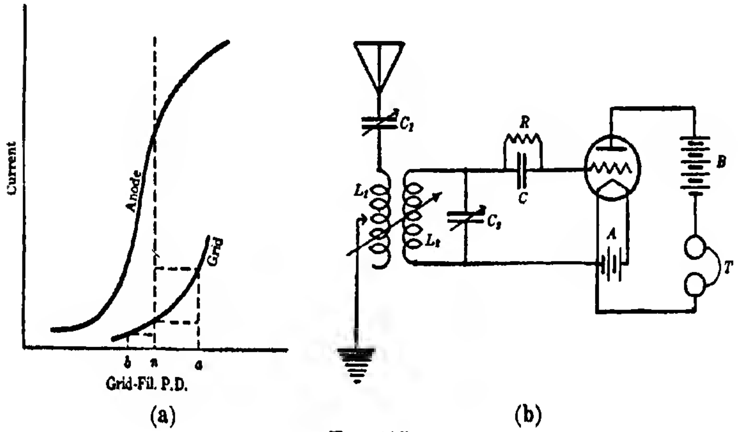


Fig. 195

From an examination of the grid current-grid potential curve in Fig. 195*a* it will be evident that a change in grid potential represented by *n–a* will cause a *greater* increase in grid current than an equal, but negative, change *n–b*. Any alternating E.M.F. established in the circuit L_2C_2 (Fig. 195*b*) will produce, through the condenser *C*, a corresponding cyclic change in the potential difference between the grid and filament of the tube. During that part of each cycle when the grid is positive with respect to the filament, additional electrons will pass from the filament to the grid, and this grid charge cannot escape except through the very high resistance *R*, which has a value of 1 to 10 megohms. During the succeeding half cycle the normal electronic current to the grid will decrease but by an amount less than the corresponding increase during the previous positive half cycle. During succeeding positive half cycles, then, the grid will acquire a negative charge, the magnitude of this charge depending upon the ampli-

* The term "carrier" indicates the high frequency alternating current which in various ways is modulated (changed in amplitude) for the purpose of signaling or conveying speech.

tude of the incident oscillations. The mean potential of the grid will hence be lowered. This decrease in grid potential will in turn decrease the anode (plate) current, as long as the oscillations persist, with the result that a sound will be heard in the telephone receiver. Since the action of the tube is thus to integrate the effect of the individual oscillations, the note or sound heard in the receiver will correspond to the envelope of the graph representing the exciting oscillations. This will be evident from an examination of the graphs shown in Fig. 196. The "grid leak" resistance R serves to allow the accumulated negative grid charge to escape from the grid just fast enough so that the tube will not "block" (reduce plate current to zero) due to excessive negative grid potential. If the negative charge could not escape from the grid between groups of oscillations, or gradually, in the case of a continuously modulated wave, the tube would not be in a condition to respond to succeeding oscillations. From what has been said it will be evident that working conditions should be so arranged that the tube will be operating on that part of the grid current curve where the curvature is greatest. With the tubes now in common use, this condition will obtain if the grid return is connected to the positive side of the battery supplying the filament. The characteristics of present "receiving" tubes are such that a "grid condenser" (C, Fig. 195b) of about 0.00025 mf. capacitance, and a grid leak resistance R of 2–5 megohms, give optimum results.

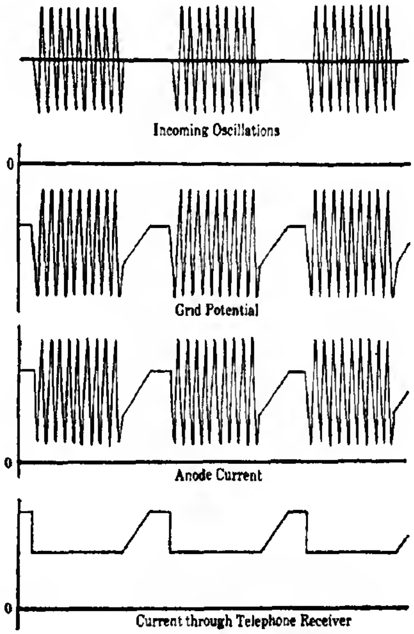


Fig. 196

148. The Electron Tube as an Oscillograph.—In the study of alternating currents it is important to be able to observe, either

visually or photographically, the wave form of both the pressure and the current. There are various ways of accomplishing this but a method which has proven useful, particularly in high frequency work, makes use of the electronic discharge in vacuo. Since a stream of electrons constitutes an electric current and hence *if these electrons be free, the stream may be deflected by* either an electrostatic or a magnetic field. Due to the extremely small mass of an electron, the response of such a stream of electrical entities to a deflecting force is, for all practical purposes, instantaneous. Hence it will follow a cyclic deflecting force, which may have a frequency of the order of several million per second. If an electronic stream impinges on certain chemical

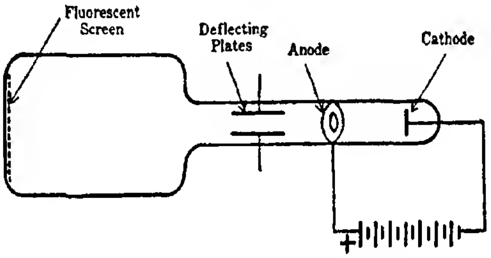


Fig. 197

substances, such for instance as willemite, a screen so prepared *will fluoresce and thus the position of the stream may be visually* observed and studied. Dr. F. Braun in 1897 was probably the first to make use of a cathode ray tube for oscillographic purposes.

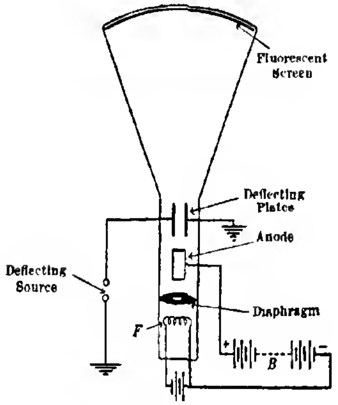


Fig. 198

In the original Braun tube (as it has come to be called) the electron stream was developed by applying an exciting and accelerating potential of the order of 10,000–50,000 volts between the cathode and anode. Electrons were liberated from the residual gas atoms by impact of the few free electrons moving under the action of the high potential gradient. The resulting electrons bombarded the metal cathode and liberated additional

electrons. A diagrammatic sketch of the original type of Braun tube is shown in Fig. 197.

The disadvantage of the high excitation potential required for the operation of the original Braun form of oscillograph has been eliminated in a new type of tube which utilizes a heated filament as a source of electrons. With the new model the operating conditions are considerably more stable and a greater sensitivity is attained. An oscillographic tube of this character has recently been developed along lines suggested by Dr. Hull of the Bureau of Standards.* The construction of this type of tube is shown diagrammatically in Fig. 198, and a photograph of an actual tube is shown in Fig. 199.

(Courtesy Western Electric Co.)

FIG. 199.—CATHODE RAY OSCILLOGRAPH TUBE

The electrons liberated from the heated filament (in this case oxid coated), under the action of the field existing between the filament and the anode, pass through the metallic diaphragm and along the axis of the cylindrical anode to the fluorescent screen at the large end of the tube. The diameter of the aperture in the diaphragm, and of the tubular anode, is of the order of one mm.; hence the fluorescent spot on the sensitive screen is of very small dimensions and, under proper operating conditions, is sharp and bright enough to be photographed as it moves in response to deflecting forces. Two sets of deflecting plates are provided (only one pair is shown in the diagram), the two pairs being at right angles to one another, thus making it possible to trace a curve in two dimensions. In order to deflect the cathode beam by means of an electromagnetic field, two small coils may be placed one on either side of the narrow part of the tube in the region of the plates. One of each of the pairs of plates is con-

* See Bulletin, Radio Laboratory, Bureau of Standards, March 1, 1920.

nected to the anode and earthed. This is for the purpose of removing the charge which would otherwise collect on the plates and cause the spot on the screen to drift. All connections are brought out through the base, a special vacuum tube socket being used.

The hot cathode oscillograph tube may be used in the study of a variety of alternating current problems, and is particularly adapted to serve as a tool in the examination of high frequency phenomena. It has the disadvantage that the life of the tube is comparatively short. When this feature has been improved, it is probable that the hot cathode oscillograph will come into extended use in research and engineering laboratories.

CHAPTER XXIII

RONTGEN RAYS

149. Cathode Rays.—In the preceding chapter we have dealt with the various means by which electrons may be detached from atoms, and utilized for certain purposes. We now come to a study of a series of phenomena which have their genesis in the movement of electrons at high velocities. We have already seen (Sec. 143) that electrons may be detached from a substance if the potential of the body is made sufficiently high with respect to its surroundings. The electrons so separated may, under the action of a suitable potential gradient, be caused to move with a definite velocity. If such an electronic movement takes place between electrodes in a vacuum of the order of 1/100 mm. pressure, the stream of electrons, or "cathode rays," * issuing from the negative terminal, possesses certain well-defined and highly important properties. The control of such an electronic stream by means of an electrostatic field has been discussed in connection with the description of the three-electrode tube (Sec. 144). Since the cathode stream is made up of electrons, it constitutes an electric current and hence gives rise to a magnetic field. The cathode rays may therefore be deflected by an *exterior* magnetic field. Indeed it is by the deflection of the rays by means of a magnetic and an electrostatic field that the ratio of the mass of the electrical entities constituting the stream to their charge has been determined. Sir J. J. Thomson has found this ratio to be 1.3×10^{-7}. Since we know the value of the electronic charge to be 4.774×10^{-10} e.s.u., it is apparent that the mass of the electron must be 9.01×10^{-28} gram, which turns out to be 1/1845 of the mass of the hydrogen atom. There is some evidence for believing that the mass is not a fixed quantity but is a function of the velocity at which the electron is moving. The magnitude of the charge however is constant and is the same as that carried by the hydro-

* The term "rays" in this connection is more or less of a misnomer but has become well established in the phraseology of the subject. In other connections "ray" has the connotation of a wave motion. In this relation however we are not dealing with a periodic disturbance in the ether.

gen atom in electrolytic processes. *Both the mass and the charge are independent of the nature of the cathode and of the nature of any residual gas in the tube.* The velocity of the moving electrons is a function of the potential gradient in the tube and may reach a value as high as one-third of the velocity of light. The electrons leave the cathode at right angles to its surface and their natural trajectories are straight lines. The moving electrons constituting the cathode stream possess kinetic energy and hence produce heating effects when they impinge upon any surface. The rays also have the property of exciting fluorescence in certain bodies, notably in barium platino-cyanide, willemite, zincblende, and kunzite.

By arranging a thin aluminum "window" in the wall of a vacuum tube Lenard in 1894 showed that the cathode rays can be projected outside the tube in which they originate, and that they retain their characteristic properties when passing through the air. The name Lenard Rays is sometimes given to that part of the cathode stream which exists outside the tube in which it originates. Recently Dr. W. Coolidge has succeeded in securing a copious supply of Lenard rays from a thermionic tube constructed for that special purpose. Preliminary experiments carried out by Dr. Coolidge and others indicate that Lenard rays when produced by modern methods exhibit very curious and striking properties.

Sir Wm. Crookes studied the characteristics of the cathode rays within the tube, but it remained for Professor Wilhelm Konrad Röntgen of Worzburg, Bavaria, in 1895 to discover another and by far the most important property of cathode rays. While working with a Crookes tube which was enclosed in heavy black paper, Professor Röntgen noticed that a sensitive screen which was lying on a table about three meters away fluoresced brightly. Investigation disclosed the fact that the source of the unknown radiation was the area on the glass wall of the tube which was bombarded by the cathode stream. The exact nature of the newly discovered rays being unknown, Röntgen designated them *x-rays*. However they have since come to be called *Röntgen rays*, after their discoverer.

150. Röntgen or X-rays.—Shortly after the discovery of the new form of radiation it was found that x-rays originate wherever cathode rays encounter matter. As they impinge upon a body a

part of the energy of the swiftly moving electrons is converted into heat and the remainder manifests itself in the form of a disturbance in the ether which we know as x-rays. The original tubes utilized for the production of Röntgen rays were essentially modified Crookes tubes and consisted of a concave cathode of aluminum and a target or "anti-cathode" of platinum or tungsten, as shown in Fig. 200. If there be an appreciable amount of residual gas in the tube, the supply of electrons is relatively great with the result that a comparatively small difference of potential between the electrodes will cause a large current to pass through the tube. Under these conditions the speed of the electrons will be slow and the resulting x-rays will have small penetrating power. A tube operating under such conditions is said to yield "soft" radiation. If there be little residual gas in the tube (high vacuum), there will be fewer electrons and hence a higher potential will be required to produce a given current through the tube. Under these conditions the electronic velocity will be relatively high and the x-radiation resulting from the bombardment of the anti-cathode will accordingly be more penetrating. When these conditions obtain, the bulb is said to be "hard" and the radiation "hard" rays. Rontgen's original observations disclosed the fact that x-rays pass through matter which is opaque to ordinary light and ultraviolet radiation, while practically all glass and other mineral matter, except soda glass, is more or less opaque to the rays. The rays are strongly actinic as shown by their effect on a photographic plate and their ability to excite fluorescence in certain chemical bodies. Röntgen rays also produce marked ionizing effects (Sec. 139), this property having been extensively utilized in recent investigations connected with study of the constitution of matter, and in the study of crystalline structure. The utilization of the ultrapenetrating power of Röntgen radiation in connection with surgery is too well known to need further description. X-rays are also used extensively in connection with the testing of metallic and other

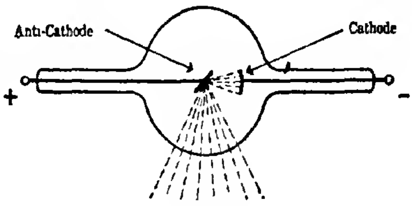


FIG. 200

materials used in the manufacturing industries.* Figure 201 shows a typical x-ray shadow picture or *skiagraph*. Röntgen rays possess certain therapeutic properties and are utilized in the treatment of a number of superficial pathological conditions of the skin. The rays will produce severe burns if allowed to act upon

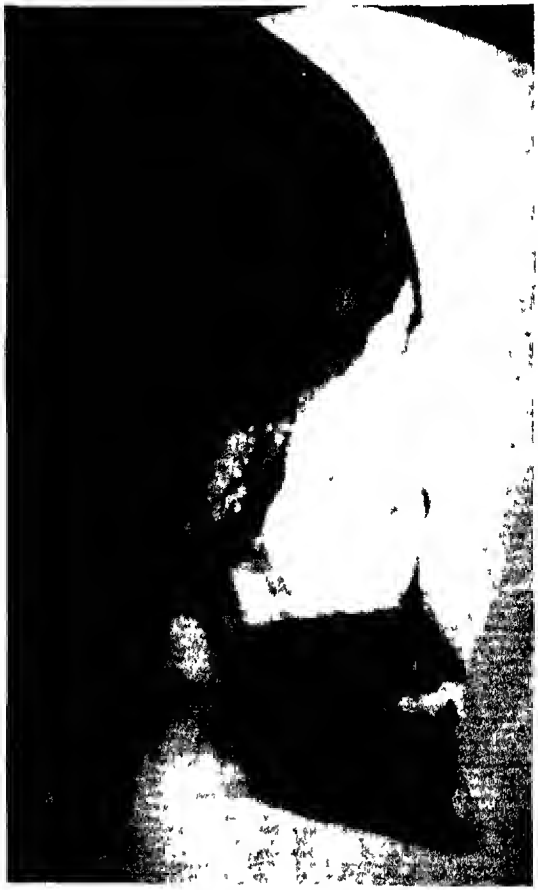


FIG. 201.—SKIAGRAPH OF HUMAN HEAD

the tissues for any great length of time; hence x-ray operators must be protected from direct radiation from the tube by means of metallic lead or lead glass. In the early days of research in this field a number of workers unfortunately lost their lives as a result of x-ray burns.

* See *Applied X-rays*, by Prof. G. L. Clark; also *X-rays, Past and Present*, by V. E. Pullin and W. J. Wiltshire.

As indicated above, the penetrating power of the rays is a function of the potential gradient between the tube electrodes, which in turn depends upon the degree of vacuum in the tube; the higher the vacuum the more penetrating the radiation. In the older type of tubes the vacuum was a variable quantity and hence the photographic and treatment results were apt to be uncertain.

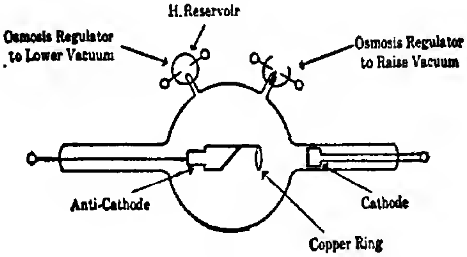


FIG. 202

Various attempts were made to design a tube in which the vacuum would be automatically regulated. The most successful tube of this type was designed by Dr. N. Clyde Snook. In the Snook tube the residual gas is pure hydrogen and the vacuum is automatically regulated by the transfer of hydrogen through the walls of two regulating tubes, one of platinum and the other of palladium, sealed into the wall of the x-ray tube proper. When heated by the discharge hydrogen is admitted to or rejected from the tube. Figure 202 is a diagrammatic sketch of the Snook tube.

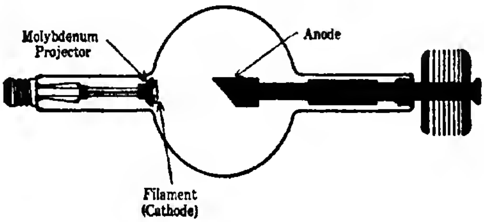


FIG. 203

A tube designed by Dr. W. Coolidge is however rapidly displacing other types of x-ray generating units and its advent, in fact, may be said to mark a distinct advance in x-ray technique. The Coolidge tube (Fig. 203) is a thermionic device, the source of electrons being a spiral of tungsten heated from an auxiliary source of current. The tungsten filament is surrounded by, and

electrically connected to, a molybdenum tube which serves to focus the electronic stream on the anticathode with the result that the point of impact of the cathode stream does not wander or change in size. The vacuum in the tube is extremely high, the amount of residual gas being exceedingly small. As a result of this condition, the vacuum of the tube does not change during the operation and hence the intensity of the radiation is constant. The intensity of the x-ray output in this type of tube is entirely under the control of the operator, being regulated by the temperature of the electrically heated cathode or filament. Coolidge tubes are now made in various capacities, varying from relatively small units employed in dental work to large tubes capable of handling several kilowatts. In the larger units heavy water-cooled anticathodes are employed. Since the Coolidge tube is a thermionic device, it possesses unilateral conductivity and hence the necessary alternating potential is commonly supplied by a specially designed transformer, and in the larger tubes may be as high as 100,000 volts. Because of the relatively large current-carrying capacity of the modern tube and the use of intensifying screens, it is now possible to make instantaneous skiagraphs of the vital organs of the human body. An intensifying screen consists of a fluorescent surface having as an active chemical compound such a material as tungstate of calcium. Such a screen fluoresces with a highly actinic bluish light when subjected to x-rays and in use is placed in close contact with the film of the photographic plate. By the use of such a screen the exposure necessary to secure satisfactory skiagraphs may be reduced to one-twentieth of the time formerly required.

151. Nature of X-rays.—Thus far in our discussion the question of the nature of Röntgen rays has purposely been held in abeyance. It is now in order to raise this interesting and important query and to examine the evidence leading to modern conclusions about the character of this form of radiation.

Following the discovery of the new form of radiation various suggestions were advanced as to the nature of the dark Röntgen rays. We will not review these in detail but only note in passing that the tentative theory which was given most serious consideration at the time was that due to Stokes and was to the effect that x-rays consist of irregular pulses in the ether. Owing to the fact that, until a comparatively recent date, investigators had been

unable to reflect or refract x-rays, it was extremely difficult to arrive at a satisfactory explanation of their nature.

Proceeding on the assumption that Röntgen rays were similar to light waves but of extremely short wave length, Professor Max Laue * of the University of Zurich suggested that the regular arrangement of the atoms in a crystal might possibly serve as a "grating" for use in producing interference effects with x-rays, and thus yield data from which it would be possible to accurately determine the magnitude of the wave length. According to this

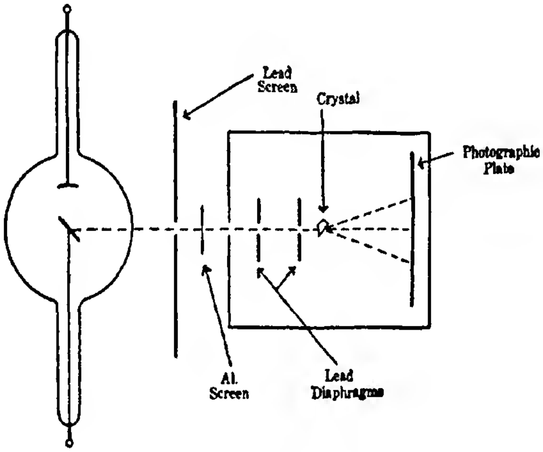


Fig. 204

suggestion the spacings between the atoms in the crystal were to serve as the grating spaces. Dr. Laue investigated the problem mathematically and formulated laws which pointed to the fact that one should be able to secure an interference pattern. Knipping and Friedrich † in 1912 carried out the necessary experiments to test Laue's analytical results. These experiments were brilliantly successful and clearly confirmed Laue's predictions, and thus established the fact that x-rays are in no respect different from visible light except in the magnitude of their wave length. These and other experiments show that Röntgen rays lie between 10^{-7} cm. and 10^{-9} cm. in wave length. The arrangement of apparatus necessary for the production of a Laue diffraction photograph is indicated in Fig. 204; Fig. 205 is a reproduction of

* M. Laue, *Phys. Zeitschr.*, 14, p. 421, 1913.

† W. Friedrich, P. Knipping, and M. Laue, *Le radium*, 10, p. 47, 1913.

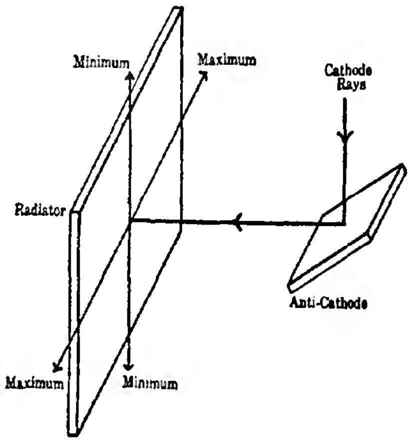
a typical photograph made by the use of a crystal "grating." The central spot is due to the rays transmitted directly through the crystal while the spots surrounding this central area are due to the interference caused by the atoms and interatomic spaces acting as a transmission grating. Each spot in the pattern represents the diffraction of the incident x-rays by a certain plane of the crystal structure. By means of an instrument known as an x-ray spectrometer (Sec. 152) it has been possible to expend the work of Knipping and Friedrich in such a manner that photographic records of x-ray spectra may be readily made. Not only is it possible to produce interference effects with x-ray, but we now know that the rays may be polarized and refracted as is the case with visible radiation. In fact the study of x-rays has now come to be considered a branch of optics. Incidentally the diffraction of x-rays by crystals has also led to a knowledge of the mechanical arrangement of the atoms in crystalline structures. For a complete discussion of the last mentioned subject, the student is referred to *X-rays and Crystal Structure*, by W. H. Bragg and W. L. Bragg.

FIG. 205.—DIFFRACTION PHOTOGRAPH MADE BY X-RAYS PASSING THROUGH A CRYSTAL OF SODIUM CHLORIDE

152. Characteristic X-rays.—In general when x-rays are incident upon a substance three effects are produced; there is a certain amount of scattering, or what might be called "diffused reflection," a secondary radiation is developed, and electrons are liberated from the substance. The relative magnitude of the several effects depends on the nature of the substance and the quality of the primary or exciting radiation. We are not, for our present purposes, particularly concerned with the scattering effect except to note that a study of scattered x-rays by Barkla (1905) disclosed the fact that both the scattered and the primary radiation is polarized. The relation of the plane of polarization to the

direction of the original cathode beam is shown in Fig. 206, which is a reproduction of a diagram due to Kaye.*

In general the radiation from a fairly hard x-ray tube will be more or less heterogeneous in character, that is, it will consist of a mixture of various wave lengths. Barkla and Sadler, however, in 1908 made the highly important discovery that many substances (probably all) may be caused to emit a homogeneous radiation which is *characteristic of the particular substance.* This characteristic x-radiation may be produced by bombarding the substance with electrons or with primary x-rays. If x-rays are utilized as the means of stimulation, the characteristic rays will be "softer" (less penetrating) than those constituting the primary beam. The hardness of the characteristic radiation is a function of the atomic weight of the element being stimulated; the higher the atomic weight, the more penetrating the rays. It has been found by Barkla and others that many elements give out two principal characteristic radiations. These two types of radiation have been designated as the K series and L series, the former being perhaps 300 times more penetrating than the latter. Both the K and L series of x-radiation increase in hardness as the atomic weight of the substance increases.



(Courtesy Longmans, Green & Co.)
FIG. 206

The X-ray Spectrometer.—Proceeding along the line originally suggested by Laue it is possible to utilize crystalline structures as reflection gratings and thus assemble an x-ray spectrometer. If x-rays are reflected from crystals having well-defined cleavage planes, the direction of the reflected radiation will be a function of the wave length of the incident x-radiation. One is thus able to produce for examination a spectrum of Röntgen rays. Bragg and Bragg devised an x-ray spectrometer which has been exten-

* *X-rays*, Kaye.

sively used by them and others in the study of x-ray spectra, particularly in the investigation of characteristic radiation. A diagrammatic sketch of the Bragg apparatus is shown in Fig. 207. The x-ray spectrometer is quite similar to the corresponding optical instrument. Indeed an ordinary spectrometer may be adapted for use in this connection. The crystal which is used as a reflection grating is mounted on the prism table by means of soft wax. The observing telescope is replaced by an ionization

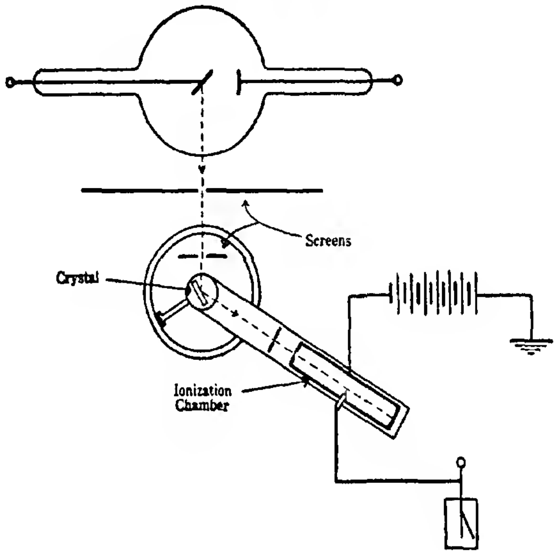


FIG. 207

chamber, if the electrical method of observation is to be used, or by an arrangement for carrying a photographic plate, if the photographic method is employed. The x-ray tube is inclosed in a lead-lined box and one or more lead slits serve to give a well-defined pencil of rays incident on the crystal as shown. In order to secure a stable beam it has been found desirable to utilize a tube in which the anticathode is normal to the incident cathode beam and make use of the x-radiation which leaves the target at nearly grazing angle. In the electrical method the ionization chamber is usually filled with a gas such as SO_2 which strongly absorbs the radiation and thus yields a relatively large ionization current. The electroscope, being connected to the insulated electrode projecting into the chamber, will show a change in deflection when the exploring chamber (or the crystal) is moved to a position where

reflected radiation obtains. The spectrum lines may thus be located and plotted against angular displacement as read from the attached vernier.

Moseley's Experiments.—Utilizing the x-ray spectrometer and the photographic method Moseley * (1913–1914) carried out a very comprehensive study of a number of elements and uncovered some highly important facts. The elements to be examined were mounted as anticathodes and the characteristic radiation was examined by means of a potassium ferrocyanide crystal. Figure 208 is a reproduction of one of Moseley's illustrations and shows the x-ray spectra of a number of the elements examined. It will be noted that the wave length varies inversely as the atomic weight. It is also to be observed that each spectrum consists of two lines, the one of the lower frequency being the more intense (*K* characteristic radiation). The *L* radiation consists of several lines. Recent study has shown that the *K* lines are doublets. The wave lengths of the *K* lines for iron are 1.946 and 1.765 $\times 10^{-8}$ cm. The most significant aspect, however, of Moseley's work

(Courtesy Phil. Mag.)

FIG. 208.—PHOTOGRAPH OF X-RAY SPECTRA, BY MOSLEY

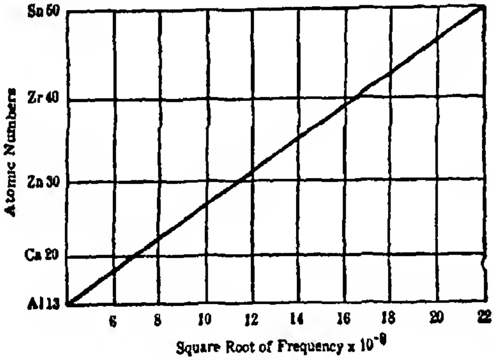


FIG. 209

* *Phil. Mag.*, Dec. 1913, April 1914.

was his discovery of the fact that the *square root of the frequency of the x-radiation varies directly as the atomic number* (order of the element in the periodic table). Figure 209 shows this important relation. As a result of Moseley's work it is now possible to arrange the elements on an atomic number-frequency graph, and by so doing it becomes evident that atomic numbers are of the utmost significance. These numbers represent the number of electrons in an atom which are not held within the nucleus. It will thus be seen that x-ray spectroscopy is yielding valuable data bearing on the constitution of matter.

153. Positive Rays.—Before leaving the study of electronic currents in vacuo the phenomenon of the so-called positive "rays" should be mentioned. As early as 1886 Goldstein * discovered that, in addition to the cathode "rays," there also exists a stream of bodies which manifest a positive charge and which move in a direction opposite to that of the cathode stream, that is, *toward* the cathode. Goldstein investigated these rays by means of a specially designed discharge tube having a cathode perforated with small round holes, and an enlarged region "behind" the cathode. It was found that positive entities made their appearance a short distance from the cathode in the region of the cathode dark space, and moved toward the cathode, passing through the perforations and into the region behind. That these bodies are positively charged is shown by the direction of the deflection when either an electrostatic or magnetic field is applied. The magnetic deflection is however very slight with field strengths which would produce marked effects on the cathode stream. The positive stream appears to consist of positive ions resulting from the ionizing effect of the electrons ejected from the cathode. Goldstein called this positive stream "*Kanalstrahlm*" or "*Canal rays*." Like the cathode stream, the positive rays travel in straight lines and cause fluorescence, the fluorescence being of a different color than in the case of cathode rays. They do not however give rise to x-rays. They cause the residual gas to emit visible radiation but of a different color than that produced by the cathode stream. For instance, if the residual gas in a tube be neon, it will exhibit a beautiful red color in that region through which the positive rays pass and a pale blue in that part of the tube traversed by the cathode stream. W. Wein,† employing a

* Goldstein, *Berl. Sitz. Ber.*, V. 39, p. 691, 1886. *Wied. Ann.*, V. 54, p. 38, 1898.
† W. Wein, *Wied. Ann.*, V. 65, p. 440, 1898.

combined electrostatic and magnetic field, determined the value of the velocity of the entities which go to make up the positive stream and also the ratio of their mass to the charge $\left(\frac{m}{e}\right)$. He found that the velocity is of the order of 1/100 (about 3.6

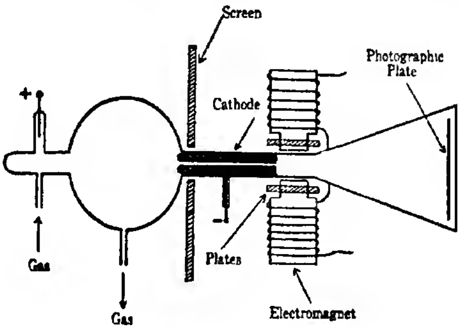


FIG. 210

$\times 10^{-7}$ cm. per sec.) that of the cathode rays, and that the ratic $\frac{m}{e}$ might have various values, the mean being of the order of 1.3×10^{-3}. It is thus evident that the bodies which make up the positive stream have much greater mass than the electrons composing the cathode rays.


(Courtesy Royal Society)
FIG. 211.—TYPICAL POSITIVE RAY PHOTOGRAPH

Sir J. J. Thomson * has made an extensive study of positive rays and found that the nature of at least certain of the positive rays depends upon the residual gas in the discharge tube. Thomson originally utilized the fluorescent effects but later employed a photographic method of investigation, the set-up of his apparatus being shown diagrammatically in Fig. 210, the photographic plate occupying a position at the enlarged end of the conical part of the tube at the right of the illustration. The positive rays pass through a tube of very small bore inserted in the cathode and the beam of these positive ions is simultan-

* Sir J. J. Thomson, *Rays of Positive Electricity.*

eously subjected to an electrostatic and a magnetic field, the two fields having the same direction.

Figure 211 is a reproduction of a typical Thomson photograph. It will be seen that the curves made by the impact on the photographic plate of the positively charged particles take the form of a series of parabolas. An analysis of these curves shows that a given curve is due to a particle having a certain value of $\frac{m}{e}$ and therefore probably corresponds to a definite and particular kind of particle. Professor Thomson studied the parabolic curves corresponding to many ions, including those of carbon, oxygen, neon and mercury. He was thus able to determine the atomic weights of these elements, and in fact discovered several hitherto unknown substances. It was found that only an exceedingly small quantity of gas is needed in order to carry out such investigations; much less than commonly figures in chemical analytical methods. It will be evident that there is thus made available a new and extremely sensitive method of chemical analysis. It is probable that further study of the positive rays will disclose other and useful properties.

CHAPTER XXIV

RADIOACTIVITY

154. Becquerel Rays.—The discovery of radioactivity in 1896 had its genesis in the discovery of x-rays. Since the cathode rays caused various substances to fluoresce and had been found to give rise to x-rays the question naturally arose as to whether fluorescent substances, of which a number are known, might not also prove to be a source of x-radiation. Becquerel * in studying this question, by great good fortune, examined among other substances a number of the compounds of the element uranium. His method was a photographic one and consisted in inclosing the

FIG. 212.—AUTO-SKIAGRAPH MADE BY URANIUM ORE. (Note the Shadow Cast by a Copper Coin.)

substance to be examined in light-proof paper and placing it immediately adjacent to the film of a photograph plate for a period of several days. A metal coin was placed between the fluorescent substance and the film of the plate. Upon developing the plate it was evident that some form of radiation had emanated from the uranium compound, and formed a shadow picture on the plate.

* Henry Becquerel of the Conservatoire des Arts et Métiers is the son of Edmond Becquerel and the grandson of A. C. Becquerel, both of whom were eminent physicists.

Figure 212 is a reproduction of a skiagraph made according to Becquerel's method.

In addition to the photographic effects, Becquerel found that the radiation from uranium causes fluorescence, and also gives rise to ionization. Becquerel checked his results in various ways and found that the *uranium,* per se, was the essential substance in the production of the observed effects, and what is even more important he learned that the previous condition or treatment of the uranium had no bearing on the result. Though on the wrong track so far as the discovery of a possible source of x-rays in fluorescent bodies was concerned, he had however, as a result of careful research, been led to uncover a tremendously important truth. Becquerel had in fact made a discovery which marked the beginning of an epoch in the advancement of scientific thought; a discovery the results of which were destined to profoundly modify certain fundamental concepts in the domain of both physics and chemistry.

As soon as Becquerel announced his findings, other investigators promptly and vigorously took up the search for other bodies which might possibly possess the recently discovered property exhibited by uranium, a property which soon came to be designated by the term *radioactivity.*

FIG. 213—AUTO-SKIAGRAPH OF A GAS MANTLE

155. Radioactivity of Thorium.—In 1898 Schmidt * in searching for other radioactive bodies discovered that the element

* G. Schmidt, *Wied. Ann.*, V. 65, p. 141, 1898.

thorium also possesses this property, and to about the same degree as uranium. The radioactivity of thorium can be quite easily demonstrated by the photographic method. Compounds of thorium are used in the manufacture of Welsbach gas mantles. If a mantle (one which has been burned for a time will answer) is laid on a photographic plate, which has been wrapped in light-proof paper and put away for a week or more, it will be found, upon developing the plate, that the radioactive emission has produced an appreciable effect. Figure 213 is an auto-skiagraph showing the result of such a test. We shall have occasion to refer again to the radioactivity of both uranium and thorium.

156. Discovery of Radium.—Among the workers who were studying in the new field mention may now be made of M. and Mme. Curie, who had begun a series of laborious but brilliant experimental researches. The Curies proved that the radioactivity of uranium salts is directly proportional to the uranium content, thus establishing the fact that the radioactivity of uranium is an atomic property.

As a result of an examination of the natural minerals from which uranium is derived, the Curies found that certain specimens of pitchblende were more radioactive than could be accounted for on the basis of their uranium content. It was therefore natural to conclude that the ore contained another substance which was strongly radioactive. The Curies accordingly subjected a quantity of radioactive pitchblende to a systematic chemical analysis in an attempt to isolate this unknown radioactive body. Barium is present in pitchblende and it was found on precipitating that element as the carbonate that the precipitate was strongly radioactive. It was however found to be impossible to make any further separation by means of reagents and recourse was had to the method of fractional crystallization. The barium was converted to the chloride and, as the fractional crystallization progressed, those parts which separated out first proved to be more and more radioactive. By a long and extremely laborious procedure M. and Mme. Curie were thus able to secure a concentrated radioactive product whose activity was of the order of a million times the activity of uranium. The end product of the analysis was a chemically pure body, which proved to be a new element, which they called *Radium.* It is interesting to note in passing that it was necessary to chemically treat several hundred

kilograms of pitchblende ore in order to secure a few milligrams of the element radium. Radium was found to be an element having properties closely resembling those of barium, and having an atomic weight of 226.4. It is commonly handled in the form of the bromide (RBr_2) or the chloride (RCl_2). Its radioactive properties will be discussed later.

157. Method of Measuring Radioactivity.—Before proceeding further it will be well to glance at the means employed in the estimation of the degree of radioactivity which a given substance exhibits.

It has already been noted that radioactive bodies produce photographic effects, give rise to fluorescence and bring about ionization. The last-mentioned effect may be utilized in making quantitative measurements of radioactivity. M. and Mme. Curie employed the ionizing property of the substance under test as a measure of its radioactivity and made use of the gold leaf electroscope as an indicator of the degree of ionization. Figure 214 is a diagrammatic sketch of the apparatus used by the Curies in their original investigation. If the radioactive material be spread on the plate P, the electroscope first having been charged to a given potential, the air between the plates P and P' will be ionized and the electroscope gradually discharged; the rate of discharge will be a function of the degree of radioactivity. The rate of movement of the gold leaf is observed by means of a suitable optical arrangement. Various other forms of the gold leaf electroscope have been devised for use in connection with radioactive investigations, one designed by Wilson * and improved by Kaye † being particularly sensitive and convenient. A modern form of an electroscope of this type is shown in Fig. 215. The movement of the leaf is observed by means of an attached reading microscope in the eyepiece of which is a transparent scale.

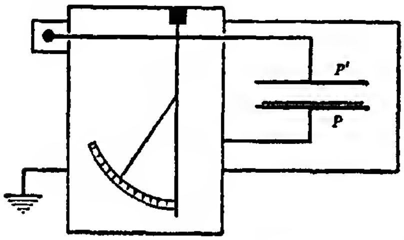


FIG. 214

The quadrant electrometer (Sec. 21) is also used for the precise

* C. T. R. Wilson, *Camb. Phil. Soc. Proc.*, V. 12, p. 135, 1903.

† G. W. C. Kaye, *Proc. Phys. Soc.*, V. 23, p. 209, 1911.

measurement of the ionization produced by radioactive materials. A typical circuit arrangement is indicated in Fig. 216. A well-

(Courtesy Central Scientific Co.)

FIG. 215.—THE LIND ELECTROSCOPE USED AT THE UNITED STATES BUREAU OF MINES

insulated chamber is utilized, the plate P, carrying the material to be tested, being connected to one terminal of a battery of, say,

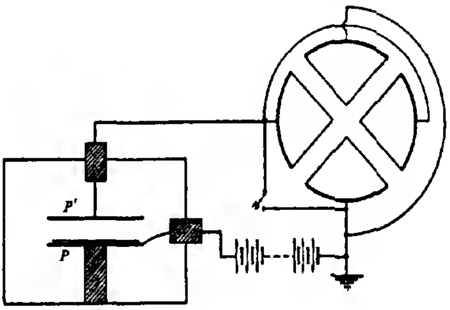


FIG. 216

100 volts pressure; the plate P' is connected to one pair of quadrants, while the other set of quadrants and the other battery

terminal are grounded. Ionization in the region between the plates in the chamber will cause the electrometer quadrants to acquire a charge. The rate at which this charge accumulates may be observed by noting the deflection of the electrometer during a given interval of time, this giving an index of the degree of radioactivity. In order to secure dependable results a number of experimental precautions must be observed. For a detailed account of the technique of radioactive measurements, the student is referred to *Practical Measurements in Radioactivity*, by Makower and Griger.

158. The Radioactivity of Polonium and Actinium.—In working on the preparation of radium from pitchblende Mme. Curie was successful in separating from the antimony-bismuth chemical group a second radioactive body which is several times more active than uranium and which she named *Polonium*, after her native country, Poland. In more recent years polonium has been found to be identical with RaF, which later will be shown to be one of the disintegration products of radium. Debierne * also succeeded in obtaining from the iron group in pitchblende still another radioactive substance known as *Actinium*. The radioactivity of actinium is comparable to that of radium.

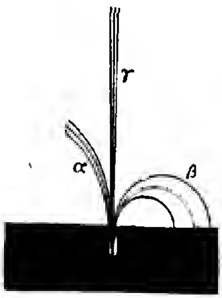


FIG. 217

159. The Nature of Radioactivity.—In general, there are three types of "rays" emitted from a radioactive body, being, for reference purposes, designated by Rutherford as α, β, γ rays. Let us now consider the nature of each type of emission, seriatim.

If a quantity of strongly radioactive substance be placed in a lead block, as shown in Fig. 217, and a strong magnetic field be brought to bear on the emission, the direction of the field being at right angles to the plane of the figure, the rays will be separated into three parts as shown, and the several groups of rays can be separately detected and identified. A part of the emission is strongly deflected and the direction of deflection indicates that the entities which go to make up the beam of radiation manifest a negative charge. These are the so-called beta (β) rays.

Another part of the radiation is deflected to a lesser degree by

* A. Debierne, *Comptes rendus*, V. 30, p. 906, 1900.

the magnetic field, and the direction of movement shows that the bodies composing this part of the radiation carry a positive charge. They are the alpha (α) rays.

A third portion is undeflected and shows no evidence of an electric charge. These are the gamma (γ) rays.

Without going into detail concerning the method whereby the data have been secured, the properties and nature of the three types of rays may be summarized as follows:

Alpha Rays.—They are the least penetrating of the three types, the penetrating power being proportional to the cube of their velocity. The velocity depends upon the source and lies between 15×10^8 and 20×10^8 cm./sec. The "range" in air of α-rays emanating from radium is about 7.5 cm. Even those of highest velocity are completely stopped by an aluminum screen 0.06 mm. in thickness. The α-rays produce marked ionization, being the most effective in this respect of the three types of emission. They also give rise to fluorescence, particularly in the diamond and in zinc sulphide. The α-entities show a positive charge whose magnitude is *twice* that of the charge carried by the hydrogen ion in electrolysis.

Alpha rays are now known to be positive ions of the element helium, the original atom having lost two electrons. The energy possessed by the α-rays is very much greater than in the case of the positive rays in a vacuum tube. The kinetic energy of a moving α-particle is sufficient to disrupt an occasional atom of certain elements, and it has been utilized as a "projectile" for the purpose of stripping atoms of one or more electrons and thus "transmuting" certain elements into others.

Beta Rays.—This part of the radiation from radioactive bodies is something of the order of 100 times more penetrating than the α-rays. They give rise to fluorescence when incident upon platino-barium cyanide and certain other substances. *The β-rays are identical with the cathode rays in a vacuum tube and consist of electrons*, that is, free entities or "pieces" of negative electricity, the magnitude of the charge being 1.59×10^{-19} coulomb. The velocity of the β-rays is greater than that of the cathode rays, being of the order of 10^{10} to 3×10^{10} cm./sec. Experiment has shown that the mass of the entity making up the β-rays is a function of their velocity, increasing with increasing velocity. For a relatively slowly moving β-ray entity, the mass is $\frac{1}{1845}$

of the mass of the hydrogen atom, which is the same as that of the electrons forming the cathode stream in a vacuum tube.

Gamma Rays.—The γ-radiation is the only part to which the term rays strictly applies. It has been established that the γ-radiation is essentially the same as x-rays, the only difference being that the γ-rays are very much more penetrating than x-rays. An eighth of an inch of metallic lead will stop the hardest x-rays, but *γ-rays will pass through a foot of the metal.* This part of the radiation from radioactive bodies is, in short, a periodic electromagnetic disturbance in the ether, which, except in point of frequency, is the same as visible light. It is important to note that γ and β-rays always occur together while α-rays may exist alone. Gamma rays produce marked ionization and also affect a photographic plate.

160. Radioactive Transformations and Their Significance.—That changes are taking place in radioactive bodies, particularly in the case of radium, is evidenced by the fact that radium compounds are always at a higher temperature than their surroundings, thus indicating that thermal energy is constantly being spontaneously liberated. A portion of radium compound may be as much as 2° C. above the temperature of its surroundings. It has been estimated that one gram of radium liberates heat at the rate of approximately 130 gram-cal. per hour. This is sufficient heat to raise 130 grams of water 1° C. and, so far as observations have gone, this liberation of heat has shown no appreciable diminution. This appears to be an anomalous phenomenon, but is found to be explicable in the light of results secured in recent investigations, as we shall see shortly.

Basing their reasoning on a large amount of experimental data, Rutherford and Soddy several years ago brought forward a theory which is to the effect that the radioactive elements are in a more or less unstable condition, the atoms undergoing a process of disintegration. This disintegration or transformation process follows a perfectly definite law. It has been found that radioactive elements of higher atomic weight, as the result of the spontaneous emission of α-, β-, and γ-rays, change to other radioactive elements of lower atomic weight. There are three distinct series of such changes, these groups being commonly referred to as the uranium series, the actinium series, and the thorium series. The uranium series involves radium and may be taken as typical.

The following diagram (Fig. 218) serves to give a graphic representation of the transformation steps beginning with uranium and ending with one of the isotopes (Sec. 162) of lead.

From an examination of the chart it will be seen that uranium is the original parent body of radium and that a form of lead, known as radium-lead, is the stable end product of the complete series. Whenever an α-particle (positive helium ion) is expelled from a given element the atomic weight of the transformation product is less by four than its parent body, four being the atomic weight of helium. Radium itself comes into being when an ionium atom expels a single alpha entity, that is,

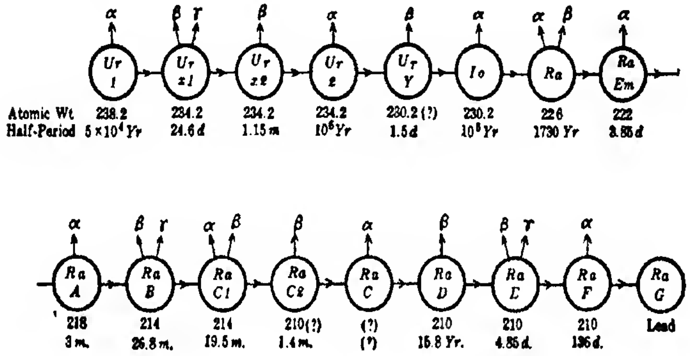


Fig. 218

a helium ion. In common with its atomic "ancestors," radium itself is an unstable element and as a result of its atomic disintegration a helium ion (α-ray) is expelled and an electron (β-ray) liberated. The remainder of the atomic "wreck" constitutes an atom of gas, sometimes spoken of as *niton*, but more commonly referred to as *radium emanation*. The element niton is a relatively heavy inert gas which boils at 65° C.; it may be liquified at 150° C. A gaseous element corresponding to niton is also evolved as one of the steps in the transformation which occurs in both the actinium and thorium series. We have cited the particular case of the radioactive elements radium and its emanation, niton, and the product helium, in order to call attention to what may be considered as a *transformation*, or, as it would have been called in the days of the alchemists, *transmutation*, of certain elements. And perhaps the most remarkable thing about this strange transformation process is that it is

entirely beyond our control; it proceeds quite independently of local physical and chemical conditions. In such a process of atomic transformation we have a change which transcends hitherto known chemical laws. In fact radioactive changes constitute an ultraatomic phenomenon.

It should be noted that these radioactive changes follow perfectly definite laws. The rate of disintegration of any radioactive element may be expressed by the relation

$$M_t = M_0 e^{-\lambda t},$$

where M_0 represents the initial mass of radioactive material, M_t the mass existing t seconds later, e the base of the Napierian logarithms, and λ the transformations constant for that particular material, that is, the proportion of active material which undergoes change each second. Since the above relation is an exponential expression, it has been found convenient to deal with what is known as the half-period of transformation, that is, the time required for one half of the particular body to be transformed. If we modify the above expression to make it cover this half-period concept it becomes

$$\frac{M_0}{2} = M_0 e^{-\lambda T},$$

where T is the time required for one half of the substance to be transformed. This reduces to

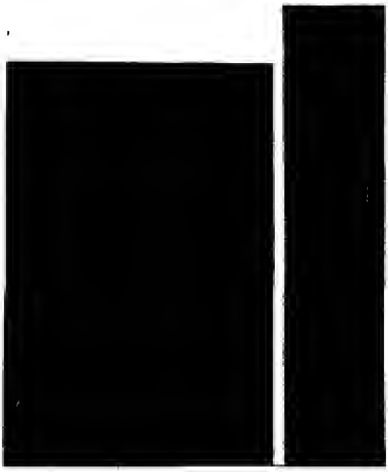
$$T = \frac{\log e^2}{\lambda},$$

or

$$T = \frac{.6931}{\lambda} \qquad \text{and} \qquad \lambda = \frac{.6931}{T}.$$

The values of the half-period, T, are indicated for each radioactive body in the uranium series shown in Fig. 218. The half-period for radium has been determined to be about 1700 years, while for niton (radium emanation) it is only 3.86 days. To put the case differently, a quantity of radium will not depreciate more than 4 per cent in a hundred years. The fact that the disintegration process is so slow in the case of radium explains why the evolution of heat by a radium salt has not shown any appreciable diminution during the period of time over which observations have thus far been made.

161. Artificial Transformation of Elements.—Notwithstanding the fact that we cannot change the rate of disintegration in the case of the known radioactive elements, it has been found possible to artificially decompose, on a very small scale, a number of the nonradioactive elements. The method by which this has been accomplished is both interesting and significant. It was observed by Sir Ernest Rutherford that when a group of α-particles move through a gas or through a metal foil a few of them are sharply deflected at a point in their path. Such deviations from an otherwise straight path are illustrated in Fig. 219, reproduced from a paper by Prof. C. T. R. Wilson. An examination of the evidence has led to the conclusion that most of the particles pass directly through the atomic structure without encountering any of the constitutent parts of the atom. However, an occasional α-particle apparently passes near enough to the nucleus of an atom of the substance through which the α-particles are moving to be affected by the repelling electrostatic force due to the charge on the nucleus. In fact there is evidence which points to the conclusion that an α-particle does occasionally actually strike a nucleus and as a result a part of the nucleus is detached and moves out of the atom with a velocity comparable to the exciting α-particle. Alpha particles may thus be utilized as high velocity projectiles with which to break up certain nuclei. But the most remarkable and significant aspect of this unique process is the fact that, in the cases examined, *the detached part of the atomic nucleus proves in all cases to be a hydrogen nucleus.* Rutherford * and Chadwick have found that "*hydrogen nuclei* are



(Courtesy Royal Society)

Fig. 219.—Photograph of Alpha Ray Tracks

* "The Electrical Structure of Matter," Sir Ernest Rutherford, *Science*, Sept. 21, 1923. This paper gives an excellent résumé of the development up to that date. The student is urged to read the entire article.

released from the elements boron, nitrogen, fluorine, sodium, aluminum, and phosphorus, when they are bombarded by swift α-particles, and there is little room for doubt that these hydrogen nuclei form an essential part of the nuclear structure." Thus far hydrogen nuclei have only been liberated from elements having *odd atomic numbers*, a fact which may in time prove to be significant. In the same paper from which the above quotation is taken Rutherford remarks that "these experiments suggest that *the hydrogen nucleus or proton must be one of the fundamental units which build up a nucleus,* and it seems highly probable that *the helium nucleus is a secondary building unit composed of the very close union of four protons and two electrons.*" (The italics are the author's.) Other evidence also tends to confirm Rutherford's conclusion and to show that all atoms are composed of protons and electrons, that is, of positive and negative units of electricity.

It is interesting to note in passing that Dr. Prout, an English physician, in 1815 suggested that hydrogen was the basic material from which all elements are made. The facts outlined above indicate that Prout's hypothesis was essentially correct, although it was rejected when originally suggested.

162. Isotopes.—It will be recalled that the so-called positive rays (Sec. 153) consist of a stream of positive gas ions travelling in a direction opposite to that of the cathode stream. It has already been pointed out that the ions which make up this stream consist of atoms or groups of atoms each of which has lost one of its outer electrons. Hence these ions contain the nucleus and most of the electrons of the atoms involved. Mention has been made of the fact that these positive "rays" may be utilized for the purpose of identifying gases which exist in extremely small quantities. When studying the gas neon by the positive ray method (magnetic deflection) Sir J. J. Thomson found that this gas may exist in two quite distinct forms. One of these forms has an atomic weight of 20 while that of the other is 22. The atomic weight when determined by chemical means is 20.2. Both forms of the gas exhibit the same chemical properties. Since Thomson's original discovery of the fact that an element may exist in two distinct forms, F. W. Aston,* also of the Cambridge Laboratory, by using an improved method has found that a

* *Isotopes*, F. W. Aston, 1922.

number of elements may exist in more than one form. For example, ordinary chemically pure magnesium is a mixture of atoms whose atomic weights are 24, 25 and 26; chlorine is a combination of a form of atomic weight 35 and one whose weight is 37; potassium, 39 and 41. Such forms of elements are called *isotopes.* It is significant that, except in the case of hydrogen, no isotopes have yet been identified having an atomic weight which is *not a whole number.* Since many of the atomic or combining weights as formerly determined are not whole numbers it has been suggested that the elements commonly dealt with in chemical reactions are mixtures of several isotopes, and that the combining weight of an element is the mean of the atomic weights of its isotopes.

But the aspect of isotopes which most concerns us here has had to do with the physical disposition of the electrical entities which go to make up the nucleus of an atom. It seems probable that isotopes represent different arrangements of the charges which constitute the nucleus. In the case of chlorine, for instance, the atoms of one isotope might possibly have a nucleus made up of 35 protons and 18 electrons with 17 "free" electrons in the outer structure of the atom, while the other isotope might have 37 protons and 20 associated electrons making up the nucleus with 17 electrons in the outer region.

While many of the chemical and physical properties of the isotopes of an element are the same, it has been found that the isotopes of certain elements may exhibit properties which differentiate them quite clearly from one another. This is particularly true of the isotopes of the radioactive elements and to some extent of the isotopes of other elements. It is possible that we will in time be able to separate the isotopes of an element, in substantial quantities, and that the several constituents of the element thus made available may prove to have properties which will be significant in the applied arts. As a result of the discovery of radioactivity, and the light it has shed on the constitution of matter, the time-honored definition of an element must be amended. Owing, however, to the rapidity with which new facts in this field are being uncovered, a new definition can not be easily formulated.

163. Atomic Structure and Radiation.—The subject of atomic structure is one of such compelling interest and is so pregnant

with possibilities that one is tempted to discuss such a theme at considerable length. Such a course would however lead us too far afield and hence we must limit ourselves to a brief résumé of the theories current at the time of writing concerning the nature of the sub-atomic world and the phenomena having their origin in that organization.

We have seen (Sec. 161) that the nucleus of the atom appears to be composed of elementary positive quantities of electricity called protons, and that this elementary positive charge is identical with the nucleus of the hydrogen atom; further, that the proton is equal in electrical magnitude to the electron. It has been pointed out that a number of electrons are usually "embedded" in the nucleus, thus neutralizing a part of the total protonic charge. The number of unneutralized protons in the nucleus determines the *net nuclear charge*, which in turn fixes the number of electrons which exist in orbital motion in the outer regions of the atom. The atom of hydrogen consists of a single elementary positive charge as a nucleus and one electron moving about it in orbital motion. The nucleus of the helium atom is composed of four protons (probably hydrogen nuclei) and two intimately associated electrons, leaving two electrons in the outer spaces of the atom structure. Lithium has 3 and uranium 92 orbital electrons. The atomic numbers, it will be recalled (Sec. 152), correspond to the number of electrons revolving about the nucleus. All of the physical and chemical properties of the atom appear to be determined by these outer electrons.

As a result of a central force action and the force of mutual repulsion between the several outer electrons these electrons assume a more or less stable "planetary" configuration about the nucleus. The arrangement of the electronic orbits and the manner in which the moving electrons receive and radiate energy have been and are subjects of much speculation and concentrated study. The question of how energy is received by, and radiated from, the atom is, obviously, of the utmost importance. It was originally thought that the electrons in their orbital motion gave rise to an electromagnetic disturbance in the ether in the form of radiant energy and that energy was thus radiated in a continuous "stream." It was believed that energy was also received in an uninterrupted flow. Further, it was held that the frequency of the radiated energy corresponded to the frequency of orbital

revolution. The electromagnetic theory of radiation as applied to electronic activity accounted in a very satisfactory manner for most of the then known phenomena. There was however one serious defect in the electromagnetic theory as applied to subatomic processes; it did not explain how a moving electron can continuously give out energy and still maintain the same orbital velocity and a fixed distance from the force center.

In 1901 Max Planck, an eminent German mathematical physicist, proposed a radically new theory of radiation. Planck assumed that the process of radiation consists in the giving out of small indivisible grains or units of energy which he called *quanta.* He maintained that the size of the quantum depends upon the frequency of radiation and that its magnitude is given by the product $h\nu$, where ν is the frequency of the emitted radiations and h *is an absolute constant now known as Planck's constant.* (The frequency of a radiation is of course the ratio of the velocity, 3×10^{10} cm./sec., to the wave length.) The numerical value of Planck's constant is 6.56×10^{-27} erg-seconds. The product, $h\nu$, that is, the amount of energy in a given quantum, may be expressed in ergs or in calories, and in all instances is of exceedingly small magnitude. For the radiation employed in radio communication (Ch. XXV) it has a value of the order of 6.554×10^{-22} ergs or 1.566×10^{-29} calories, while in the case of x-rays it is millions of times greater, being 1.966×10^{-8} ergs or 4.721×10^{-15} calories.

The quantum theory of radiation was originally proposed by Planck to account for the way in which the energy is distributed in the radiation spectrum of a black body, and while it is, in its fundamental concepts, a decided departure from the Maxwellian theory of radiation, it has now come to be more or less generally accepted. In fact Planck's constant is coming to be looked upon as one of the three or four basic constants of nature.

As implied above the quantum theory has been extended to include all radiation phenomena. The most recent and perhaps the most striking of all adaptations of this theory was made in 1913 by Dr. Niels Bohr, a Danish physicist. In order to more satisfactorily account for the radiation and reception of energy by an atom Bohr, by an ingenious but comparatively simple analytical process, applied the quantum theory to Rutherford's nuclear atom. He assumed an atomic structure of the

nuclear type as proposed by Rutherford and begins his argument with the hypothesis that the electron is *not* subject to the laws governing electromagnetic radiation.

As a result of the combined work of Balmer (1885), Rydberg (1890) and Ritz (1908) an equation exists by which the frequency of various series of spectrum lines can be predicted. For the case of hydrogen, which has the most simple atomic structure, this equation has the form

$$\nu = Rv\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right), \tag{i}$$

where ν is the frequency of the radiations giving rise to a given spectrum line, v the velocity of light and R a constant known as Rydberg's constant. The terms n_1 and n_2 may be a series of values expressed as whole numbers. Bohr multiplied both sides of this equation by Planck's constant and got

$$h\nu = \frac{Rhv}{n_1^2} - \frac{Rhv}{n_2^2}. \tag{ii}$$

It will be evident that the left hand side of this equation now represents a quantum of energy for the particular frequencies determined by the special values assigned to n_1 and n_2. In other words it is possible by this equation to compute the energy represented by each of the lines in the hydrogen spectrum. It will be noted that the right hand side of eq. (ii) consists of the difference of two energy terms. According to Bohr's interpretation of the analysis, each of these two terms represents a stationary energy state of the atom. In the light of this interpretation eq. (ii) might be written $h\nu = w_1 - w_2$. If the atom gives up energy in the form of radiation it will change from the energy state designated by w_2 to another energy state determined by w_1, and *in making this transition a quantum of energy equal to $h\nu$ will be liberated.* If the transition takes place in reverse order the atom is *receiving* energy from an outside source. In terms of electronic movements this means that the electrons of a given atom rotate in certain definite orbits and while so moving do not radiate energy. An electron may however suddenly change its orbit to a path nearer the nucleus and while this orbital change is taking place a definite quantum of energy is liberated in the form of radiant energy. If sufficient energy is passed from an outside source to

an electron it may abruptly shift from an orbit comparatively near the nucleus to a path at a greater distance from the force center. The energy resident in an atom is due to the energy of the electrons moving in their orbits, the energy contribution of any particular electron varying directly as the size of its orbit. In general the electronic orbits are supposed to be elliptical and the undisturbed motion of the electron is considered to be governed by the Newtonian laws of mechanics, the positive nuclear charge giving rise to the central force. By making certain justifiable assumptions as to the angular momentum of the electron as it moves in its orbit Bohr's theory will account for the hydrogen spectral lines; the theory has also been found to conform with known experimental facts in other connections. The Bohr theory however does not explain what happens as the electron jumps from one orbit to another, or why this transition occurs suddenly. But the *result* of the abrupt shifting from outer to inner orbits is, according to the theory, the giving out, in "pulses" as it were, of energy, this liberated energy taking the form of what we are accustomed to call light *waves*.

The Bohr-Rutherford atomic structure is undoubtedly more or less imperfect, but it is serving as a useful tentative working explanation, and can be modified as new data come to light. In fact other atomic models have been recently suggested, notably one first proposed by G. N. Lewis and later extended by Dr. I. Langmuir. Probably never in the history of all science has so extensive and concentrated attention been given to a single problem as is now being focused on the sub-atomic processes. Indeed a discovery which may prove to be of considerable fundamental importance has but recently been announced by Dr. C. J. Davisson and Dr. L. H. Gerner of the Bell Telephone Research Laboratories. These investigators have found that a stream of electrons in vacuo is diffracted by a crystal as if it were a train of waves. Undoubtedly discoveries of far reaching importance lie just ahead.

164. Cosmic Rays.—In addition to the invisible radiation described in this and the preceding chapters, there is another form of radiation which should be mentioned and this radiation appears to have its origin somewhere in the region beyond the earth's atmosphere. For a considerable period of time it has been known that the discharge of electrified bodies, particularly an electro-

scope, could not be wholly accounted for on the basis of the ionization due to known local causes, and repeated efforts have been made to discover the cause of what might be termed a residual ionization effect which is sometimes encountered. Rutherford and McLennan in 1903 first noticed that the rate of leakage of the charge from an electroscope in an airtight box could be decreased if the entire apparatus were inclosed in a metal case the walls of which were one cm. thick. It was thus made evident that the leakage of the charge was not due to faulty insulation but to ionization produced by radiation from an outside source. At first it was thought that this general ionization was due to the radioactive material contained in the earth's crust. In 1910–11 the experiments of a German physicist, Gockel, showed the source of radiation to be extraterrestrial. Gockel took an electroscope in a balloon to an altitude of 13,000 ft. and found that the ionization at that height was of about the same magnitude as on the earth's surface. Two other German investigators, Hell and Kohlhorster, in 1912–14 repeated the balloon experiments, going to a height of 5.6 miles (9 km.) and found the ionization to decrease slightly for the first two miles and then to increase, the maximum value being eight times what it was at the earth's surface. In 1922 Millikan and Bowen at Kelly Field near San Antonio, Texas, sent up captive balloons carrying recording electroscopes and found that the ionization was greater than at the surface but not as large as might have been expected from the German results. The Millikan test extended to a height of 15.6 km., or nearly 10 miles. Later Millikan and his co-workers made some observations on Pikes Peak near Denver, Colorado, and found a rather large ionization value. This, however, they ascribed, in part at least, to local causes.

Millikan and Cameron in the summer of 1925 conducted an extensive series of experiments at Muir Lake on Mt. Whitney at an elevation of 11,800 ft. above sea level. The purpose of the test was to determine the penetrating effectiveness of the cosmic radiation. The snow-fed lake was free from radioactive contamination and hence ionization from local causes was absent. Electroscopes were sunk in the lake to a depth of 45 ft. and it was found that the discharge rate decreased with depth. It was estimated that the atmosphere had an absorption equivalent to 23 ft. of water; hence the extraterrestrial radiation was capable

of penetrating 65 ft. of water. This would be the equivalent of six feet of lead. It may therefore be concluded that the cosmic radiation is more penetrating than gamma rays. It was found that the radiation being studied was present at all times of day and night and hence came from all directions. The rays appear to be non-homogeneous, and Millikan has estimated the wave lengths to lie between the limits of 0.0004 Å and 0.00067 Å, the longest being only 1/50 that of the hardest known gamma rays. The results secured at Muir Lake were checked at another snow-fed lake 300 miles distant and at an altitude 7000 ft. lower. More recently Professor Millikan has carried out similar experiments at high altitudes in the Andes Mountains of South America. The results of these later tests confirm the earlier observations.

In dealing with observations of this character ionization currents of the order of 10^{-15} amperes are involved, but currents of this magnitude can be measured with a fair degree of accuracy. There is therefore no doubt of the existence of a highly penetrating form of radiation, having its origin entirely outside of the earth and its atmosphere. As to the source of this radiation we as yet have no data on which to base conclusions, though there is some evidence which appears to indicate that this radiation may possibly be due to a nuclear change in which helium is being formed from hydrogen. *Where* such a possible atomic condensation process is going on is not known, though it may possibly be in the stars. In this connection it would be interesting to observe the radiation from different directions *simultaneously*.

CHAPTER XXV

ELECTROMAGNETIC WAVES AND SOME APPLICATIONS

165. Electromagnetic Radiation of Energy.—We have in previous chapters considered the electrostatic field and the magnetic field; consideration has also been given to the elements of alternating current theory. In our discussion thus far a transfer of energy has always been brought about by means of electronic displacement between the points in question. We now are to consider a process whereby energy may be transferred between the point where the original electronic movement takes place and other points far removed from this. We shall find that this end is accomplished by means of the combined effect of a moving electrostatic field and its concomitant magnetic field; in other words by means of electromagnetic waves. It would be beyond the scope of this book to give a complete analysis of the theory of the propagation of electromagnetic waves. However, an elementary review of the essentials involved is given in the following paragraphs.

To begin with it will be useful to set down several well-established facts in connection with electric and magnetic fields.

1st. An electric and a magnetic field represent potential energy, probably due to the strained condition of the ether.

2d. A moving electric field creates an accompanying magnetic field, and the direction of the latter is at right angles to the former and also to the direction of motion. Further the electric field shares half its energy with the magnetic field it creates.

3d. A magnetic field in motion gives rise to an electric field and the direction relations are as indicated above.

4th. Any electric or magnetic disturbance in the ether travels with the velocity of light.

As previously pointed out (Sec. 3) a stationary charge Q, Fig. 220*a*, will be surrounded by a radial electrostatic field as shown. If a constant current is flowing in the conductor YY' (Fig. 220*b*), as indicated, a magnetic field represented by the circles will obtain in the region of the conductor (Sec. 89). Simultaneously there will also exist a moving electrostatic field

as shown by such radial lines as XX'. If now the current flowing in the conductor be caused to decrease in value to zero both the magnetic and the electric fields will shrink and thus return the energy of their respective fields to the conductor. If however the

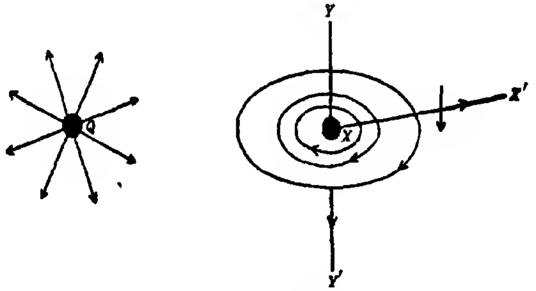


Fig. 220

direction of the current changes quite a different phenomenon presents itself. In this case the lines of electric force are distorted. If we fix our attention on a single electron and follow the behavior of a single electric line we may get an approximate picture of what happens.

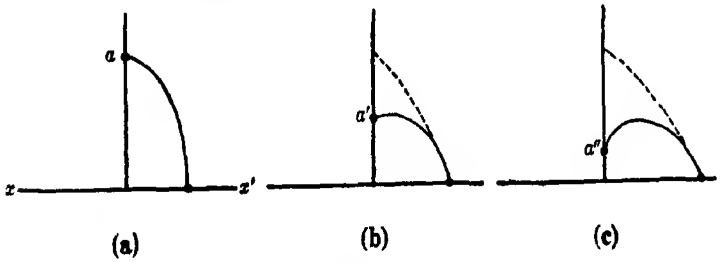


Fig. 221

Suppose an electron is moving rapidly back and forth between the point a and the conducting plane xx', Fig. 221, at the instant illustrated in (a). Electrostatic lines will exist between the electron and corresponding positive charges on the conducting plane, one such line being shown. If the electron moves to the position a' as shown in (b) the line of electric flux will be bent as shown. Still further displacement toward the plane xx' is shown in (c). The distortion of the ether caused by the electronic motion will spread outward from the source with the velocity of light. Since

we have assumed the to and fro movement of the charge to be of a high frequency *all of the energy resident in the electrostatic field cannot return to the source* before the electron is again moving through any given point in the same direction. Hence a part of the energy given up to the field never returns to the source but continues to move outward as a wave disturbance. At low (commercial) frequencies most of the energy in the field has time to return to the source before reversal occurs; hence there is little radiation under those circumstances.

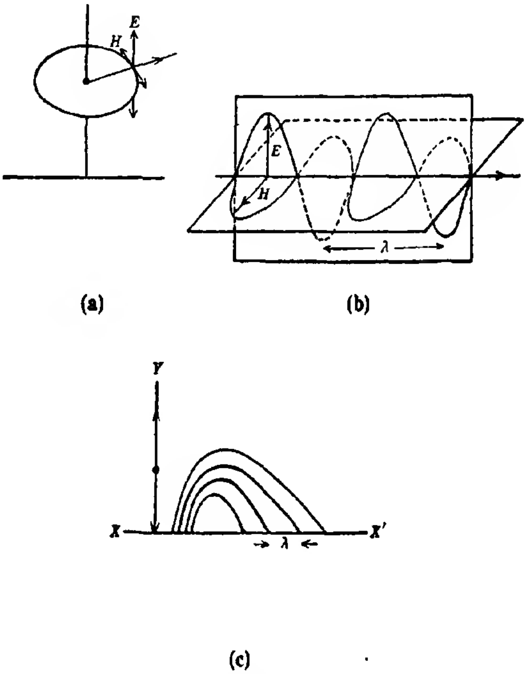


FIG. 222

But this is only a part of the story. As a line of electric flux sweeps through space following the electron it gives rise to a magnetic field and gives over to this created field half its own energy. This generated magnetic field, in accordance with the principles laid down at the beginning of this section, will be at right angles to the electric field and to the direction of its motion. The vector relations corresponding to both of these fields at a given point in the field is indicated in Fig. 222a where E represents the electric component of the field and H the corresponding magnetic element.

This created magnetic field is in time phase with the electric field but in space quadrature as shown in (*b*). As with the electric field, the magnetic field cannot return all of its energy to the source between reversals of the electron and so it continues to travel outward from the source as does the concomitant electric pulse. Indeed these two fields are not to be thought of as separate and distinct but rather as two aspects of the same disturbance. If then we provide means whereby our electron will continue to oscillate, that is, continue to supply energy to the moving charge, it is evident that we may cause energy to be radiated into space in the form of a periodic ether disturbance known as electromagnetic radiation. A vertical section of a detached group of the electrostatic pulses is shown in (*c*). If one were to view the plane XX' from the point Y the magnetic aspect of the field would consist of a series of expanding concentric circles. The wavelength (distance between two points in like phase) is indicated in both (*b*) and (*c*).

166. Radiation from, and Absorption by, Antennas.—From the preceding discussion it will be apparent that electromagnetic radiation over the earth's surface may be brought about by maintaining an alternating current in a conductor (called an antenna) placed normal to the earth's surface. Alternating currents of frequencies between 30 and 60,000 kilocycles have been found to produce effective radiation and it seems probable that even higher frequencies may come into commercial use. Under these conditions lines of electrostatic force will connect the oscillating electrons with corresponding positive charges in the superficial layers of the earth's surface. The movement of the charges in the antenna will therefore give rise to a concomitant to and fro movement of the charges of opposite sign in the earth; this, in fact, amounts to a displacement current in the earth's surface. At these high frequencies the electronic displacements occur only in the most superficial layers of the conductor and since much of the earth's surface is a relatively poor conductor the advancing waves will suffer more or less attenuation due to earth resistance; hence the amplitude of the electromagnetic disturbance will diminish with distance from the source.* The electric and mag-

* The exact relation which exists between the energy represented by the advancing wave and the distance from the source is still a matter of debate. For a discussion of this point see any standard text on radio communication, such as the excellent treatise by Professor J. H. Morecroft.

netic flux constituting the wave travels in all horizontal directions from the radiating antenna, except as noted later (Sec. 169), and it probably extends to heights measured in miles above the earth's surface. In fact there is some evidence for believing that owing to the presence of an ionized (Sec. 139) layer of gases in the upper regions of the atmosphere * some of the electromagnetic flux is reflected back to the earth and thus recombines with the original advancing wave in such a way as to produce interference effects. This point is however still a matter of discussion.

As the displacement current with its attendant alternating electromagnetic field spreads out over the earth's surface any vertical conductor (a receiving antenna) having the proper natural electrical period will have established in it an alternating E.M.F. and a corresponding current. (See Sec. 101.) This means an absorption of a certain amount of energy from the incident wave disturbance. This absorbed energy in the form of a high frequency alternating current may be caused, by the use of suitable apparatus (Sec. 145), to become manifest to the senses, and thus utilized as a means of communication. Since the amount of energy which can be abstracted from a passing electromagnetic wave depends chiefly on the application of the fundamental principle of electrical resonance (Sec. 118) to the receiving circuit provision is made in practice for the adjustment of the capacitive and inductive reactance of the antenna system, as shown in Fig. 195*b*. In dealing with either a transmitting or receiving antenna system it should be borne in mind that the antenna, regardless of its particular form, constitutes in effect a condenser, the conductors composing the antenna system forming one electrode of the condenser and the earth the other. Any horizontal portions added to the vertical part are inserted chiefly in order to afford added capacitance to the system. The methods by which a high frequency alternating current is established and maintained in a transmitting antenna system are briefly described in the following section.

167. Radiotelegraphy.—The scope of this volume will not admit of even a brief résumé of those types of equipment which are already obsolete or are rapidly becoming so. Our discussion

* See "Further Studies of the Kennelly-Heaviside Layer by the Echo-Method" (and a previous paper), by L. R. Hafstad and M. A. Tuve, *Proc. I. R. E.*, Sept. 1929, V. 17, No. 9.

will therefore be confined to a description of the essential features of equipment which is representative of that in general use at the time of writing.

In radio communication the tendency at the present time is toward the use of high power water-cooled tubes as generators of high frequency alternating current (Sec. 146) and also as high frequency power amplifiers. Until quite recently large high frequency electromechanical generators or arc converters were utilized for transoceanic radiotelegraphy; but equipment of this type is being rapidly replaced by tube installations.

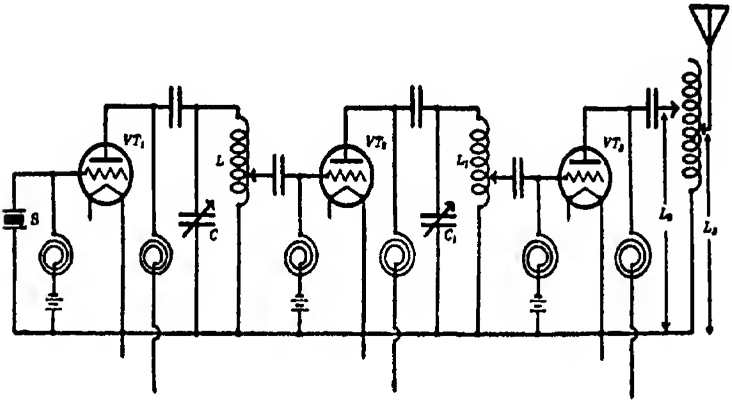


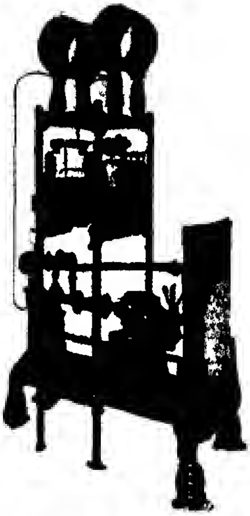
FIG. 223

A schematic diagram of the essential elements of a typical high power tube radiotelegraphic transmitter is shown in Fig. 223. This consists of what is known as a "master oscillator," the frequency of which is accurately controlled by a quartz crystal. The thermionic tube VT_1 and its associated circuit CL serves to generate an alternating current as described in Chapter XXII. This particular circuit differs from the one discussed in Sec. 144 in that the grid circuit consists of a thin plate of quartz S loosely held between two metal plates, instead of the usual inductance coil. Due to the piezoelectric property (Sec. 75) of quartz a plate of this material ground to a certain thickness and incorporated in an audion circuit as indicated above will permit the circuit organization to generate oscillations *of only one frequency*, thus acting as an automatic stabilizing device.

The output of the master oscillator, or "driver circuit" as it is sometimes called, is fed to the grid of a second or "intermediate"

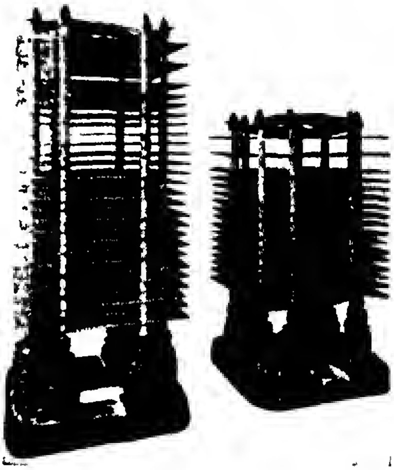
high frequency amplifier circuit (Sec. 143) the tube of which is designated as VT_2.

The output of the first amplifier circuit is in turn utilized to excite the grid of a second high frequency amplifier organization the tube of which is shown as VT_3. This latter amplifier stage is coupled directly (as shown) or inductively to the radiating (antenna) circuit. Each succeeding tube has a larger energy capacity than the one preceding; sometimes several audions are operated in parallel, thus giving a pyramidal arrangement of the



(Courtesy I R E)

FIG. 224.—20-KW. WATER-COOLED TUBE UNIT



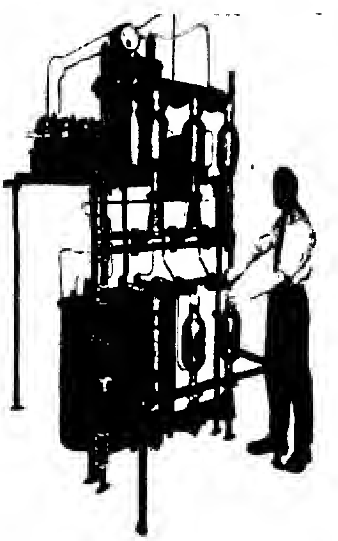
(Courtesy I R E)

FIG 225—HIGH VOLTAGE AIR CONDENSERS

tube units and thereby affording a relatively high power output. In some instances the complete transmitter circuit may contain more than two high frequency amplifier stages. In any event the constants of the anode circuit of each amplifier tube are adjusted so that that circuit is in electrical resonance (case of parallel resonance; see Sec. 120) with the frequency being delivered by the master oscillator, and thereby producing the maximum output from the amplifier organization as a whole. In practice various other adjustments are necessary but these do not affect the fundamental principles already outlined.

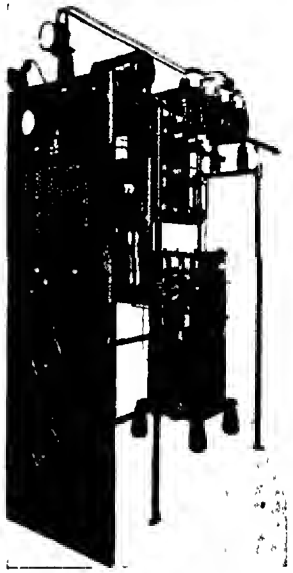
In order to carry on communication some means must be employed whereby the high frequency alternating current (carrier

wave) supplied to the antenna may be "modulated," i.e., modified by the signals which are to be transmitted. In radiotelegraphy this is usually accomplished by causing a telegraphic key to modify or completely interrupt the flow of energy from one amplifier stage to the next. Figures 224 to 227 inclusive show the component units of a typical modern radiotelegraphic set.



(Courtesy I. R. E.)

FIG. 226.—HIGH POTENTIAL 20-KW. THERMIONIC RECTIFIER UNIT



(Courtesy I. R. E.)

FIG. 227.—MASTER OSCILLATOR USED TO EXCITE 20-KW. TRANSMITTER

168. Radiotelephony.—The high frequency power circuits employed in radiotelephony do not differ essentially from those already described in connection with radiotelegraphy. The chief difference lies in the method by which the output of the organization is controlled or modulated. In order to more clearly understand the electrical process whereby voice-generated potentials are caused to control the output of a high frequency tube generator and its associated high frequency amplifier system reference may first be had to Fig. 228. Sound waves striking the microphone give rise to an alternating E.M.F. of complex wave form and small magnitude. This voice-generated E.M.F. is amplified by means of one or more audions and then passed either to the oscillator unit or to one of the high frequency power amplifier

stages as shown, the latter procedure being the better practice in the case of transmitters of 1000 watts and over.

The detailed manner in which the above outlined general plan of modulation is affected differs somewhat in different installations but the scheme most commonly employed is that originally

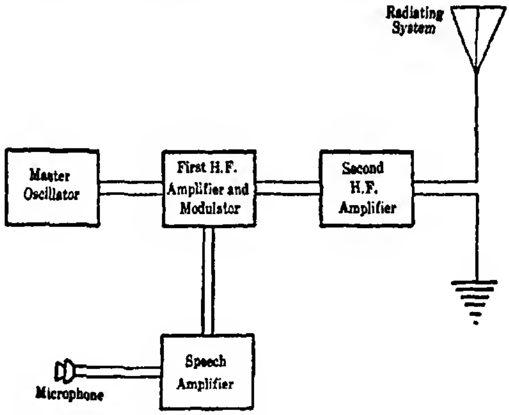


Fig. 228

devised by R. A. Heising, and frequently referred to as the "constant current" system. The fundamental principles involved in the Heising system will be understood by reference to Fig. 229.

From one point of view a three-electrode tube may be thought of as a variable resistance, the grid acting as the control member. Bearing this in mind we may represent two audions connected in parallel by two resistances R_1 and R_2, these two resistances being connected to a common source of electromotive force E. In series with this source of E.M.F. is a coil having a self-inductance value of such magnitude that its inductive reactance is practically infinite for those frequencies commonly encountered in music.

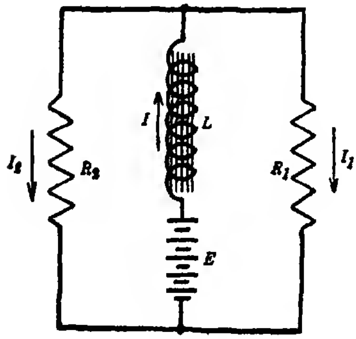


Fig. 229

This inductance will therefore serve to prevent any rapid changes in the value of the *total current* passing through it. With R_1 and R_2 each of fixed value the current I will divide into I_1 and I_2 as

determined by the individual resistances. If R_2 is suddenly decreased in value I_2 will increase accordingly, and *this change will cause a decrease in I_1 due to the fact that I cannot change in value,* for the reason above cited. Hence if R_2 be caused to vary rapidly in value the current through R_1 will change in a corresponding manner. The resistance R_1 may be a high frequency power amplifier tube and R_2 a so-called modulator tube, *the anode circuits of both audions being supplied by a common source of E.M.F. through the "choke coil" L.* If then R_2 is caused to change in value at voice or musical frequencies the energy supplied to the amplifier (R_1) will be modified accordingly and its output will thus be "modulated" in conformity with the changes impressed on the grid of the modulator tube. The practical manner in which this process may be carried out is indicated in Fig. 230,

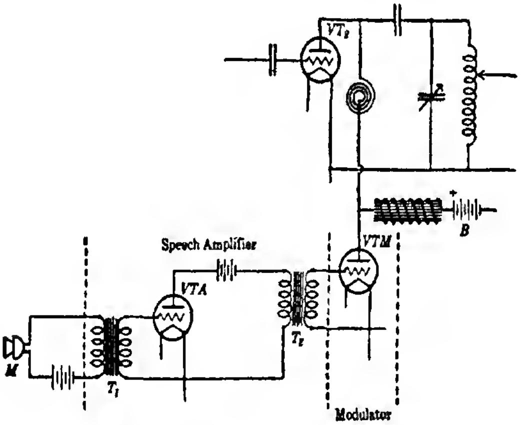


FIG. 230

where VT_2 corresponds to the first high frequency amplifier stage in Fig. 223. The alternating E.M.F. produced by sound waves incident on the microphone M is amplified by the tube VTA, and the output of this "audio" or "speech amplifier" is impressed on the grid of the modulator tube VTM, thus varying its internal impedance. The result on the output of the high frequency amplifier unit is then as outlined above. When the master oscillator is in operation and no modulation of the amplifier output is being effected the alternating current in the antenna and the so-called "carrier wave" being radiated from the antenna have the general form as shown in the first part of (*b*), Fig.

231. When, however, modulation is taking place the antenna current and the resulting radiated wave have the form indicated in the remainder of (*b*); in other words, the high frequency carrier wave is being modulated at audio frequency. This modulated electromagnetic wave will establish in any suitably "tuned" receiving antenna an alternating current having

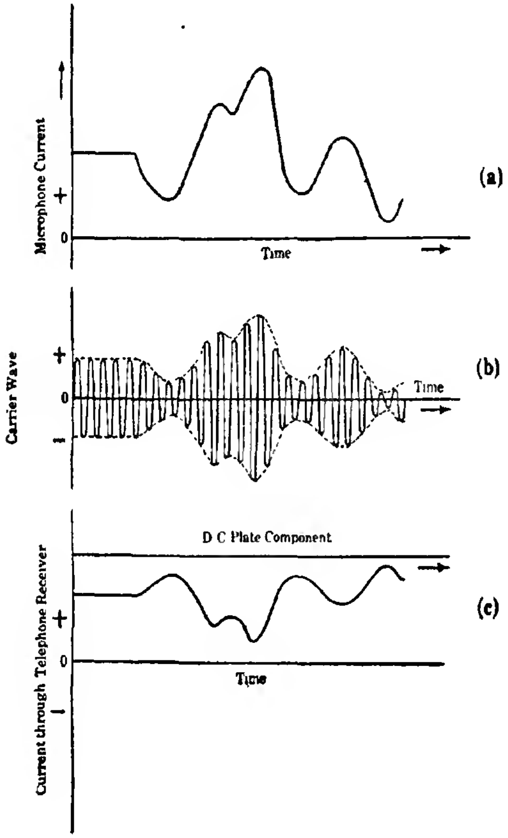
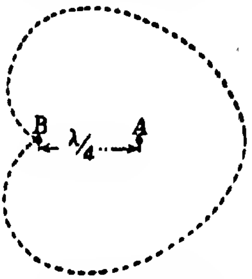


Fig. 231

a like wave form. As a result of the action of a suitably connected detector tube (Sec. 145) a varying current may be secured which will have a wave form (Fig. 231*c*) corresponding to the envelope (dotted in the figure) of the modulated wave. This rectified and integrated current from the detector may be utilized for the purpose of operating telephone receivers or a loud speaker, and thus voice communication may be established.

169. Beam Radio Communication.—When employing an antenna system of the ordinary type, the energy is radiated to a greater or less extent in all directions; hence a comparatively small part of the total energy reaches any given receiving point. It has recently been found possible to work out a system whereby a relatively large percentage of the total energy emanating from a transmitting plant can be concentrated into a beam and thus the radiation directed toward a definite point and the overall efficiency materially increased. A brief résumé, based on a paper by B. H. White,* of the theory of "beam" radio transmission follows.*

Imagine two simple vertical antennas (Fig. 232) located one fourth wave length apart and that we are looking at these aerials from above. Further, assume that energy is supplied to antenna A, and that B is tuned to the same frequency as A but receives no energy except by radiation from A. B will be set into oscillation as a result of the incident energy received from A and will therefore reradiate a part of this energy. The energy reaching some distant receiving point to the right of A will consist of two parts, energy coming directly from A and energy reradiated from B. A phase difference will however exist between these two wave trains. A wave in traversing the distance from A to B will change (lag) in phase by 90 degrees. In the process of generation in B there will be another loss of 90 degrees and in reradiation a third change of 90 degrees will take place. In addition to these three phase shifts there will be a loss of 90 degrees because of the time consumed by the wave in passing back from B to A. We therefore have a total phase change of 360 degrees, which means that the reradiated wave will be *in phase* with the wave which originally leaves A. The reflected waves will therefore serve to *augment* the energy being radiated directly from A and the field strength at any given receiving point to the right of A will thus be increased.



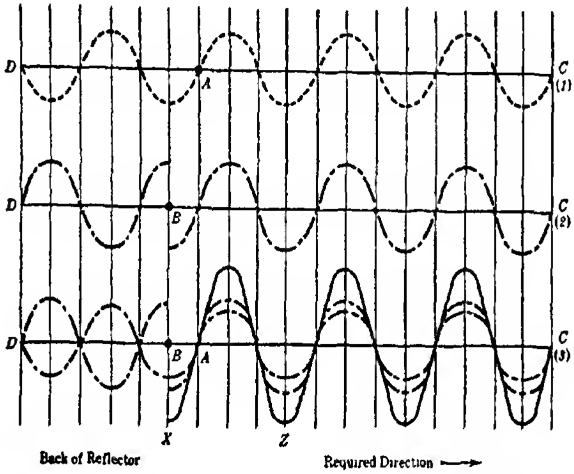
(Courtesy *The Electrician*)

FIG. 232

If we examine the phase relations *behind* the reflecting antenna

* *The Electrician*, April 3, 1925. Figures 232, 233, 234 are reproduced from Mr. White's paper.

wire, that is, to the left of B, we find that the energy radiated directly from A and that reradiated from B arrive at any given point 180 degrees *out of phase*, and hence the direct waves and the reradiated wave trains tend to nullify one another. This is due to the fact that the reradiation from B is 270 degrees out of phase with the wave emerging from A, and that the wave in passing from A to B loses 90 degrees in phase and will therefore differ by 180 degrees from the wave leaving B. Figure 233 shows diagrammatically the phase relations above referred to, and should be carefully studied.



(Courtesy *The Electrician*)

FIG. 233

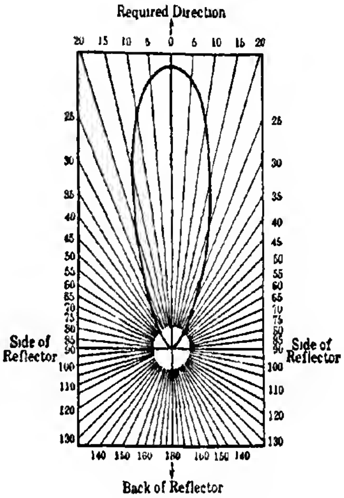
If the combined effects of a single antenna wire and one reflecting wire in directions other than those already discussed be examined and plotted, it will be found that the polar curve will take the well-known form shown in Fig. 232. If however a number of reflector wires are placed behind the antenna proper, the energy will be concentrated within a much smaller area as shown in Fig. 234.

It will be apparent that if we concentrate the energy which would ordinarily be radiated throughout 360 degrees within a beam whose limits are, say, 36 degrees, the energy required to produce a given field strength at a certain receiving point would be but one tenth that necessary without the reflecting system.

But the gain in efficiency does not end here. A reflecting

system may also be utilized behind the receiving antenna. If the gain at the receiving end also be on the basis of 1 to 10, the overall efficiency of the system would be increased as the square, or one hundred fold. Experience has shown such an increase in efficiency to be entirely practicable.

In October 1926 a two-way beam channel between England and Canada was put into commercial operation, utilizing a wave length of slightly over 26 meters. This channel is arranged for duplex operation at a speed in excess of 100 words per minute. With this new system a 20-kw. transmitting plant will produce the same strength of signal at the receiving station as a 200-kw. installation employing the older nondirectional radiating system. Since the Canada-England circuit was installed several other transoceanic beam channels have been established.



(Courtesy *The Electrician*)

FIG. 234

In the first beam stations established parabolic reflecting grids were employed, but in more recent installations a plain grid is used. The antenna system consists of a number of antenna conductors supported in a vertical plane, each elemental aerial being energized by a common transmitting unit. One fourth of a wave length behind this composite antenna is placed the plane vertical reflecting grid, both the antenna conductors and the reflecting wires being attached to cross arms supported by common masts. The antenna conductors commonly have a physical length equal to some fraction of a wave-length. Directive antenna arrays are widely used in short-wave transmission.

Not only is substantially less power required for the beam system than with a nondirectional arrangement, but atmospheric disturbances are very much less pronounced at the higher frequencies (shorter waves). Further, the frequency band from 3000 to 30,000 kilocycles is much less crowded than is the case at lower frequencies. The beam system is utilized for telephony

as well as telegraphy. Recent results attained make it appear probable that short wave beam transmission for both voice and code will be extensively developed in the near future.

170. Guided-Wave (Carrier Current) Communication.—In 1910 Major General Squier (then Major) of the U. S. Army by a bold and ingenious adaptation of the fundamental principles and apparatus previously employed in radio transmission developed a new and highly important system of communication. Prior to Gen. Squier's invention it had of course been possible to simultaneously transmit several telegraph messages over a single circuit and while certain earlier attempts had been made to develop a system of syntonic multiplex telegraphy, it is to Gen. Squier that the world owes the production of a multiplex system of communication by which it is possible to carry on as many as ten or more simultaneous two-way telephone conversations over one electrical circuit. In other words, the possible traffic-carrying capacity of a simple metallic telephone or telegraph circuit has been increased several fold. How this is accomplished will be evident from the following description of the essentials of the Squier system.

The transmitting and receiving equipment utilized in the guided-wave * system is essentially that employed in ordinary radio communication. However, instead of being connected to a radiating or absorbing system consisting of an antenna and the

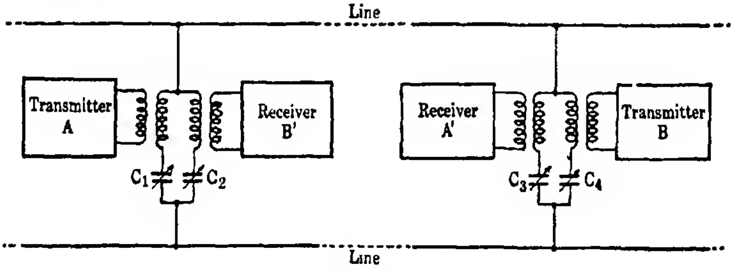


Fig. 235

ground, the transmitter and receiver are bridged across a physical telephone or telegraph pair or between a single wire and the ground. By this arrangement the energy is guided along the physical circuit in the form of a high frequency alternating cur-

* Gen. Squier prefers to designate this system of communication by the term "Wired Radio." This method of communication is frequently referred to by the term "carrier current" system.

rent instead of being radiated into space. This is shown diagrammatically in Fig. 235. The equipment there outlined would be required for a single two-way conversation. Transmitter A would be adjusted to deliver a modulated carrier wave to the line, the frequency of the carrier current being, say, 60 kcy. By means of the variable capacitance C_1 the line is then brought into resonance with A. Transmitter B would be set for some other noninterfering frequency, for instance, 40 kcy. The receiv-

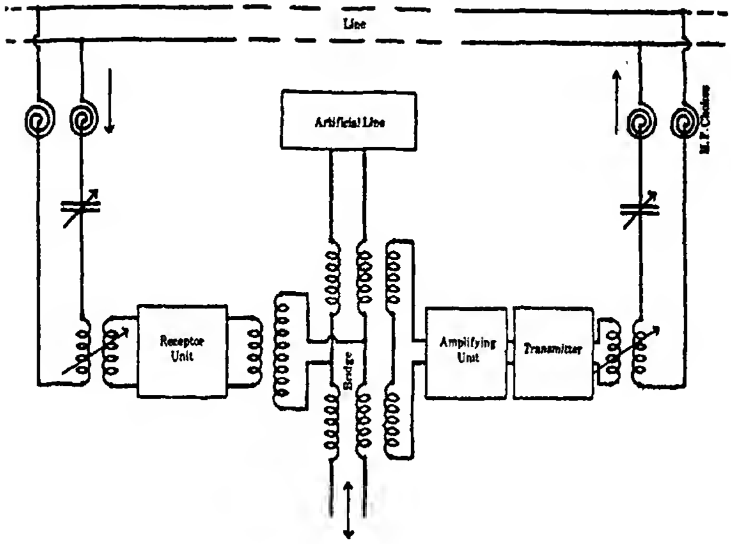
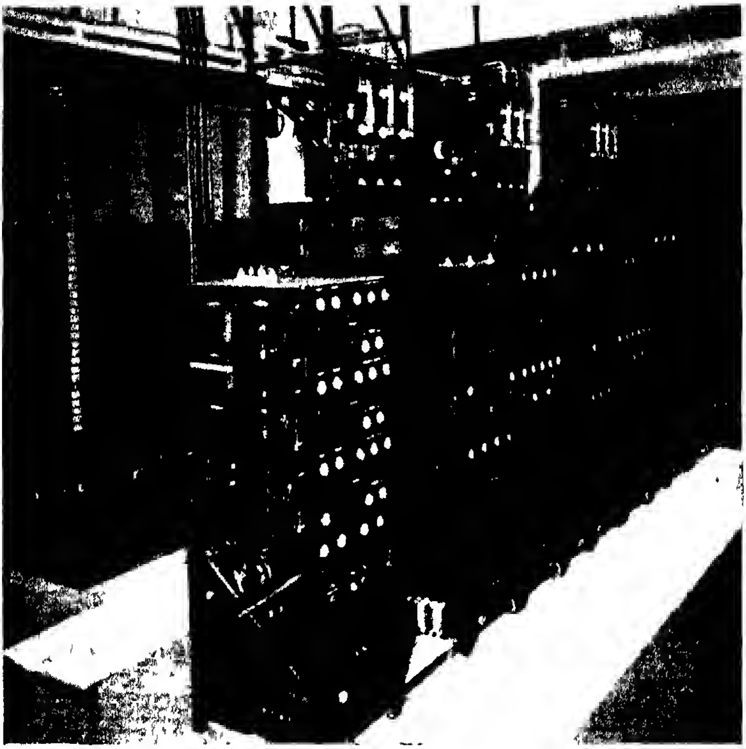


FIG. 236

ing organization A' would be adjusted to resonance with transmitter A and receiver B' to transmitter B. Each party to the conversation would thus utilize a given carrier wave. At the same time this superchannel is in operation communication may also be maintained by ordinary telephonic or telegraphic means. Additional high frequency channels may be established over the same physical pair, a definite pair of frequencies being assigned to each two-way channel.

In practice it becomes necessary to provide means whereby interaction between adjacent apparatus and also between interconnected circuits may be eliminated. In Fig. 236 is shown how reaction between interconnected circuits is avoided. The "bridge" circuit (sometimes called a hybrid coil) should be carefully noted. By means of this arrangement of circuits the energy

from the receptor is prevented from feeding into the transmitter and thus giving rise to regeneration and the consequent generation of audiofrequency oscillations ("howling"). The artificial line serves to balance the constants of the local physical telephone circuit. When a multiplicity of two-way superchannels is set up, the high frequency chokes are replaced by band filters (Sec. 121). A set-up of circuits similar to that shown in the figure is



(Courtesy Wired Radio, Inc.)

FIG. 237.—GUIDED-WAVE TELEPHONE EQUIPMENT FOR USE IN BROADCASTING OVER ELECTRIC LIGHT AND POWER LINES

required for each two-way high frequency telephonic or telegraphic channel. From the nature of the case the cost of the equipment involved is relatively high and hence it is not economical to employ a multiplex system of this character except in the case of long distance transmission. However, for distances measured in hundreds of miles the economic gain warrants its use. In fact, guided-wave multiplex telephony and telegraphy are in extensive commercial use both in this country and abroad.

The guided-wave system is employed to some extent as a means of communication over power transmission lines. When utilized for this purpose the transmitting and receiving circuits are electrostatically coupled to the power wires. Coupling is effected in one of two ways. In some installations one or more wires several hundred feet in length are supported parallel to and slightly below the power wires. In other plants the carrier current equipment is coupled to the power wire system through special high potential condensers.

Recently the guided-wave method of transmission has come to be applied to the broadcasting of speech and music over power lines leading to residential centers. Atmospheric electrical disturbances, as well as interstation interference, are thus largely avoided. Figure 237 is an illustration of a three program three phase wired radio telephone transmitter for use in broadcasting over electric light and power lines.

171. Telephone Repeaters.—A discussion of the applications of the audion in the art of communication would be incomplete without a reference to the amplifier-repeater circuits which are in extensive use on long distance telephone lines. In attempting to establish telephonic communication over a circuit several hundred miles in length the attenuation becomes so great that the energy reaching the receiving end is so small that intelligible conversation will not obtain. It therefore becomes necessary to provide some means whereby energy may be supplied to the line at some point or points on the line between the speaker and the listener. Thus far in our discussion of amplification we have had to do only with a one-way or unilateral repetition, but in a telephone circuit it is obviously necessary to provide a two-way arrangement, and this without employing two circuits (four wires). A careful study of Fig. 238 will disclose how this is accomplished. It will be seen that two tubes are used in connection with two bridges. Each of the lines is carefully balanced by an artificial line or network. As a result of this the output transformer T_1, for instance, will not give rise to a potential difference between the points x and x'. Thus the outgoing energy will not feed back into the first tube and cause regeneration with the resultant "howling." The balancing network absorbs half of the energy delivered by the tube, thus reducing the amount which would otherwise be delivered to the line. This

loss can however be provided for, if necessary, by operating several tubes in parallel. Tubes which are capable of delivering a few watts are commonly used for such purposes, and the amplifier-repeaters are installed every few hundred miles on long circuits. Transcontinental telephony would not be possible without the use of such amplifier organizations.

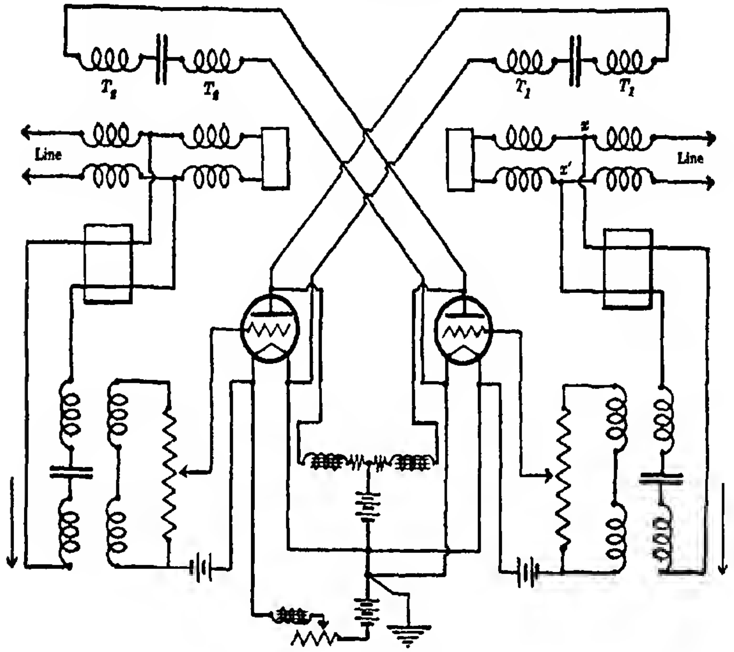


FIG. 238

172. Transmission of Pictures.—One of the most recent successful applications of the principles discussed in previous chapters is the transmission of pictures by wire and by radiotelegraphy. It is now possible to transmit photographs, including facsimiles of checks, drafts, and other similar documents, between points separated by thousands of miles. Figure 239 is a reproduction of a photograph transmitted by this means. Several forms of equipment have been worked out for accomplishing this type of transmission, but the basic units in each case are the photoelectric cell, described in Sec. 141, and the audion used as an amplifier (Sec. 145). In Fig. 240 are shown the essential elements employed in a typical system for the transmission of pictures.

The transmitting equipment consists of a motor-driven cylinder

C about which is wrapped the photograph, printed matter or other copy to be telegraphed. The cylinder is also caused to advance longitudinally as it revolves. Light from a suitable

(Courtesy Underwood and Underwood)

FIG 239.—PHOTOGRAPH TRANSMITTED BY TELEGRAPH

source S is focussed by means of the lens L to a small point on the revolving picture. The light reflected from the copy is collected by the parabolic mirror M and directed by the plane mirror M'

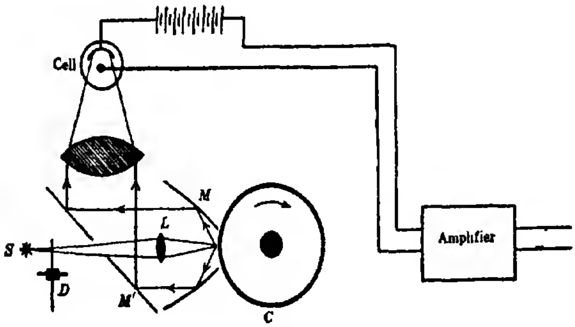


FIG. 240

into the photoelectric cell. As the pencil of light explores the surface of the picture a variable amount of light will be reflected, the quantity of reflected energy depending upon the degree of

coloring of the copy. The variations in intensity of the reflected beam of light thus incident on the sensitive surface of the photoelectric cell result in a variation in the photoelectric current. Since the output of the light-sensitive cell is very small it is necessary to increase this energy by means of an audion amplifier consisting of several stages. The output of this amplifier is fed to the grid of the modulator tube (Sec. 168) which controls the output of an audion high frequency generator. If the picture is to be transmitted over a physical channel the high frequency carrier current is impressed on the two wires composing the circuit, as outlined in Sec. 170. If the transmission is to be carried out by radiotelegraphy the audion generator is arranged to supply energy to an antenna system. In fact any radiotelephone transmitter may be utilized for picture transmission, the picture amplifier being substituted for the speech input amplifier, as shown in Fig. 230.

One additional point in regard to the picture transmitter should be noted. If and when the scanning light pencil is passing over a surface which is uniformly black or white *there will be no variation in the energy output of the photoelectric cell;* hence amplification will not occur. In order to make the potential supplied to the amplifier alternating or variable in character, under all circumstances, a rotating disk D having a row of apertures near its periphery is positioned between the light source and the lens system. This "chopper," as it is called, serves to interrupt the beam of light at a high frequency (of the order of 6000 times per sec.), thus causing the output of the light cell to be of an intermittent character. The variations in the light incident on the cell act to vary the amplitude of these regular variations imposed by the chopper.

The reception of the picture is effected in a comparatively simple manner as indicated in Fig. 241. At the receiving terminal of the channel the picture-modulated carrier wave is passed into a standard receiving amplifier (to compensate for the attenuation which occurs during transmission) and thence to a special one-stage amplifier in the plate circuit of which is a glow-tube. This glow-tube is a simple two-electrode unit containing a gas such as neon or helium at low pressure. A current will pass through such a tube when an E.M.F. of from 200 to 400 volts is applied to its terminals, the gas being rendered incandescent by the passage of

the current. The intensity of the light emitted by such a unit is proportional to the strength of the current through the tube. Variations in the voltage impressed on the grid of the amplifier tube A due to the picture signals thus give rise to corresponding variations in the intensity of the light emitted by the glow lamp. This variable light is focussed by means of a suitable optical system on a piece of photographic paper which is wrapped about a revolving cylinder C; thus the gradations in light and shade of the original copy are impressed on the light-sensitive receiving sheet. The photographic paper later undergoes chemical development in the usual manner.

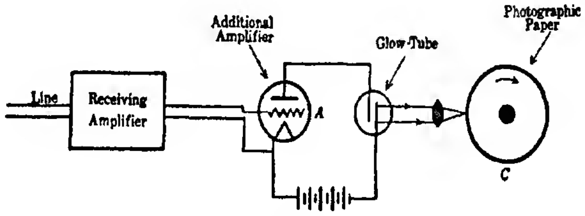


Fig. 241

One of the difficult problems in connection with facsimile picture transmission which had to be solved was that of synchronization. Obviously the cylinder at the receiving end must revolve at exactly the same speed as that of the transmitter; otherwise distortion will occur. While this aspect of the case is too complicated for discussion here, it may be said in passing that it is now possible to satisfactorily synchronize the two cylinders by several different methods. One method frequently employed depends upon the use of electrically driven tuning forks in connection with both the transmitting and receiving equipment.

Facsimile picture transmission has been developed to the point where a 5-inch by 8-inch picture or other similar copy can be transmitted in about one minute. This is equivalent to a rate of 630 words per minute in the case of a printed or typed page. It seems probable that this form of transmission will in time more or less completely replace the present code method of telegraphy.

APPENDIX

APPENDIX

APPROXIMATE VALUES FOR ANNUAL RATES OF SECULAR CHANGE IN THE MAGNETIC ELEMENTS DECLINATION (*D*), INCLINATION (*I*), AND HORIZONTAL INTENSITY (*H*) FOR THE EPOCH 1925[1]

(Because of the different intervals covered by available data and the known large accelerations in some parts, the values given for the annual secular changes at intersections of parallels and meridians indicated are approximate except for those localities near magnetic observatories; in some cases there is

Latitude	Longitude East of Greenwich								
	0°	20°	40°	60°	80°	100°	120°	140°	160°
	Annual Change in Declination (D)								
°	′	′	′	′	′	′	′	′	′
60 N	+13	+12	+ 7	0	− 3	− 4	− 6	− 6	− 4
40 N	+10	+10	+ 7	0	− 3	− 4	− 3	− 2	− 2
20 N	+ 8	+ 8	+ 4	− 1	− 3	− 2	− 1	− 1	0
0	+ 6	+ 9	+ 8	− 6	− 5	− 2	0	+ 1	+ 1
20 S	+ 2	+10	+10	−11	−14	− 6	+ 1	+ 1	+ 2
40 S	0	+10	+ 9	− 4	−12	− 7	0	+ 1	+ 3
60 S	+ 2	+ 7	+ 6	+ 2	0	− 1	− 1	+ 1	+ 4
	Annual Change in Inclination (I)								
°	′	′	′	′	′	′	′	′	′
60 N	+ 1	+ 2	+ 3	+ 4	+ 3	+ 2	+ 2	+ 1	+ 1
40 N	− 2	+ 1	+ 4	+ 7	+ 4	+ 2	+ 1	0	0
20 N	− 6	− 1	+ 5	+ 8	+ 3	0	− 1	− 2	− 2
0	−12	− 4	+ 4	+ 6	+ 1	− 1	− 3	− 4	− 4
20 S	−11	− 7	− 1	+ 2	+ 1	− 1	− 2	− 4	− 4
40 S	− 9	− 8	− 5	− 2	+ 1	0	− 1	− 2	− 1
60 S	− 6	− 6	− 5	− 3	0	0	0	0	− 1
	Annual Change in Horizontal Intensity (H)								
°	γ	γ	γ	γ	γ	γ	γ	γ	γ
60 N	−10	−30	−45	−60	−50	−40	−25	0	+10
40 N	+ 5	−10	−25	−35	−35	−20	−10	0	0
20 N	+10	+15	0	+ 5	+35	+30	+15	+10	0
0	−25	−15	−10	0	+25	+25	+10	0	−10
20 S	−55	−70	−50	−40	−30	−20	−25	−25	−25
40 S	−95	−110	−95	−70	−50	−45	−35	−30	−30
60 S	(−70)	(−80)	(−80)	(−70)	(−60)	(−35)	(−30)	(−30)	−30

[1] Prepared by H. W. Fisk, of the Department of Terrestrial Magnetism, Carnegie Institution of Washington.

great uncertainty and the values for these are enclosed in parentheses. The signs of the values given are in the algebraic sense for extrapolation considering east declination, north inclination, and horizontal intensity as positive and west declination and south inclination as negative.)

Latitude	Longitude East of Greenwich								
	180°	200°	220°	240°	260°	280°	300°	320°	340°
Annual Change in Declination (D)									
°	′	′	′	′	′	′	′	′	′
60 N	− 3	0	0	− 2	− 3	0	+ 5	+ 9	+12
40 N	0	+ 1	0	− 1	− 2	− 4	− 2	+ 2	+ 8
20 N	0	+ 2	+ 3	+ 4	+ 4	+ 1	− 6	− 4	+ 2
0	+ 1	+ 2	+ 2	+ 2	+ 3	0	−12	−12	− 2
20 S	+ 3	+ 3	+ 4	+ 3	+ 2	− 3	−10	− 9	− 5
40 S	+ 5	+ 6	+ 6	+ 4	+ 2	− 3	− 8	− 7	− 4
60 S	+ 6	+ 7	+ 6	+ 4	+ 2	+ 2	− 4	− 4	− 2
Annual Change in Inclination (I)									
°	′	′	′	′	′	′	′	′	′
60 N	+ 1	0	0	0	0	− 1	− 2	− 2	0
40 N	− 1	− 1	0	0	+ 1	+ 1	0	− 5	− 4
20 N	− 1	+ 1	+ 2	+ 1	+ 3	+ 7	+ 3	− 8	−11
0	− 1	+ 1	+ 4	+ 2	+ 4	+ 9	+ 6	−10	−15
20 S	− 2	0	+ 4	+ 2	+ 3	+ 5	0	−11	−14
40 S	− 1	0	+ 2	+ 2	+ 2	+ 2	− 1	− 7	−10
60 S	0	(+ 1)	(+ 1)	(+ 1)	+ 1)	(+ 2)	+ 1	− 2	− 4
Annual Change in Horizontal Intensity (H)									
°	γ	γ	γ	γ	γ	γ	γ	γ	γ
60 N	+10	0	−10	− 5	0	+10	+20	+20	+ 5
40 N	−10	−15	−20	−25	−30	−40	−25	+ 5	+25
20 N	−20	−30	−35	−45	−60	−90	−60	0	+20
0	−10	−15	−25	−30	−35	−25	− 5	0	−15
20 S	−20	−15	−15	−15	−20	−20	−25	−30	−40
40 S	−25	−20	−25	−25	−30	−50	−85	−70	−70
60 S	−15	−20	−20	−20	−30	−45	−50	−60	−65

INDEX

Numbers refer to pages

SUPPLEMENTARY PROBLEMS

CHAPTER II

1. Two concentrated unlike charges of 25 e.s.u. each are immersed in petroleum oil and separated by a distance of 5 cm. What is the value of the mechanical force, in grams, tending to draw them together?

2. What will be the field strength in air at a point 50 cm. from a charge of 1000 e.s.u.? What mechanical force in grams would be experienced by a body carrying a charge of 0.01 coulomb if placed at the point designated?

3. Three like concentrated charges of 5, 7, and 10 e.s.u. respectively are so located that lines joining them form an equilateral triangle whose sides are 30 cm. If a body carrying a charge of 25 e.s.u. is placed at the center of the line joining the 5 and 7 unit charges, to what mechanical force, in grams, will it be subjected? Assume the medium to be air.

4. Two small spheres, each of mass m, are suspended by weightless threads from a common point, the thread being 10 cm. long. The spheres carry equal like charges of value q. It is found that the two bodies assume a position d cm. apart. Develop an expression, in terms of d, for the charge on one of the bodies.

5. What is the total normal induction over a closed surface surrounding a charge of 35 e.s.u.? Will the distance of the closed surface from the charge have any bearing on the answer?

6. What will be the magnitude of the mechanical force, in grams, acting on a body carrying a charge of 0.01 coulomb if located 25 cm. from a long cylindrical conductor which bears a charge of 0.001 coulomb per cm. of length?

7. What is the field intensity in the region of a surface of large area, the surface density of the charge being 25 e.s.u.?

8. What will be the value of the mechanical force acting on a uniformly charged surface whose area is 10 cm^2. when located in a uniform electrostatic field whose intensity is 10 e.s.u.?

CHAPTER III

1. What is the potential, in volts, at a point in air 15 cm. from a concentrated charge of 25 e.s.u.? What would be the potential in e.s.u. and volts at the same point if the surrounding medium were castor oil instead of air?

2. Three like concentrated charges of 5, 7, and 10 e.s.u. respectively are so located that lines joining them form an equilateral triangle whose sides are 30 cm. Find the potential at the center of the line joining the 5 and 7 unit charges, assuming air to be the dielectric.

3. Two points separated by 30 meters have a P.D. of 2 e.s.u. What is the potential gradient in volts per meter?

4. A body is found to have a potential of 6000 volts due to a charge of 5

coulombs. How many foot-pounds of work could be done if all of the energy represented by the charge could be utilized?

5. How much work in foot-pounds will be done in moving a charge of ½ coulomb between two points whose P.D. is 10,000 volts?

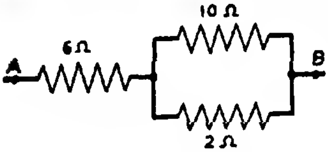
CHAPTER VI

1. What will be the resistance at 20° C of 1000 ft. of copper conductor, size No. 6 B. & S. (A.W.G.)?

2. What would be the resistance of the wire specified in Problem 1 if the temperature were 35° C?

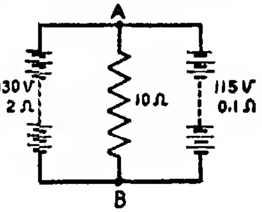
3. A current of 5 amperes flows through a resistance of 200 ohms. What will be the "drop" across the resistance?

4. A galvanometer coil has a resistance of 75 ohms. It is desired to arrange a shunt of such a resistance value that only 1/100 of the current to be measured shall pass through the galvanometer winding. What should be the resistance of the shunt?



5. Assuming the resistance values *shown in the figure, find the total resistance* between the points A and B.

6. It frequently happens in practice that two sources of E.M.F. of somewhat different magnitude are connected in parallel to a common electrical load, as shown in the accompanying diagram. Using the data given and employing Kirchkoff's law, find:



(a) the P.D. between the points A and B.

(b) the current through the load resistance.

(c) the current delivered by each source.

CHAPTER VIII

1. The resistance of a certain circuit is 50 ohms. The applied voltage is 120. What is the power consumed? What horse power does this represent?

2. What will it cost to operate a 5 HP motor for 8 hrs. daily for 30 days, the power rate being 3¢/kw.-hr.?

3. It is desired to develop 1000 B.T.U. during a period of 1 hr. If a 120-volt circuit is available, what should be the resistance of the conductor forming the heating element?

4. In a power system 1000 kw. is transmitted at 100 kilovolts pressure. The line is 50 miles long and consists of two copper conductors of No. 10 B. & S. What will be the line loss in heat units (B.T.U.)? If the service is maintained for 30 days, what will this line loss amount to at ½¢/kw.-hr.?

CHAPTER XI

1. In calibrating an ammeter by the electrochemical method, a deposit of 0.4 gram of silver was deposited from the electrolyte. If the current passed for 15 minutes, what was the value of the current?

2. If electrical energy costs 2¢/kw.-hr., what will it cost to plate out 100 lbs. of copper from a salt in which the valence of copper is 2? Assume that an E.M.F. of 5 volts is used. What would be the cost if a solution containing univalent copper were used?

CHAPTER XII

1. A bank of 55 lead storage cells connected in series supplies an electrical load having a resistance of 10 ohms. Compare the energy loss, in watts, in the external and internal parts of the circuit. Assume the E.M.F. per cell to be 2.2 volts and the internal resistance per cell to be 0.002 ohm. What would be the value of the current if the battery were short circuited?

2. Assuming a primary cell to be made up of iron and copper electrodes, the electrodes being immersed in normal solutions of their respective salts, compute, from the electrochemical energy relations, the E.M.F. which will be developed, and compare the result with that computed by means of the table given in Sec. 60, p. 124.

3. If, in determining the potential of a given electrode when immersed in its normal salt solution, a calomel electrode were used, and the potentiometer gave a reading of 0.71 volt, what would be the electrode potential of the material under test?

CHAPTER XIV

1. One pole of a magnet when placed in a known field whose strength is 10 gausses is found to be subjected to a force of 0.5 gram. What is the strength of the magnetic pole? How many lines of magnetic flux emanate from such a pole?

2. Assuming the distance between the two poles of the magnet indicated in Problem 1 to be 20 cm., what will be its magnetic moment?

3. If the cross section of the magnet discussed in the two preceding problems is 0.5×1 cm., what will be the intensity of magnetization?

CHAPTER XVI

1. What will be the value of the mechanical force experienced by a magnetic pole whose strength is 50 c.g.s. units if placed 2 cm. from a long wire in which a current of 5 amperes is flowing?

2. What will be the magnetic field intensity due to the current in a conductor in the form of a circular loop at a point on the axis of the loop 20 cm. from the center of the loop, the radius of the loop being 10 cm. and the current 5 amperes?

3. What will be the value of the mechanical force experienced by a conductor carrying a current at right angles to the direction of a uniform magnetic field whose intensity is 10 c.g.s. units? The conductor is 25 cm. in length and the current is 2 amperes.

4. Examine Eq. 109 and point out how the sensitivity of a galvanometer of the D'Arsonval type may be increased.

CHAPTER XVII

1. A conductor whose length is 20 cm. moves at right angles to a magnetic field, where the flux density is 6000 lines per cm., at a velocity of 1000 cm. per sec. What is the average value of the E.M.F. induced in the conductor in e.m.u.? In volts?

2. A rectangular coil of wire consists of 25 turns each of which measures 20×10 cm. The coil revolves at 1800 R.P.M. about an axis parallel to the long sides and in a magnetic field where the flux density is 5000 lines per cm. Compute the E.M.F. for every 15 degrees in one revolution and plot the values thus obtained against the angular position of the coil.

3. If the terminals of the revolving coil specified in Problem 2 be short circuited, what will be the maximum current that will flow in the winding? Assume the coil to have a resistance of 0.1 ohm.

4. A certain helix consists of 25 turns having a mean diameter of 16 cm. The length of the helix is 20 cm. Determine the coefficient of self-inductance, applying Nagaoka's correction in making the computation.

CHAPTER XVIII

1. The effective value of the difference in potential between two parts of an A.C. circuit, as shown by a voltmeter, is 220 volts. What is the maximum or "peak" voltage to which the insulation will be subjected?

2. The ammeter in an A.C. circuit reads 10 amperes. In any given cycle what are the maximum and minimum values?

3. The self-inductance of a choke coil is 30 henrys. What is its reactance at 60 cycles? At 1000 cycles?

4. The resistance of a certain series circuit is 12 ohms and the self-inductance is 1.2 henrys. If a 60-cycle alternating E.M.F. of 120 volts is applied to the circuit, what will be the value of the current?

5. What will be the angle of current lag in the case cited in Problem 4? What will be the power factor?

6. The capacitance of a condenser is 2 mf. What is its reactance at 60 cycles? At 1000 cycles?

7. The resistance of a series circuit is 5 ohms; its capacitance is 4 mf. If a 25-cycle alternating E.M.F. of 110 volts is applied in series, what will be the value of the current?

8. In the case cited in Problem 7, will the current lead or lag the E.M.F.? What will be the value of the phase angle?

9. In a series circuit the resistance is 5 ohms, the coefficient of self-inductance 2 henrys, and the capacitance 6 mf. What E.M.F., at 60 cycles, must be applied to the circuit in order to maintain an r.m.s. current of 10 amperes?

10. What value of capacitance must be connected in series with an inductance of 60 microhenrys in order that the circuit may be resonant at a frequency of 1250 kilocycles?

11. An inductance has a value of 20 henrys and carries a current of 0.5 ampere. What will be the P.D. developed between its terminals if the frequency of the current is 60 cycles? 1000 cycles?

12. A circuit which includes a series condenser carries a current of 10 amperes at a frequency of 850 kilocycles. The value of the capacitance is 0.01 mf. What P.D. is developed between the terminals of the condenser? What would the P.D. be if the frequency were 500 kilocycles? What would the P.D. be at the two frequencies mentioned if the capacitance were 0.001 mf.?

13. It is desired to determine the power factor in an inductive circuit. A wattmeter connected in the circuit reads 500 watts. An ammeter in the circuit shows a reading of 6 amperes, and the applied voltage is 115. Compute the power factor.